Universität Bremen
Zentrum für Technomathematik

Prof. Dr. Dr. h.c. Peter Maass

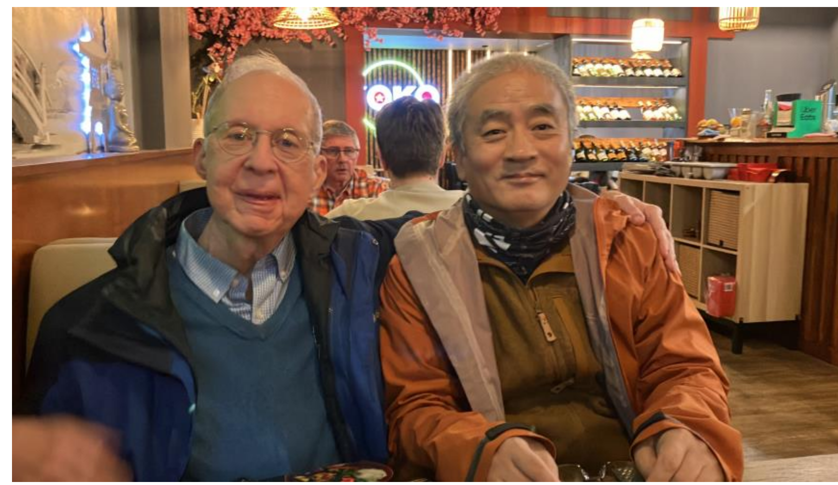
A Tribute to Todd Quinto's 'Shower' Theorem

Meira Iske, Ming Jiang, Peter Maass

- Todd Quinto
- A remark on learned reconstruction methods
- From Mathematics to Innovation

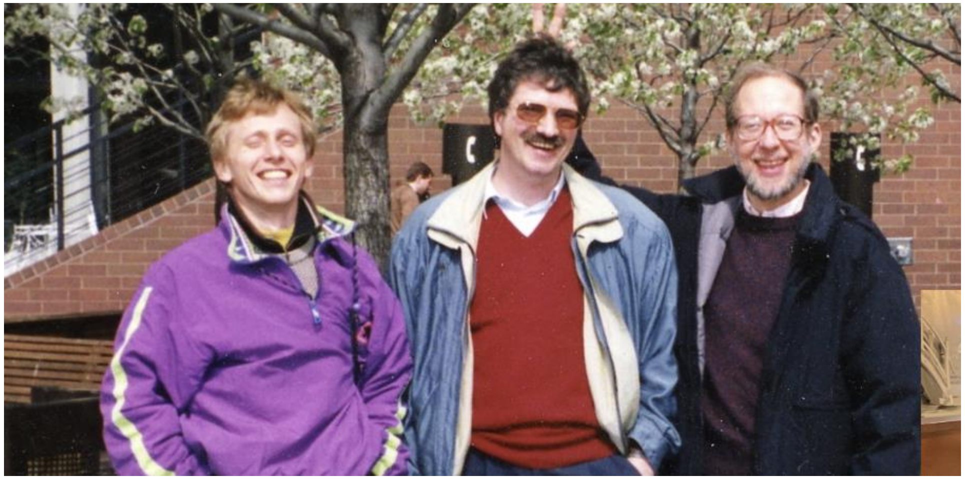
Todd75
ESI, Vienna, June 8 2026

Todd Quinto



Last year in Saarbrücken

Todd Quinto



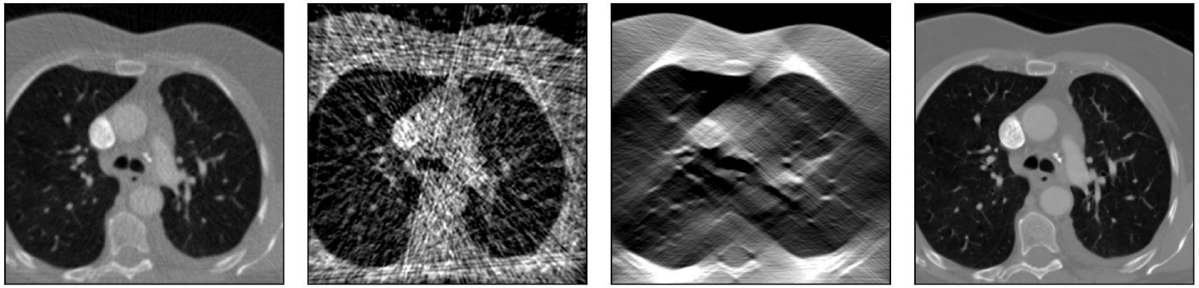
… a few weeks earlier in Boston

Last year in Saarbrücken

Micro local analysis, wavefront sets, support theorems, generalized Fourier integral operators, limited data problems,…

Distances between measures from 1-dimensional projections
MG Hahn, ET Quinto
Zeitschrift für Wahrscheinlichkeitstheorie und Verwandte Gebiete, 1985

Todd Quinto's ‚Shower' Theorem or Utilizing the Invisible



given data $g = \mathcal{R}_N f^+$

$g_i = \mathcal{R} f^+(s_i, \omega_i)$

$i = 1, .., N$

$$f^+ = f_{rec} + f_{cor}$$

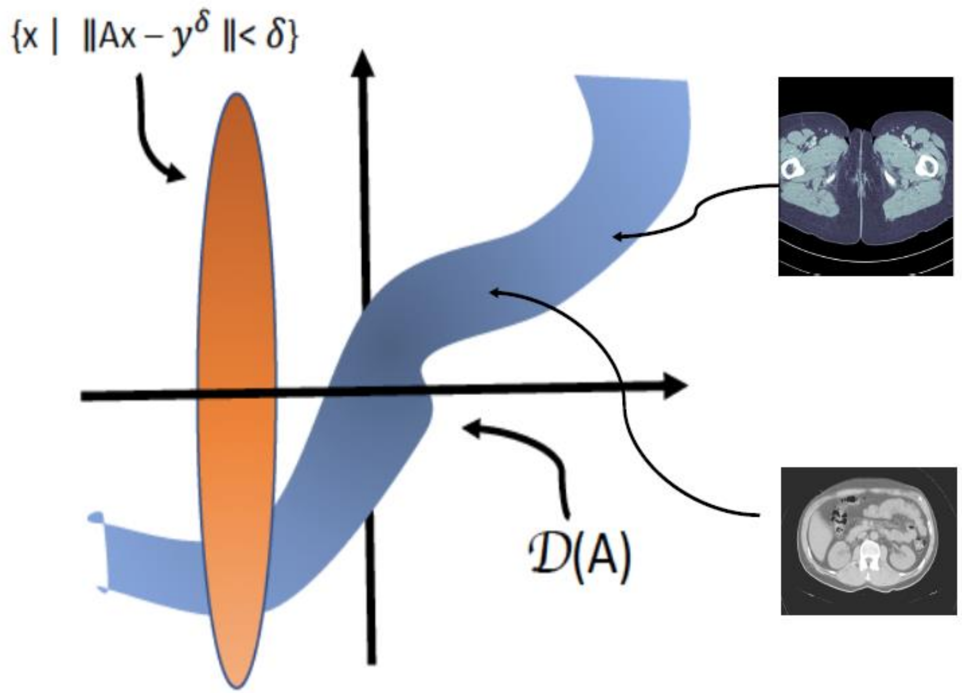
with $\mathcal{R}_N f^+ = \mathcal{R}_N f_{rec}$ hence $\mathcal{R}_N f_{cor} = 0$

