What if String Theory has a de Sitter Excited State?

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The search for de Sitter has become pretty intensive in the last few years with debates ranging from yes to no with equal passion. The neutral ones are the ones working with < 0 or $= 0$ spaces. Our answer will turn out to be yes and we will give a proof to justify our statement. Our result will be based on the following papers.

- **What if string theory has a de Sitter excited state? Joydeep Chakravarty, K. D 2404.11680**
- **Coherent States in M-Theory: A Brane Scan using the Taub-NUT, Joydeep Chakravarty, K. D, Diksha Jain, Dileep P. Jatkar, Archana Maji, Radu Tatar, 2308.08613**
- **Resurgence of a de Sitter Glauber-Sudarshan State: Nodal Diagrams and Borel Resummation, Suddhasattwa Brahma, K. D et al, 2211.09181**

- What if string theory has no de Sitter vacua? Ulf H. Danielsson, Thomas Van Riet, 1804.01120

- Four dimensional de Sitter space is a Glauber-Sudarshan state in string theory I, II, Suddhasattwa Brahma, K.D, Radu Tatar et al 2007.00786, 2108.08365; de Sitter space is a Glauber-Sudarshan state in string theory, 2007.11611
- Crisis on infinite earths: short-lived dS vacua in the string theory landscape, Heliudson Bernardo, Suddhasattwa Brahma, KD, Radu Tatar, 2009.04504
- Quantum Break-Time of de Sitter, G. Dvali, C. Gomez and S. Zell, 1701.01776; Quantum Breaking Bound on de Sitter and Swampland, G. Dvali, C. Gomez, S. Zell, 1810.11002

Other relevant papers related to my talk are as follows.

- **de Sitter vacua in type IIB string theory: Classical solutions and quantum corrections,** K.D, Rhiannon Gwyn, Mohammed Mia, Evan McDonough and Radu Tatar **1402.5112.**
- $\overline{D3}$ **and dS,** Eric Bergshoeff, K.D, Renata Kallosh, Antoine Van-Proyen, Timm Wrase, **1502.07627**
- **Quantum Corrections and the de Sitter Swampland Conjecture,** K.D, Maxim Emelin, Evan McDonough, Radu Tatar, **1808.07498**
- **de Sitter vacua in the string landscape,** K.D, Mir Mehedi Faruk, Maxim Emelin, Radu Tatar, **1908.05288; How a four-dimensional de Sitter solution remains outside the swampland, 1911.02604**

Summary of the results

- **Problems of realizing de Sitter as a vacuum solution**
- **How to realize it as a Glauber-Sudarshan state instead of a vacuum**
- **Why is this a hard problem?**
- **Although hard, it may still be a doable problem!**

The problems start early on. To see why we would like to view four-dimensional de Sitter space-time as as an excited state, we need to take a step back and start by asking some basic questions. With this in mind, let us take the following four-dimensional action:

$$S = \int d^4x \sqrt{\mathbf{g}_4}\ \frac{1}{16\ G_N}\ \mathbf{R}_4\ \ \overline{2}\ + \frac{1}{4g_{YM}^2}\mathbf{F}\wedge\ \mathbf{F} + ...$$

where G_N is the four-dimensional Newton's constant, g_{YM} is the gauge coupling for gauge field F,   is the positive cosmological constant, and the dotted terms denote higher order interactions.

If the above action is considered, then there is nothing to show! We can as well end the talk right here! The theory admits, in four-dimension, a positive cosmological constant solution which we can identify with the de Sitter space-time.

The issue here is that little term that is proportional to $\sqrt{g_4}$ **: string theory or M-theory** *does not* **come equipped with a term like this!**

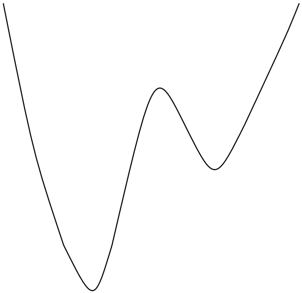
The question then shifts to the following. Can this little term come from (say) the dimensional reduction of an action of the form:

$$
\begin{aligned}
S = & \, M_p^9 \int d^{11}x \sqrt{\mathbf{g}_{11}} \left(\mathbf{R}_{11} + \mathbf{G}_4 \wedge * \mathbf{G}_4\right) \\
& + M_p^9 \int \mathbf{C}_3 \wedge \mathbf{G}_4 \wedge \mathbf{G}_4 + M_p^3 \int \mathbf{C}_3 \wedge X_8 + \ldots,
\end{aligned}
$$

Here we denote the three-form flux as $\mathbf{C}_3$ with $\mathbf{G}_4 = d\mathbf{C}_3 + \ldots$, and X_8 is a fourth order curvature polynomial. Note that the above action has no scale other than M_p and the *size* of the internal manifold; and therefore both G_N and g_{YM}^2 in the 4d action should come only from the two aforementioned scales.

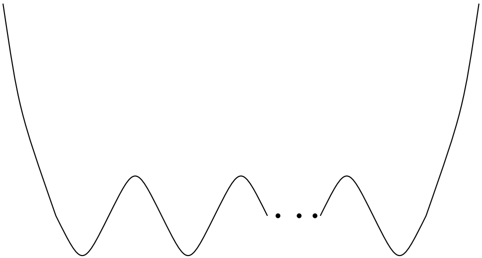
The answer is yes if is negative or zero and no if is positive!