$supp\ f^+ = \Omega$ or $f^+ = 0$ outside Ω $\Rightarrow f_{cor} = -f_{rec}$ on Ω^C

$$min\ \|f_{cor}\| \text{ subject to } f_{cor} \in \mathcal{N}\mathcal{R}_N\ ,\quad f_{cor} = -f_{rec} \text{ on } \Omega^C$$

Exploiting the ‚invisible‘ data manifold

$Ax \sim y^\delta$, "No model is perfect", "Not every matrix is an image"

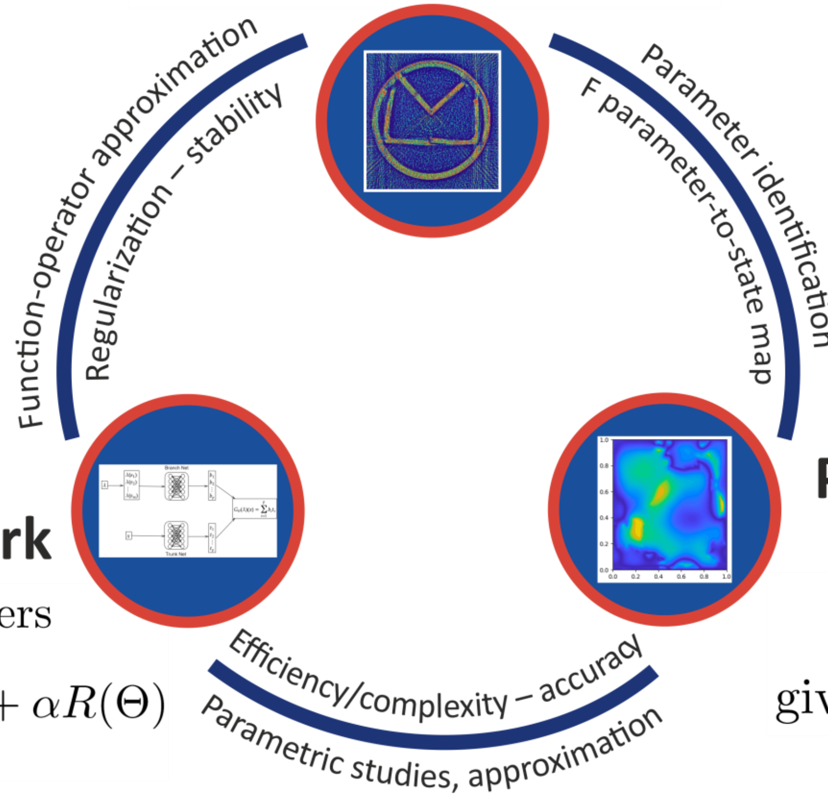


Mallat, Haltmeier, Adler, Öktem, Lunz, Schönlieb, Arridge, Hauptmann, Grasmaier, Harrach, Dittmer, Otero, ...

Ill-posed inverse problem

given $y^\delta = F(x) + \eta$

$$\min_x \|F(x) - y^\delta\|\|^2 + \alpha R(x)$$



Data driven neural network

Θ network parameters

$$\min_\Theta \sum_i \|\varphi_\Theta(x_i) - y_i\|^2 + \alpha R(\Theta)$$

Partial differential equation

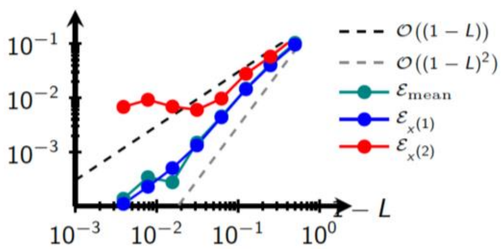
$$-\nabla(\lambda \nabla u) = f$$

given u^δ determine λ

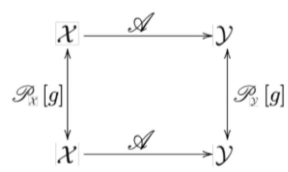
Regularization by architecture

Network architectures for inverse problems

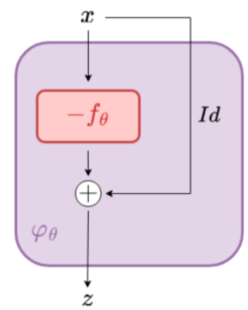
Operator approximation | **Equivariance** | **Invertible networks** | **Deep prior concepts**



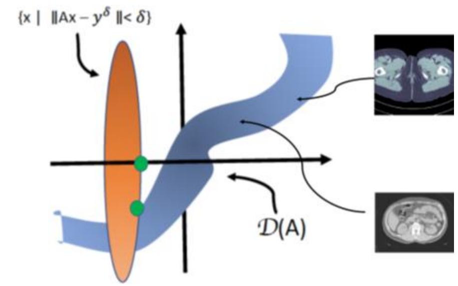
Janek Gödeke
quantifiable approx.
general activation



generalized CNN
group transform
stability



iResNet architecture
$y_{j+1} = y_j + \Phi_\Theta(y_j)$
Optimal convergence



$min\|A\Phi_\Theta(z) - y^\delta\|^2$
no training data
bi-level optimization

D. Nganyu Tanyu, J. Ning, T. Freudenberg, N. Heilenkötter, A. Rademacher, U. Iben, P. Maaß. Deep learning methods for partial differential equations and related parameter identification problems. Inverse Problems, 39(10), 2023.