You may say that this is a bold claim and maybe the quantum terms, perturbative or non-perturbative ones, should produce a positive solution. In other words quantum corrections should give us a potential like:



So that fluctuations around the positive energy will produce de Sitter vacuum in string theory.

Unfortunately it seems that string theory may not allow a potential like that! What it allows now appears to be:

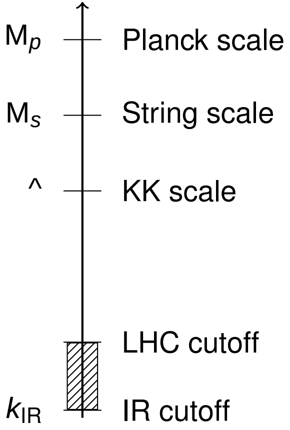


All the minima are Minkowski with possible AdS minima although scale-separated AdS minima also do not appear.

Why is that?

To study this we will use M-theory. You may ask: we are claiming to study de Sitter space from M-theory. But we know nothing of M-theory, except maybe the low energy dynamics. How are you going to tackle that?

Indeed we know nothing of the UV behavior of either string theory or M-theory, but we do know one thing for sure. No matter how complicated the short distance behavior of these theories are, or what degrees of freedom reside there, the far UV dynamics are well-defined ***without*** **any pathologies. This is important for the consistency of these theories.**



The point is that, no matter what degrees of freedom exist in the far UV, an Exact Renormalization Group (ERG) procedure will tell us that once we integrate from $_{\text{UV}}$ till , all the information of the UV can be encoded in the *coefficients* of the infinite towers of marginal, relevant and irrelevant interactions of the massless states!

Of course this means that we have to keep track of all possible operators constructed from the massless degrees of freedom. This way we lose no information of the UV, but the subtlety is that some of these massless states could be off-shell.

There is a long story of how to deal with the off-shell states that I might briefly mention if I have some time.

This means for a metric like:

$$
\begin{aligned}
ds^2 &= \frac{1}{H^2(y)|t|^2}\left(dt^2 + dx_1^2 + dx_2^2 + dx_3^2\right) \\
&+ H^2(y)\, g_{MN}(y;t)dy^M dy^N
\end{aligned}
$$

to appear as a vacuum solution in type IIB theory, the M-theory uplift of it leads to the following consistency condition:

$$
\begin{aligned}
&\frac{1}{12}\int d^8x\sqrt{g}\,\mathbf{G}_{MNPQ}\mathbf{G}^{MNPQ} + 12\int d^8x\sqrt{g}\,H^2 + 2\;{}^2T_2(n_3 + n_3) \\
&+ \int d^8x\sqrt{g}\,H^{4/3}\sum(\text{quantum pieces}) = 0,
\end{aligned}
$$

Since the first line is positive definite, solutions exist in the absence of quantum terms for $\quad = 0$ or $\quad < 0$.

For $\quad > 0$ there are no solutions in the absence of quantum terms: no branes, fluxes or planes can give rise to a de Sitter vacuum solution.

Solution would exist iff the following condition is maintained:

$$\int d^8x \sqrt{g}\, H^{4/3} \sum (\text{quantum pieces}) \; < \; 0;$$

which calls for some inherent hierarchies in the quantum series.

This may sound like a good news: if we can somehow allow some hierarchies, and satisfy the above condition we might, just might, get a de Sitter vacuum solution!

An even better news: with time-dependent degrees of freedom one appears to find some hierarchy, so what's stopping us to declare victory and finish the talk?

Well, recall that we are studying everything at far IR, so the aforementioned quantum terms must have come from integrating out the high energy modes and heavy DOFs.

What if we could not integrate out these high energy DOFs? Then everything we said about the hierarchy collapses!

You may ask when could such a situation arise. Such a situation arises:

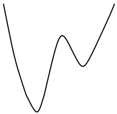
If there is no well-defined UV completion

Or if the frequencies themselves are changing with respect to time.

We can clearly rule out the former because, as I described a moment ago, no matter how complicated the short-distance behavior of string or M-theory is, the far UV is by definition a well-defined theory.

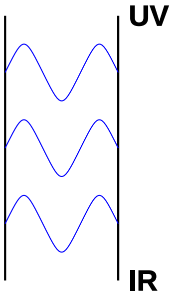
However the latter is now an issue: one may easily check that the fluctuating frequencies over a four-dimensional de Sitter spacetime do become time-dependent.

Such time-dependence would rule out the Wilsonian integrating-out procedure because as the frequencies are constantly red-shifted there is no meaning of the integrating out process now!



You may immediately ask: what about in static patch?

Unfortunately the dynamics over a static patch is highly deceptive. Let me give you an example using a cartoon.



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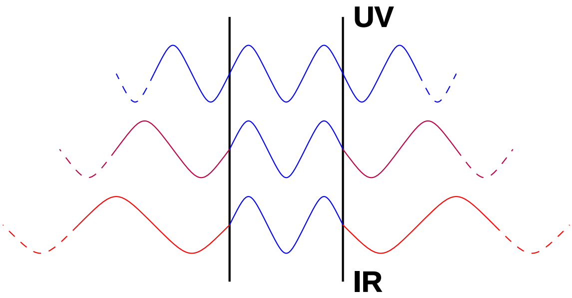


This means the frequencies are also changing inside the static patch, but the observer will not know about it!