C. Arndt, A. Denker, S. Dittmer, N. Heilenkötter, M. Iske, T. Kluth, P. Maaß, J. Nickel. Invertible residual networks in the context of regularization theory for linear inverse problems. Submitted for publication, 2023.

M. Beckmann, N. Heilenkötter. Equivariant Neural Networks for Indirect Measurements. Submitted for publication, 2023.

Learning with a false noise level

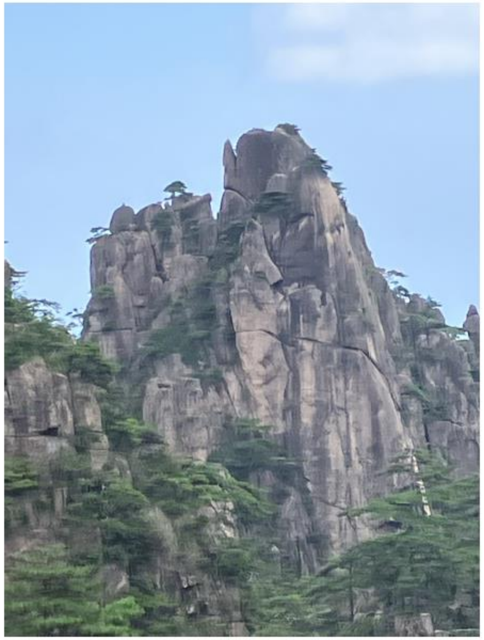
Sebastian Neumayer, ICSI 2024, Huangshan

„Train a network for denoising
and apply it to inverse problems "

„Training is done with a
fixed noise level $\bar{\delta}$ "

Denoising $A = I$ not ill-posed

Apply to data with $\delta \neq \bar{\delta}$?
Convergence $\delta \to 0$ or $\delta >> \bar{\delta}$



Learned *Regularization*[1]

Generalized LASSO objective

$$J_\alpha(x;y^\delta) = \|Ax - y^\delta\|^2 + \alpha(\delta)\|Wx\|_1$$

Approach

1. Universal training denoising task to determine W
Ingredients: Fixed $\bar{\delta}$ and dataset $(x_i^{\bar{\delta}}, x_i)_{i=1,\dots,N}$

2. Fine tuning determine regularization parameter $\alpha(\delta)$ (grid search)

3. Reconstruction compute $\mathrm{argmin}_x \; J_\alpha(x;y^\delta)$

Idea Learn penalty once, then suitable for generic reconstruction settings.

- noise level during training is fixed ⟶ reconstruction generalizes to other **noise levels**
- denoising task during training ⟶ reconstruction generalizes to other **operators**

[1] Goujon, Neumayer, Bohra, Ducotterd, Unser. *A neural-network-based convex regularizer for inverse problems*. In: IEEETransactions on Computational Imaging 9 (2023), pp. 781–795

Tikhonov reconstruction with noise level mismatch[2]

Worst-Case Errors

$$T_\alpha y^\delta = \mathrm{argmin}_x \ \frac{1}{2}\|Ax - y^\delta\|^2 + \frac{\alpha(\delta)}{2}\|x\|^2$$

- True noise level δ
- Optimal reg. parameter $\alpha := \alpha(\delta) = \frac{\delta}{C}$ depends on δ

What if we choose a mismatched $\alpha := \alpha(\bar{\delta}) = \frac{\bar{\delta}}{\mathrm{C}}$ for $\bar{\delta} \neq \delta$?

Error Mismatch

$$\|T_\alpha y^\delta - x^\dagger\| \leq \underbrace{\frac{\delta}{2\sqrt{\alpha}} + \sqrt{\alpha}\, C}_{=:\, \mathrm{wc}(\alpha,\delta)}$$

"worst-case error"

Relative Error Mismatch

Let $\delta = \lambda\bar{\delta}$ for $\lambda \gg 1$. Then

$$\mathrm{wc}(\alpha(\bar{\delta}),\delta) = \mathcal{O}(\lambda) \qquad \frac{\mathrm{wc}(\alpha(\bar{\delta}),\delta)}{\mathrm{wc}(\alpha(\delta),\delta)} = \mathcal{O}(\sqrt{\lambda})$$

⟶ Error order is less severe than expected for noise mismatch!

[2] Behrens, Iske, Jiang, Maass, Neumayer. A remark on an error analysis for classical and learned Tikhonov regularization schemes, arXiv:2604.00759 (2026).

Numerical Results

MNIST image

Setting[3],[4],[5]

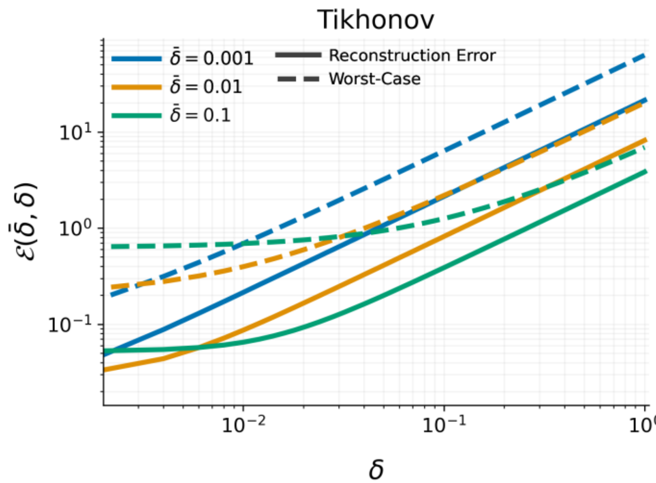
- $A \in \mathbb{R}^{28\cdot 28\times 30\cdot 41}$ discretized Radon operator
- $(x_i)_i \in \mathbb{R}^{28\cdot 28}$ vectorized MNIST images
- $y_i^{\bar{\delta}} = Ax_i + \eta_i$ with $\eta_i \sim \mathcal{N}(0, \bar{\delta}^2 I)$