You may say that's it: since the observer doesn't know he doesn't have to worry about it. This is of course wrong because a static patch is observer dependent (unlike BHs).

A different observer, with a slightly different static patch will notice this. A somewhat similar observation Banks had recently: there appears to be energy leakage in a static patch!

Moreover in a static patch one can show that the dual IIA coupling $g_s >> 1$, so instanton series with $\exp\left(\frac{1}{g_s^a}\right)$ cannot be made convergent. Going to M-theory doesn't help.

All in all a static patch is not a way out of this mess. You may ask: how did people deal with this?

I know three ways this was handled.

Ignore it, and continue with the static patch story, or just do AdS/CFT and forget that nature has positive .

Use open QFT, where energy is not conserved and using Markovian techniques.

Propose a trans-Planckian censorship criterion to block the temporally varying UV modes to escape the horizon.

The second one, namely the open QFT, is traditionally described by isolating relevant degrees of freedom from an "environment" which allows them to gain or lose energies to the environment. Since neither energy is conserved, nor an EFT description exists, the dynamics of the theory typically follows some Markovian process which may be quantified in certain settings.

The problem with this picture appears when we try to use it in string theory : in a UV complete theory, like string theory or M-theory, there appears to be no clear demarcation between the relevant degrees of freedom and the environment, and therefore a concrete realization over a temporally varying background is equally hard.

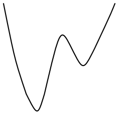
For the third case, the origin of such a “censorship" arose from the usage of the UV modes in GR. Clearly these modes are not well defined because the short-distance behavior of GR is not well-defined.

In string theory, although we may not know what is the UV behavior, or what DOFs exist there, there are no pathologies from the UV modes. Thus the usual argument of censoring the UV modes doesn’t make much sense. Although the temporal domain advocated from TCC appears here from demanding convergent instanton series!

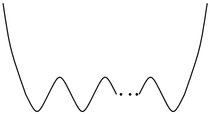
Thus all the three arguments we gave above are unsatisfactory to deal with this situation.

This is where de Sitter as an excited state over a supersymmetric Minkowski background appears.

The claim is that then there is no potential like:



But only a potential like:



in string theory with possible SUSY AdS non-scale-separated minima.

de Sitter excited state cannot be a coherent state near any of the Minkowski minima, because we expect the de Sitter state to have all the quantum corrections somehow “in-built" in it, otherwise it is going to clash with whatever we said earlier.

Plus the coherent states are simply a fancy way to saying that there are nearby de Sitter positive energy minima because of Bogoliubov transformation.

So the proposal for the excited state, which we will henceforth call as the Glauber-Sudarshan state, is:

$$j \quad i \qquad \mathrm{D}(\quad)j \quad i$$

Where $|\quad\rangle$ is the **interacting vacuum** in the **full** potential showed earlier, and $D(\quad)$ is the non-unitary “displacement" operator.

We have assumed that:

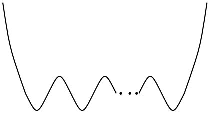
- The vacuum is non-degenerate.
- There is an energy gap between the first excited state(s) and the vacuum.

To extract the metric of spacetime from the GS state, we use the following strategy:

$$\langle \mathbf{g} \quad \rangle \qquad \frac{\langle \quad | \mathbf{g} \quad | \quad \rangle}{\langle \quad | \quad \rangle}$$

How does that work over a given Minkowski minimum?

Choose you favorite Minkowski minimum in the potential



We will call it $|0\rangle_{\rm min1}$. Now express this as a linear combination of the Eigenstates of the exact potential as:

$$|0\rangle_{\rm min1} = c_o|\Omega\rangle + \sum_{\rm N=1}^{\infty} c_{\rm N}|{\rm N}\rangle$$

Now hit the LHS and RHS by $e^{-i{\bf H}_{\rm tot}{\rm T}}$ and take ${\rm T} \to \infty(1-i\epsilon)$. This will kill off all states $|{\rm N}\rangle$ for ${\rm N} \ge 1$. This implies $|\Omega\rangle \propto \lim_{{\rm T}\to\infty(1-i\epsilon)} e^{-i{\bf H}_{\rm tot}{\rm T}}|0\rangle_{\rm min1}$ and $\langle\Omega| \propto \lim_{{\rm T}\to\infty(1-i\epsilon)} {}_{\rm min1}\langle 0|e^{-i{\bf H}_{\rm tot}{\rm T}}$.

You might worry that we have no idea what the total Hamiltonian is.

Of course we do know that it exists because we have integrated away all the high energy DOFs, so H_{tot} is at least expressed in terms of the IR (and massless) DOFs. Plugging this we get:

$$\langle \mathbf{g} \quad \rangle \qquad \frac{\langle \quad | \mathbf{g} \quad | \quad \rangle}{\langle \quad | \quad \rangle} = \frac{\langle \quad | \mathrm{D}^{\dagger}(\quad) \mathbf{g} \quad \mathrm{D}(\quad) | \quad \rangle}{\langle \quad | \mathrm{D}^{\dagger}(\quad) \mathrm{D}(\quad) | \quad \rangle}$$

$$= \frac{\int [Dg_{MN}][DC_{MNP}][D\bar{\quad}_M][D \quad_N]\, e^{\quad S_{tot}}\, \mathrm{D}^{\dagger}(\quad) g \quad (x;y) \mathrm{D}(\quad)}{\int [Dg_{MN}][DC_{MNP}][D\bar{\quad}_M][D \quad_N]\, e^{\quad S_{tot}}\, \mathrm{D}^{\dagger}(\quad) \mathrm{D}(\quad)}$$

In the M-theory uplift of the IIB scenario over a given Minkowski minimum. Note that we haven't specified what S_{tot} is!

If what we wrote above works out then the claim is that the spacetime de Sitter metric is an emergent concept!

$S_{\rm tot}$ should get contributions from the kinetic terms, the complete series of perturbative corrections, the complete non-perturbative corrections, the non-local corrections, the gauge fixing terms, and the FP and derivative ghosts.

We can avoid the FP and the derivative ghosts by sticking to a simpler scalar field model with polynomial interactions. This is good, but then how about the non-perturbative and the non-local interactions?

Moreover where are they coming from, since far IR limit of M-theory is known to be a local theory?

First thing first: how to quantify the non-perturbative terms?

The answer is magical. Remember we said that we have to take all order IR interactions after Wilsonian integration over a given Minkowski minimum?

Such a perturbative series is asymptotic! And therefore gives us a reason to incorporate the non-perturbative effects in a very natural way.

In the language of the multi-minima Minkowski potentials, these NP effects come from both real and complex instantons.

Which means for a given Minkowski minimum, $\mathbf{S}_{tot}$ can be expressed as a trans-series. This literally means:

$$
\begin{aligned}
\mathbf{S}_{tot} \quad & e^{\mathbf{S}^{(0)}_{saddle}}\,(\text{fluc det}) + e^{\mathbf{S}^{(1)}_{saddle}}\,(\text{fluc det}) + \ldots \\
= & \sum_{n=0}^{\infty} e^{\frac{n}{g^b}\hat{\mathbf{S}}^{(n)}_{saddle}} \sum_{m=0}^{\infty} g^m\,\mathbf{S}^{(n;m)}_{pert},
\end{aligned}
$$

Where $\mathbf{S}^{(n)}_{saddle}$ and $\hat{\mathbf{S}}^{(n)}_{saddle}$ are related to the instanton saddles over the Minkowski minimum, $g = M_p^{\,ive}$ and $\mathbf{S}^{(n;m)}_{pert}$ are perturbative interactions for a given choice of n and m.

Since we took simple scalar fields we don't see the g_s effects, and in particular the instanton D-branes saddles. We only see NP effects associated with M_p.

Thus for a given Minkowski minimum, we see that the effective action is given in terms of an infinite series of instanton saddles accompanied by the corresponding fluctuation determinants.

In full string theory, if we are able to do this, then there should be contributions from the D-branes and other possible g_s related instantonic saddles.

The D-brane instantons should reproduce Shenker's original motivation for introducing D-branes in string theory.

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The D-brane instantons should reproduce Shenker's original motivation for introducing D-branes in string theory. As I mentioned earlier, the asymptotic nature of the perturbative series provides a reason why nature has NP effects.

However even with scalar DOFs we can quantify the story further so let's see how far we can get.

First, as you may have noticed, we really do not have any "on-shell" DOFs as everything emerges from off-shell computations.

For example in the M-theory uplift of the following IIB background:

$$
\begin{aligned}
ds^2 &= \frac{1}{H^2(y)|t|^2} \quad dt^2 + dx_1^2 + dx_2^2 + dx_3^2 \\
&+ \quad H^2(y)\, g_{MN}(y;t) dy^M dy^N
\end{aligned}
$$

Which is given by:

$$
\begin{aligned}
ds^2 &= \quad \frac{1}{H^2(y)|t|^2}^{\frac{4}{3}} \quad dt^2 + dx_1^2 + dx_2^2) \\
&+ \quad H^2(y) \quad \frac{1}{H^2(y)|t|^2}^{\frac{1}{3}} g_{MN}(y;t) dy^M dy^N + \quad \frac{1}{H^2(y)|t|^2}^{\frac{2}{3}} \quad dx_3^2 + dx_{11}^2
\end{aligned}
$$

The emergent “on-shell" metric configurations are:

$$h\mathbf{g}\quad i = {\frac{1}{\mathrm{H}^2(y)jtj^2}}^{\frac{4}{3}} \quad ; \quad h\mathbf{g}_{ab}\, i = {\frac{1}{\mathrm{H}^2(y)jtj^2}}^{\frac{2}{3}} \quad {}_{ab}$$

$$h\mathbf{g}_{\mathrm{MN}}\, i = \mathrm{H}^2(y) \;{\frac{1}{\mathrm{H}^2(y)jtj^2}}^{\frac{1}{3}} g_{\mathrm{MN}}(y;t)$$

This would be an example how an off-shell computation creates emergent on-shell components. This goes to show that there is nothing classical in our analysis.