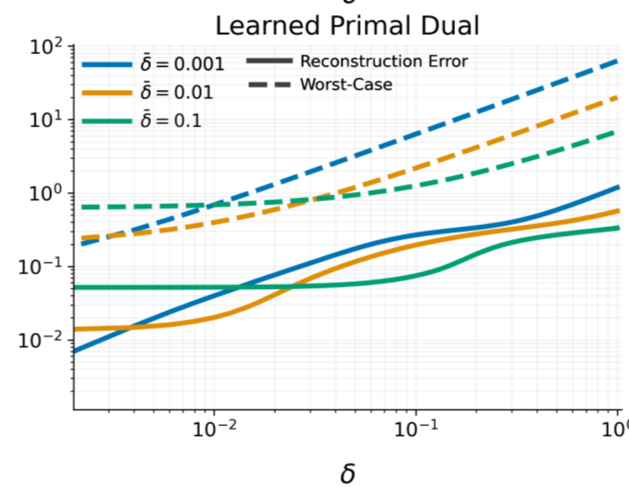
Approaches

- Tikhonov: noise level mismatch incorporated via $\alpha(\bar{\delta})$
- LPD: train data with noise level $\bar{\delta}$, test data with δ

Evaluation

$$\mathcal{E}(\bar{\delta}, \delta) = \frac{1}{K}\sum_{i=1}^{K} \|x_{i,\delta}^{\bar{\delta}} - x_i\|, \text{ with reconstructions } x_{i,\delta}^{\bar{\delta}}$$





Related (learned) "MNIST + Radon" - Settings:

[3] Adler et al. *Task adapted reconstruction for inverse problems. In:* Inverse Problems, *vol. 38 (2022).*
[4] Aspri et al. *A data-driven iteratively regularized Landweber iteration,* arXiv:1812.00272 (2018).
[5] Boink, Blume. *Learned SVD: solving inverse problems via hybrid autoencoding. arXiv:1912:10840 (2020).*

Data Dimension as Regularization

Idea

- Data may live on subspace $X_N = \mathrm{span}(b_1, \dots, b_N)$
- Use *truncated Tikhonov* with reconstruction dimension M

$$T_\alpha^M y^\delta = \underset{x \in X_M}{\mathrm{argmin}} \; \tfrac{1}{2}\|Ax - y^\delta\|^2 + \tfrac{\alpha}{2}\|x\|^2$$

- **M < N:** missing signal components
- **M = N:** best balance
- **M > N:** noise amplification

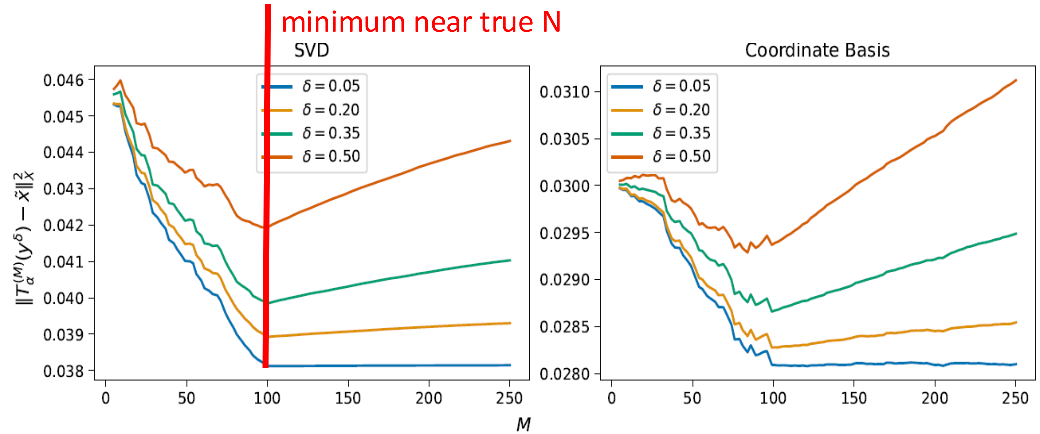


Figure: Simulated data sample x with N = 100 using different basis vectors

Theorem[2]

For $x \in X_N$ on the first N singular directions of A, a suitable choice for $\alpha = \alpha(\delta, x, A)$ minimizes the expected error, i.e.,

$$N = \mathrm{argmin}_{M \in \mathbb{N}} \; \mathbb{E}_\eta[\|T_\alpha^M y^\delta - x\|^2] \, .$$

→ Scanning over M can indicate the **effective data dimension**.

[2] Behrens, Iske, Jiang, Maass, Neumayer. A remark on an error analysis for classical and learned Tikhonov regularization schemes, arXiv:2604.00759 (2026).

Revisiting: Learned *Regularization*?

Generalized LASSO objective

$$J_\alpha(x; y^\delta) = \|Ax - y^\delta\|^2 + \alpha(\delta)\|Wx\|_1$$

Let $X_W = \mathrm{span}(w_j)$ with w_j matrix rows of W and $A : X \to Y$.

Good case	Bad case
$X_W = X$	$X_W \neq X$
W sees every direction	W has blind directions
$\|Wx\|_1$ acts like a norm	$\|Wx\|_1$ does not penalize $X_W^\perp$

Theorem[2]

If $y^\delta \in \overline{AX_W^\perp} \setminus AX_W^\perp$, there exist $(x_n)_n \subset X_W^\perp$ such that

$$\|Ax_n - y^\delta\| \to 0 \quad \text{and} \quad J_\alpha(x_n; y^\delta) \to 0 \quad \text{but} \quad \|x_n - x^\dagger\| \to \infty .$$

⟶ learned regularizers must see the right directions, otherwise the **worst-case error** is **unbounded**.

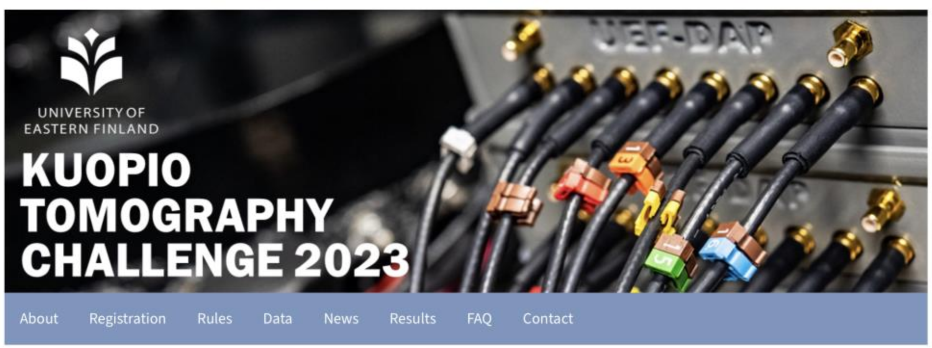
[2] Behrens, Iske, Jiang, Maass, Neumayer. A remark on an error analysis for classical and learned Tikhonov regularization schemes, arXiv:2604.00759 (2026).