On the other hand, the actual emergent “off-shell" components must satisfy

$$h\mathbf{g}\;{}_{\mathrm{M}}\, i = h\mathbf{g}\;{}_{a}\, i = h\mathbf{g}_{\mathrm{M}a}\, i = 0$$

These are truly off-shell as the actual components g $_{\mathrm{M}}$; **g** $_{a}$; **g**$_{a\mathrm{M}}$ **are non-zero. Thus the whole spacetime in either IIB or M-theory is emergent.**

You may ask how long do we expect the emergent state to survive. There are many arguments resulting to the same answer, and here is one.

We expect, in the full string theory, to allow for D-branes and other instanton saddles to go like $e^{-\frac{1}{g_s^c}}$ in the trans-series, where g_s is the dual IIA coupling. For this to be convergent, we require the IIA coupling $\frac{g_s}{\mathrm{H}(y)\sqrt{-t}} < 1$. This gives:

$$-\frac{1}{\sqrt{\Lambda}} < t < 0$$

Which is somewhat surprisingly the time period advocated by imposing the trans-Planckian bound. This result may also be got from the so-called quantum breaking time for coherent states as argued by Dvali et al.

Our state is neither a coherent state nor has trans-Planckian issues, so it's interesting that we are getting the same answer.

You may now wonder what EOMs do these emergent components satisfy?

Expectedly they are the Schwinger-Dyson equations that take, to give one example, the following form:

$$\frac{\mathbf{S}_{\mathrm{tot}}}{\mathbf{g}} = \frac{\quad}{\mathbf{g}} \log \; D^y D \qquad ;$$

Two questions arise:

1. Can we bring the equation into some useful format?

2. How are we dealing with the “off-shell" components?

The answer to the first question is yes, and for the second question is that the margin of time is too small for me to give you a satisfactory answer. You may now ask, how is this related to what we said earlier related to the trans-series? The derivation is somewhat long, so sparing everybody of the details, let me just quote the answer.

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Before that, one definition

$$\mathbf{Q}_{\rm pert}(c; \quad) = \sum_{\{l_i\};\{n_j\}} \frac{c_{nl}}{\mathrm{M}_p^{\;nl}} \left[\mathbf{g}^{\;1}\right] \prod_{j=0}^{3} [\partial]^{n_j} \prod_{k=1}^{60} \left(\mathbf{R}_{\mathrm{A_kB_kC_kD_k}}\right)^{l_k} \prod_{p=61}^{100} \left(\mathbf{G}_{\mathrm{A_pB_pC_pD_p}}\right)^{l_p}$$

where   is the set of “on-shell" fields that we discusssed earlier, and c_{nl} are dimensionless constants at the IR scale.

$$
\begin{aligned}
\mathbf{S}_{\rm tot} \;=\;& M_p^9 \int d^{11}x \sqrt{\mathbf{g}_{11}}\,[\mathbf{R}_{11} + \mathbf{G}_4 \wedge *_{11}\mathbf{G}_4] + M_p^9 \int \mathbf{C}_3 \wedge \mathbf{G}_4 \wedge \mathbf{G}_4 + M_p^3 \int \mathbf{C}_3 \wedge X_8 \\
&+ M_p^{11} \int d^{11}x \sqrt{\mathbf{g}_{11}} \sum_{s=0}^{\infty} d_s\, \mathbf{Q}_{\rm pert}(c(s)) \\
&\quad \exp\left(-s M_p^8 \int_0^y \int_0^w d^6y'\, d^2w' \sqrt{\mathbf{g}_8(Y';x)}\, \mathbf{Q}_{\rm pert}(\hat{c}(s);\, (Y';x))\right) \\
&+ M_p^{11} \int d^{11}x \sqrt{\mathbf{g}_{11}} \\
&\quad \sum_{p=0}^{\infty} b_p \exp\left(-p M_p^8 \int d^6y'\, d^2w' \sqrt{\mathbf{g}_8(Y';x)}\, F(y-y';w-w')\mathbf{Q}_{\rm pert}(c(p);\, (Y';x))\right) \\
&\quad \mathbf{Q}_{\rm pert}(c(p)) + M_p^8 \int d^6y'' d^2w'' \sqrt{\mathbf{g}_8(Y'';x)}\, \mathbf{Q}_{\rm pert}(c(p);\, (Y'';x))F(y-y'';w-w'')
\end{aligned}
$$

which is the most generic action that one could write using the on-shell fields. The M-brane terms are more subtle so won't talk about them here. The EOM for the emergent state will take the form:

$$
\begin{aligned}
&\mathbf{R}_{\rm AB}(\langle\boldsymbol{\Xi}({\rm X})\rangle_\sigma) - \frac{1}{2}\langle\mathbf{g}_{\rm AB}({\rm X})\rangle_\sigma\mathbf{R}(\langle\boldsymbol{\Xi}({\rm X})\rangle_\sigma)\\
&= \frac{2}{\sqrt{\mathbf{g}_{11}(\langle\boldsymbol{\Xi}({\rm X})\rangle_\sigma)}}\frac{\delta}{\delta\langle\mathbf{g}^{\rm AB}({\rm X})\rangle_\sigma}\left(\sqrt{\mathbf{g}_{11}(\langle\boldsymbol{\Xi}({\rm X})\rangle_\sigma)}\mathbf{G}_4(\langle\boldsymbol{\Xi}({\rm X})\rangle_\sigma)\wedge *_{11}\mathbf{G}_4(\langle\boldsymbol{\Xi}({\rm X})\rangle_\sigma)\right)\\
&- \frac{2{\rm M}_p^2}{\sqrt{\mathbf{g}_{11}(\langle\boldsymbol{\Xi}({\rm X})\rangle_\sigma)}}\sum_{s=0}^{\infty} d_s \frac{\delta}{\delta\langle\mathbf{g}^{\rm AB}({\rm X})\rangle_\sigma}\left[\sqrt{\mathbf{g}_{11}(\langle\boldsymbol{\Xi}({\rm X})\rangle_\sigma)}\mathbf{Q}_{\rm pert}(\bar{c}(s);\langle\boldsymbol{\Xi}({\rm X})\rangle_\sigma)\right]\\
&\times\ \exp\left(-s{\rm M}_p^8\int_0^y\int_0^w d^6y'\ d^2w'\sqrt{\mathbf{g}_8(\langle\boldsymbol{\Xi}({\rm Y}',x)\rangle_\sigma)}\left|\mathbf{Q}_{\rm pert}(\hat{c}(s);\langle\boldsymbol{\Xi}({\rm Y}',x)\rangle_\sigma)\right|\right)\\
&+\ {\rm M}_p^{10}\int d^6y'\ d^2w'\sqrt{\frac{\mathbf{g}_{11}(\langle\boldsymbol{\Xi}({\rm Y}',x)\rangle_\sigma)}{\mathbf{g}_{11}(\langle\boldsymbol{\Xi}({\rm X})\rangle_\sigma)}}\sum_{s=0}^{\infty} sd_s\ \mathbf{Q}_{\rm pert}(\bar{c}(s);\langle\boldsymbol{\Xi}({\rm Y}',x)\rangle_\sigma)\\
&\times\ \frac{\delta}{\delta\langle\mathbf{g}^{\rm AB}({\rm X})\rangle_\sigma}\left[\sqrt{\mathbf{g}_8(\langle\boldsymbol{\Xi}({\rm X})\rangle_\sigma)}\mathbf{Q}_{\rm pert}(\hat{c}(s);\langle\boldsymbol{\Xi}({\rm X})\rangle_\sigma)\right]\ \Theta(y'-y)\ \Theta(w'-z)\\
&\times\ \exp\left(-s{\rm M}_p^8\int_0^{y'}\int_0^{w'} d^6y''\ d^2w''\sqrt{\mathbf{g}_8(\langle\boldsymbol{\Xi}({\rm Y}'',x)\rangle_\sigma)}\left|\mathbf{Q}_{\rm pert}(\hat{c}(s);\langle\boldsymbol{\Xi}({\rm Y}'',x)\rangle_\sigma)\right|\right)\\
&-\ \frac{1}{\sqrt{\mathbf{g}_{11}(\langle\boldsymbol{\Xi}({\rm X})\rangle_\sigma)}}\sum_{p=0}^{\infty} b_p\frac{\delta}{\delta\langle\mathbf{g}^{\rm AB}({\rm X})\rangle_\sigma}\left(\sqrt{\mathbf{g}_{11}(\langle\boldsymbol{\Xi}({\rm X})\rangle_\sigma)}\mathbf{Q}_{\rm pert}(\check{c}(p);\langle\boldsymbol{\Xi}({\rm X})\rangle_\sigma)\right)\\
&\times\ \exp\left(-p{\rm M}_p^8\int_{\mathcal{M}_8} d^6y'\ d^2w'\sqrt{\mathbf{g}_8(\langle\boldsymbol{\Xi}({\rm Y}',x)\rangle_\sigma)}\left|\mathbb{F}(y-y';w-w')\mathbf{Q}_{\rm pert}(\tilde{c}(p);\langle\boldsymbol{\Xi}({\rm Y}',x)\rangle_\sigma)\right|\right)\\
&-\ {\rm M}_p^8\sum_{p=0}^{\infty}\langle\mathbf{g}_{\rm AB}({\rm X})\rangle_\sigma\int_{\mathcal{M}_8} d^6y'd^2w'\sqrt{\mathbf{g}_8(\langle\boldsymbol{\Xi}({\rm Y}',x)\rangle_\sigma)}\ \mathbf{Q}_{\rm pert}(\grave{c}(p);\langle\boldsymbol{\Xi}({\rm Y}',x)\rangle_\sigma)\mathbb{F}(y-y';w-w')\\
&\times\ \exp\left(-p{\rm M}_p^8\int_{\mathcal{M}_8} d^6y''\ d^2w''\sqrt{\mathbf{g}_8(\langle\boldsymbol{\Xi}({\rm Y}'',x)\rangle_\sigma)}\left|\mathbb{F}(y-y'';w-w'')\mathbf{Q}_{\rm pert}(\tilde{c}(p);\langle\boldsymbol{\Xi}({\rm Y}'',x)\rangle_\sigma)\right|\right)\\
&-\ {\rm M}_p^8\int d^6y'd^2w'\sqrt{\frac{\mathbf{g}_{11}(\langle\boldsymbol{\Xi}({\rm Y}',\mathbf{x})\rangle_\sigma}{\mathbf{g}_{11}(\langle\boldsymbol{\Xi}({\rm X})\rangle_\sigma}}\sum_{p=0}^{\infty}\frac{\delta}{\delta\langle\mathbf{g}_{\rm AB}({\rm X})\rangle_\sigma}\left(\sqrt{\mathbf{g}_8(\langle\boldsymbol{\Xi}({\rm X})\rangle_\sigma)}\ \mathbf{Q}_{\rm pert}(\grave{c}(p);\langle\boldsymbol{\Xi}({\rm X})\rangle_\sigma)\right)\mathbb{F}(y'-y;w'-w)\\
&\times\ \exp\left(-p{\rm M}_p^8\int_{\mathcal{M}_8} d^6y''\ d^2w''\sqrt{\mathbf{g}_8(\langle\boldsymbol{\Xi}({\rm Y}'',x)\rangle_\sigma)}\left|\mathbb{F}(y'-y'';w'-w'')\mathbf{Q}_{\rm pert}(\tilde{c}(p);\langle\boldsymbol{\Xi}({\rm Y}'',x)\rangle_\sigma)\right|\right)\\
&+\ {\rm M}_p^8\int d^6y'\ d^2w'\sqrt{\frac{\mathbf{g}_{11}(\langle\boldsymbol{\Xi}({\rm Y}',x)\rangle_\sigma)}{\mathbf{g}_{11}(\langle\boldsymbol{\Xi}({\rm X})\rangle_\sigma)}}\ \mathbb{F}({\rm Y}'-{\rm Y})\sum_{p=1}^{\infty} pb_p\ \frac{\delta}{\delta\langle\mathbf{g}^{\rm AB}({\rm X})\rangle_\sigma}\left[\sqrt{\mathbf{g}_8(\langle\boldsymbol{\Xi}({\rm X})\rangle_\sigma)}\mathbf{Q}_{\rm pert}(\tilde{c}(p);\langle\boldsymbol{\Xi}({\rm X})\rangle_\sigma)\right]\\
&\times\left[\mathbf{Q}_{\rm pert}(\check{c}(p);\langle\boldsymbol{\Xi}({\rm Y}',x)\rangle_\sigma)+{\rm M}_p^8\int_{\mathcal{M}_8} d^6y''d^2w''\sqrt{\mathbf{g}_8(\langle\boldsymbol{\Xi}({\rm Y}'',x)\rangle_\sigma)}\ \mathbf{Q}_{\rm pert}(\grave{c}(p);\langle\boldsymbol{\Xi}({\rm Y}'',x)\rangle_\sigma)\mathbb{F}(y'-y'';w'-w'')\right]\\
&\times\ \exp\left(-p{\rm M}_p^8\int_{\mathcal{M}_8} d^6y'''\ d^2w'''\sqrt{\mathbf{g}_8(\langle\boldsymbol{\Xi}({\rm Y}''',x)\rangle_\sigma)}\left|\mathbb{F}(y'-y''';w'-w''')\mathbf{Q}_{\rm pert}(\tilde{c}(p);\langle\boldsymbol{\Xi}({\rm Y}''',x)\rangle_\sigma)\right|\right)
\end{aligned}
$$

However I haven't told you how the instantons appear. It turns out that the trans-series form relies on our ability to perform Borel resummation. How do we justify this?

We will again take a scalar field model to show this, and from our experience so far, we can take one on-shell field φ. We will ignore the non-local part for simplicity here. The action and the displacement operator now will be:

$$\mathbf{S}_{\rm tot} = \int d^{11}x \left[(\partial\varphi)^2 + \frac{g}{4!}\varphi^4 \right]; \quad D(\; ; \varphi) = \exp \int d^{11}x \quad \varphi$$

If we ignore the tensor indices, φ is like one of the on-shell metric component, say $\mathbf{g}_{00}$. This means we at least expect:

$$\langle \varphi \rangle \quad \langle \mathbf{g}_{00} \rangle = \left(\frac{1}{t^2} \right)^{\frac{4}{3}} = \quad {}^{4/3} t^{\;8/3}$$

This way we might also be able to quantify .

In terms of path-integral, which we can do, the emergent quantity $\langle\varphi\rangle$ becomes (we will only show the numerator):

$$\langle\varphi\rangle = \int \prod_k d\varphi_k \prod_{k'} e^{-a_{k'}(\varphi_{k'} \quad {}_{k'}/a_{k'})^2} \Big[1 + \frac{g}{4!}\frac{1}{V}\sum_{\tilde k} \varphi_{\tilde k}^{\;4} + O(g^2)\Big] \sum_{\hat k} \frac{\varphi_{\hat k}}{V} e^{-i\hat k\cdot x}$$

where φ_k are the Fourier components, $a_k = \frac{k^2}{V}$ is the propagator, V is the volume of the space, and $k_{\rm IR} \quad k \quad$. We have ignored many subtleties as they are not very important for the present discussion. At N^{th} order this becomes:

$$\langle\varphi\rangle = \int \prod_k d\varphi_k \prod_{k'} e^{-a_{k'}(\varphi_{k'} \quad {}_{k'}/a_{k'})^2} \sum_{N=0}^{\infty} \frac{g^N}{N!(4!)^N V^{4N+1}} \left(\varphi_{k_1} + \varphi_{k_2} + \ldots\right)^{4N} \sum_{\hat k} \varphi_{\hat k} e^{-i\hat k\cdot x}$$

Now for some rules of the game.