Helsinki Tomography Challenge 2022

Winners

2nd place: **Alexander Denker**, Clemens Arndt, Judith Nickel, Johannes Leuschner, Janek Godeke, and Soren Dittmer from University of Bremen. GitHub_B

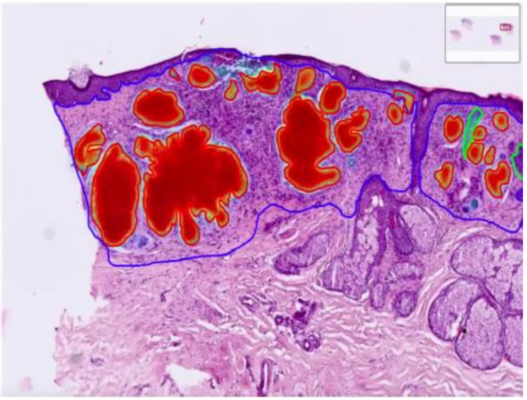
Kuopio Tomography Challenge 2023



Winners

1st place: **Alexander Denker**, Zeljko Kereta, Imraj Singh, Tom Freudenberg, Tobias Kluth, Simon Arridge and Peter Maass – from University of Bremen and University College London. GitHub_B

AI beyond ChatGPT and Computer Vision



Industrial applications
VW, Ariane Airbus, Siemens, DB, ….

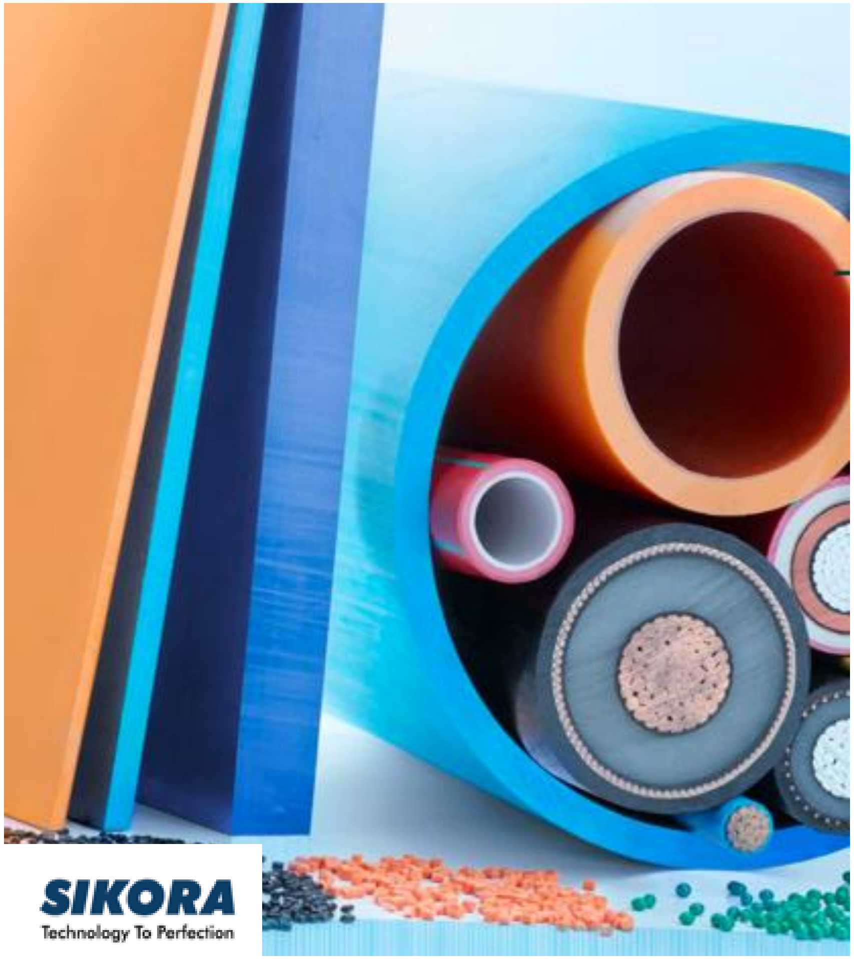
Digital pathology
1st clinical installations

CT tomography
Academic example

PDE models $$u_t + v^t \nabla u + \mathrm{div}((1 - D)\nabla u) = f$$

given measured u, determine (v, D, f)

X-Ray technology for in-process quality monitoring



03/2021 – 09/2022

Simulation of measurement process, off-line computation
Co-operation funded by WFB (local business development fund, EFRE)

10/2022-12/2022

Sikora in-house evaluation

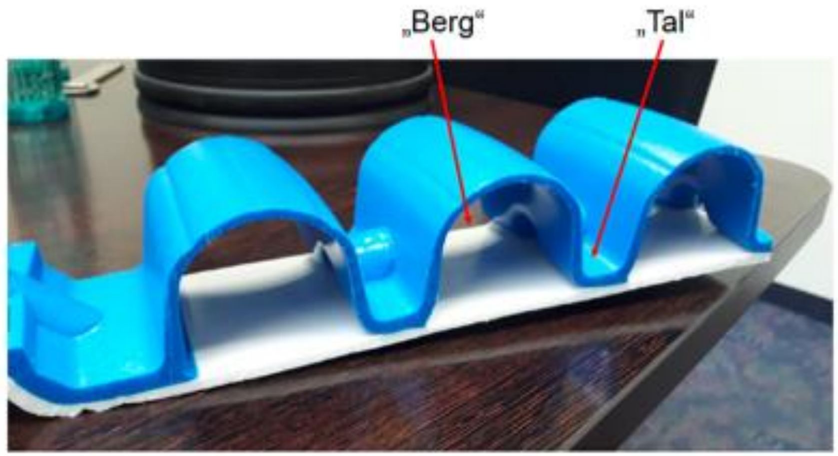
03/2023-07/2023

DLL development (WFB)

04/2024

product release

WELLROHRE

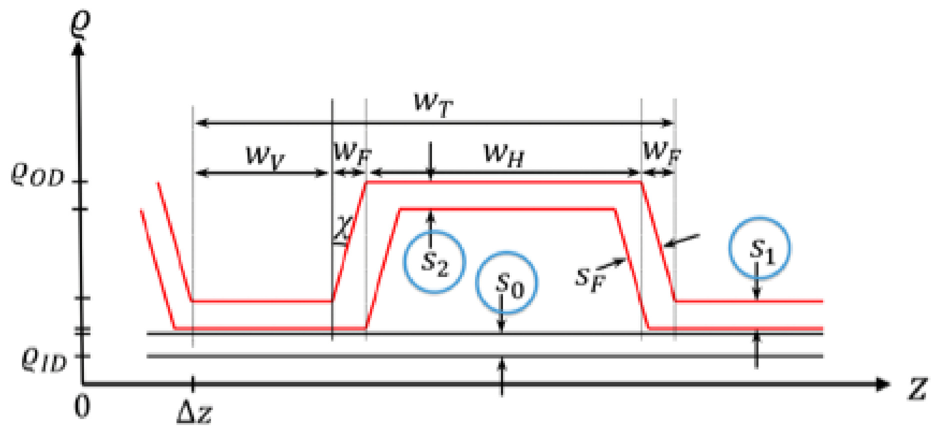


SIKORA
Technology To Perfection

Structure

- Innenhaut (weiß)
- Wellschicht (blau)

- Mono-Layer
- Single-Layer
- Multi-Layer

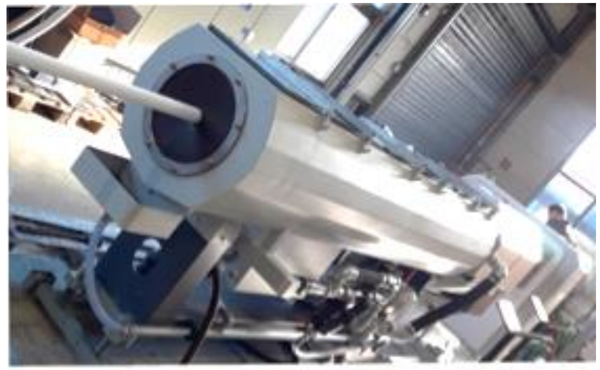


Parameters

- S0
- S2
- S0+S1

Online production monitoring

- Inprecise positioning
- Motion blur (< 9 m/s)



Kühlung & Trocknung

Abschneider

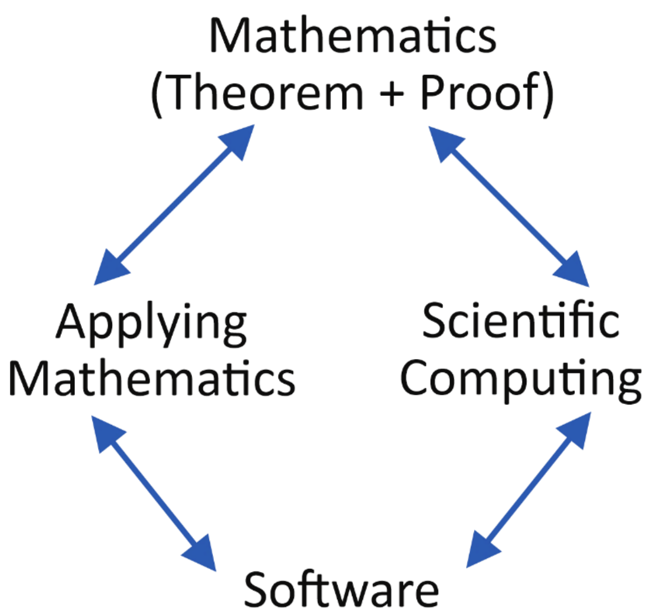
SIKORA X-RAY 6200 PRO

Basic X-Ray technology

- 2 directions
- Limited angle
- 3 μs for each scan
- Significant ray widening
- Low compute power

From Mathematical Research to Innovation

Math-technology with relevance for society/economy

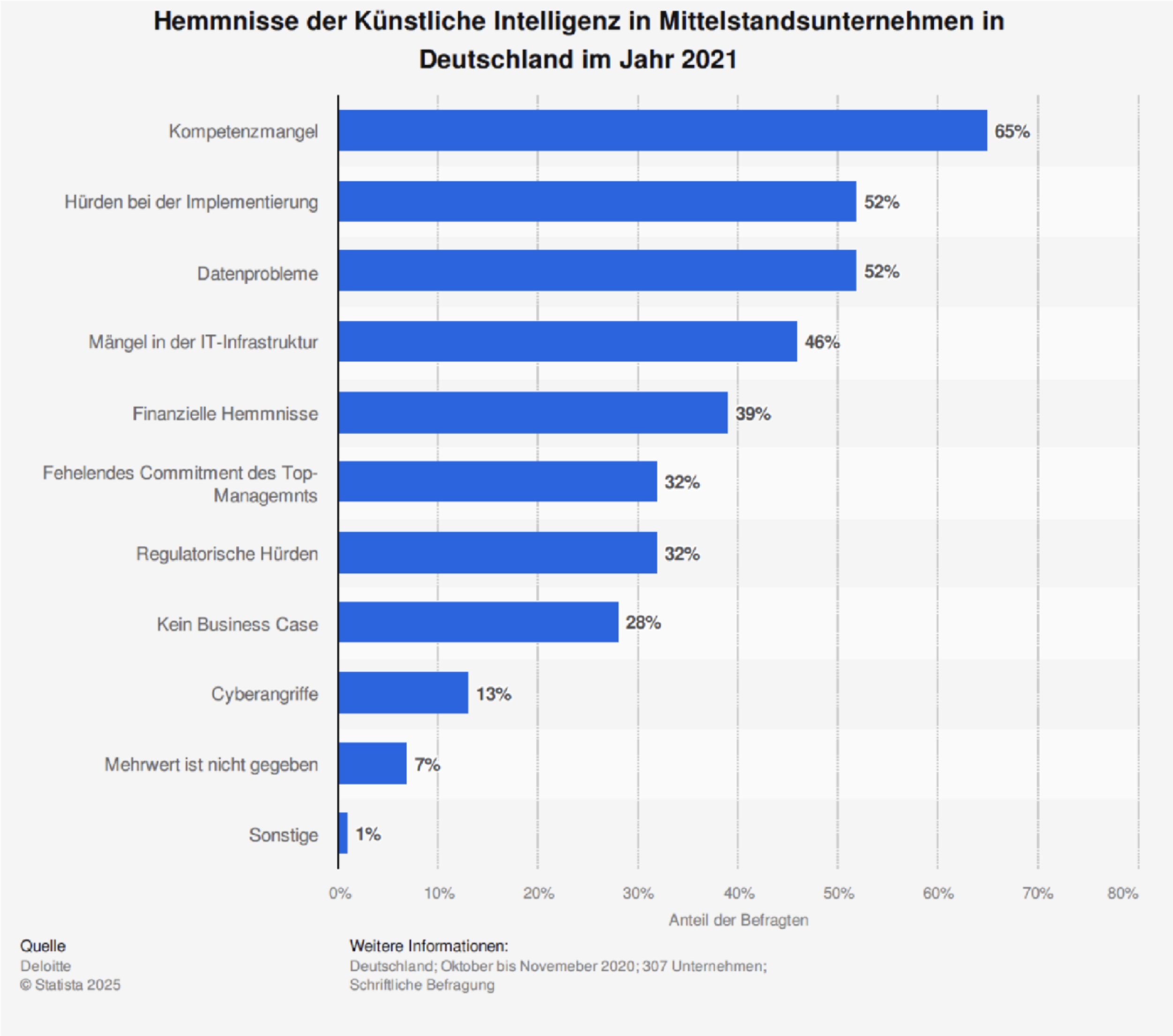


University		Company
Research Papers	Steinbeis	Problem solving
Patent		Patent
Open IP		Proprietary IP
Innovation Office	GmbH	Software
Law Office		Warranty
Non-profit Qualification		Updates
		Time to market

TRL: Mathematics needed for TRL 1-2, Mathematicians are needed on all levels

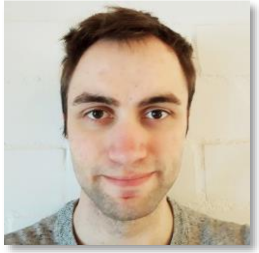
AI in industry

- Competence
- Implementation
- Data
- Business case
- Finances

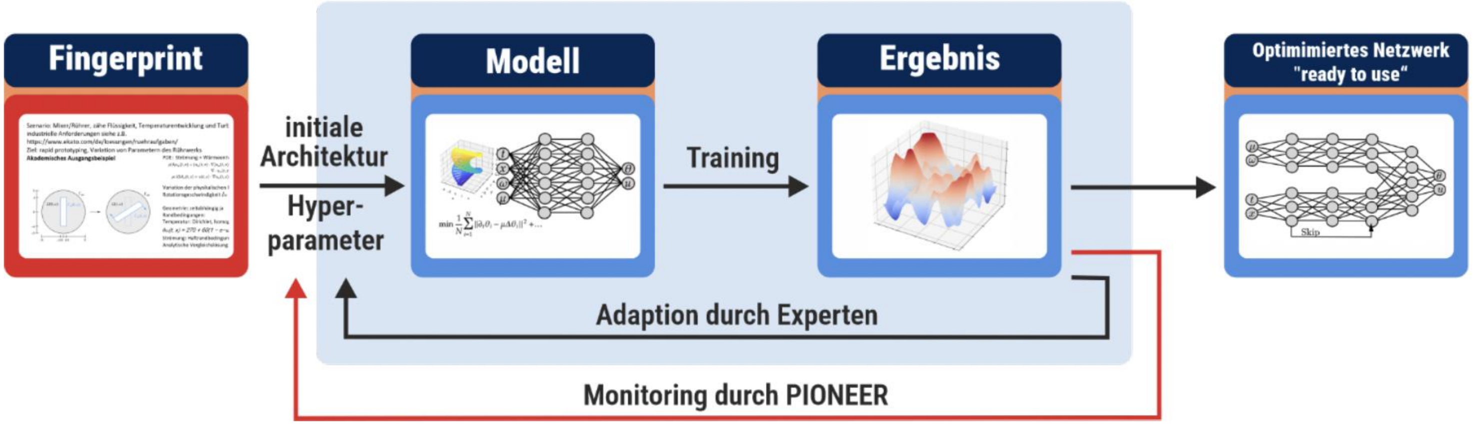


PIONEER
Physics informed optimization of neural networks for engineering and research



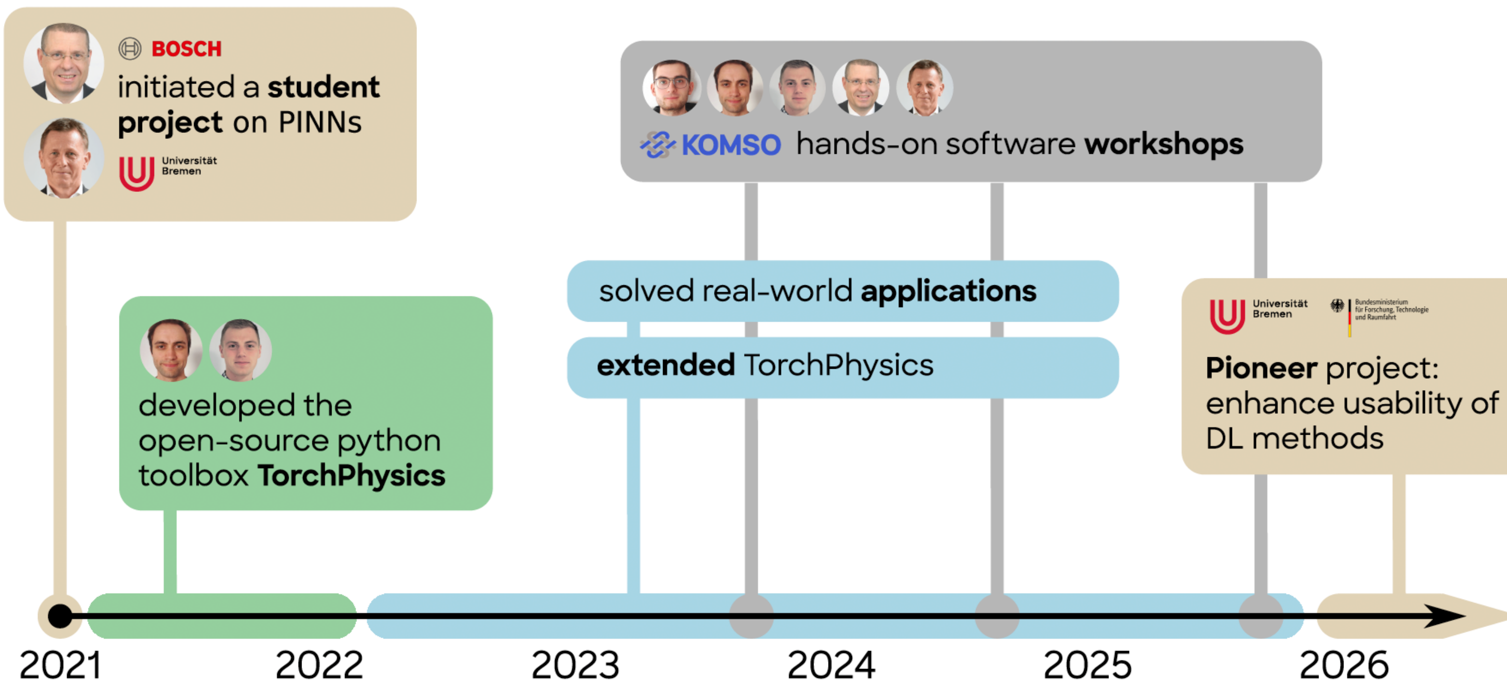


Prof. Uwe Iben (Bosch), Nick Heilenkötter, Tom Freudenberg



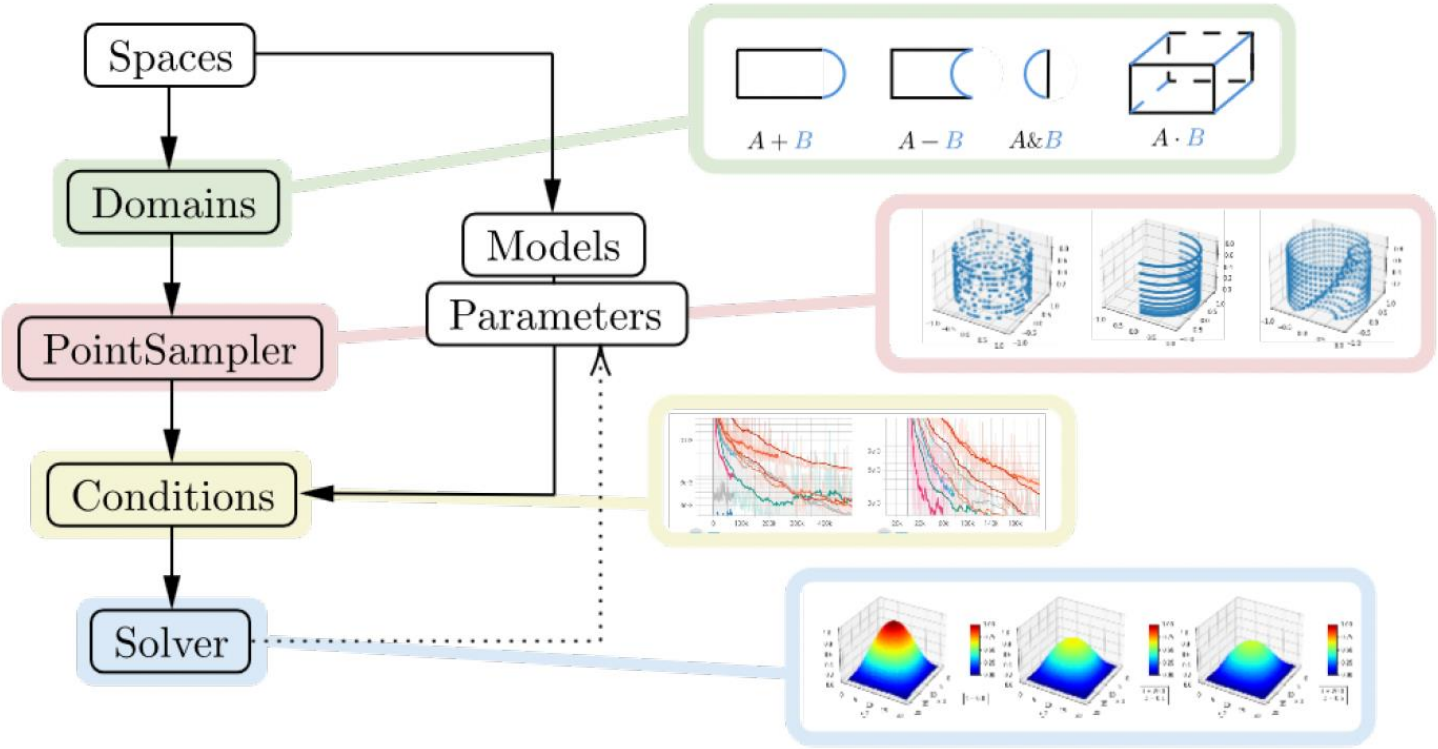
Bundesministerium für Forschung, Technologie und Raumfahrt

BMFTR: "Validierung des technologischen und gesellschaftlichen Innovationspotenzials wissenschaftlicher Forschung - **VIP+**"



TorchPhysics

Buildung Blocks



2026 - KOMSO Academy

Deep Learning Methods for Partial Differenti 1, 2026

Hanno Gottschalk is an internationally leading expert for AI based generative modelling of technical processes. His research spans from theoretical investigations for stochastic AI models to real life applications such as modelling of turbulence. In this context he recently published a highly appreciated review on how generative models help to solve parameter identification problems. He is also the KOMSO representative for national AI initiatives. Moreover, he is highlighted by NVIDIA research as one of their main collaborators, e.g. he receives generous funding from the NVIDIA Academic Grant Program for Researcher.

Hanno Gottschalk received his PhD under the supervision of Sergio Albeverio (1999, Bochum) before spending PostDocs in Rome and Bonn. From 2011 – 2023 he was Professor for Stochastic at the Bergische Universität Wuppertal before joining the TU Berlin, where he holds the Werner-von-Siemens-Professorship for 'Mathematical modelling of industrial lifecycles'.

Monday, November 30th, 2026

8:45-9:15	Arrival, Registration
9:15-9:30	**Prof. Dr. Dr. h.c. Peter Maaβ (University of Bremen)** Welcome and Introduction
9:30-10:00	**Prof. Dr. Dr. Uwe Iben (Bosch Research)** "Real and artificial intelligence for science and engineering"
10:00-10:30	**Julian Nuerk (Bosch Research)** "Enhancing numerical multiscale methods with operator learning"

Tuesday, December 1st, 2026

9:00-9:45	**Prof. Dr. Hanno Gottschalk (TU Berlin)** "Invertible Neural Networks for Mechanical Engineering Problems"
9:45-10:30	**Dr. Marius Zeinhofer (ETH Zurich)** "Geometric Optimization for Scientific Machine Learning"
10:30-10:50	Break
10:50-12:00	**Workshop Team** Lecture & Hands-on coding: "From PINNs to PINOs and PI-DeepONets"

Design ohne Titel - 1

Working group