Because of the volume factor, the most dominant terms would be the ones where all the $4N \quad 1$ momenta are different and only one of the interaction momentum will match with the source momentum.

If all of the $4N$ momenta are different from the source momentum, such diagrams will be eliminated by the denominator of the path integral. This is much like the vacuum bubble story.

However any 2 of the interaction vertices could have the same momenta and be different from the source momentum. But then no other vertices could have the same momenta as the source, otherwise they will be suppressed by the volume factor.

If more than 2 vertices have the same momenta, then the amplitude will be suppressed by the volume factor and become sub-dominant contributions.

Needless to say, this is new kind of QFT computation because we are using shifted vacuum. This also keeps the 1-point functions non-zero.

Instead of Feynman diagrams now, we have nodal diagrams. Unfortunately the margin of time is again too small to give any details on these diagrams.

At N^{th} order the interaction and the source terms, for the most dominant amplitude, scale as:

$$\frac{(4\mathrm{N})!g^{\mathrm{N}}}{\mathrm{N}!(4!)^{\mathrm{N}}\mathrm{V}^{4\mathrm{N}+1}}$$

You can clearly see some factorial growth, but is not the usual Borel one. This is called the Gevrey growth.

After we integrate out the fields in the path-integral, and take care of few other subtleties, the amplitude now grow as:

$$\sum_{N=0}^{\infty} g^{N}(2N)!A^{N} \int_{k_{IR}}^{\infty} d^{11}k \, \frac{(k)}{a(k)} \, e^{\,ik:x}$$

where A is defined in the following way:

$$A = A(k_{IR}; \quad) \prod_{i=1}^{\infty} \int_{k_{IR}}^{\infty} d^{11}k_i \, \frac{(k_i)}{a(k_i)}$$

Of course we have ignored many subtleties, including the ones with the denominator of the path integral.

Sparing us of all the details and performing the Borel-Ecalle resummation, after the dust settles we get:

$$\langle \varphi \rangle = \frac{1}{g^{1/2}} \int_0^{\infty} dS \, \exp\left(-\frac{S}{g^{1/2}}\right) \frac{1}{1 - AS^2} \, \mathrm{P.V} \int_{k_{\mathrm{IR}}} d^{11}k \, \frac{\varphi(k)}{a(k)} \, e^{ik \cdot x}$$

where P.V is the principal value of the integral over S and $a(k)$ is the massless propagator. We now make a few observations.

- **The whole series in the path-integral is summed up completely.**
- **The final answer is a wave-function renormalization of the tree-level answer.**
- **Whether or not A is even or odd, there is only one pole in the Borel axis.**

The closed form expression for the wave-function renormalization term may be re-written in the following suggestive way:

$$\int_0^\infty dS\,\frac{\exp\left(-S/g^{1/2}\right)}{1-AS^2} = \sum_{N=0}^{\infty}\underbrace{\exp\left(-\frac{N}{g^{1/2}}\right)}_{\substack{\text{instanton}\\ \text{saddle}}}\underbrace{\left[\frac{1}{1-N^2A}+\sum_{n=1}^{\infty}\frac{2^nA^nN^n}{(1-N^2A)^{n+1}}(s-N)^n\right]}_{\text{fluctuation determinant}}$$

which takes the expected form of an infinite number of instanton saddles along-with their corresponding fluctuation determinants.

However an even more surprising result appears when we identify $h\,'\,i$ with $h\mathbf{g}_{00}\,i$. In other words:

$$h\,'\,i \qquad h\mathbf{g}_{00}\,i = \frac{1}{t^2}^{\frac{4}{3}} = {}^{4=3}t^{\;8=3}$$

$$= \frac{1}{g^{1=2}} \int_0^{1} dS \exp \quad \frac{S}{g^{1=2}} \; \frac{1}{1 \quad AS^2} \int_{\mathrm{P:V} \quad k_{\mathrm{IR}}} d^{11}k \, \frac{(k)}{a(k)} \, e^{\;ik:x}$$

However a more surprising result appears when we identify $\langle \varphi \rangle$ with $\langle \mathbf{g}_{00} \rangle$. In other words:

$$\langle \varphi \rangle \qquad \langle \mathbf{g}_{00} \rangle = \left(\frac{1}{t^2}\right)^{\frac{4}{3}} = {}^{4/3} t^{\;8/3}$$

$$= \underbrace{\frac{1}{g^{1/2}} \int_0^1 dS \, \exp}_{4/3} \underbrace{\left(\frac{S}{g^{1/2}} \frac{1}{1 - AS^2}\right)_{\mathrm{P.V.}}}_{} \underbrace{\int_{k_{\mathrm{IR}}} d^{11}k}_{t} \underbrace{\frac{(k)}{a(k)} e^{ik \cdot x}}_{8/3}$$

Of course there is a more compelling reason for making this identification, but I won't have time to discuss it.

This not only fixes the form for (k) but also provides a closed form expression for the 4d cosmological constant ${}_{4d}$ M_p^2 !

This means the 4d CC is given by the following expression:

$$\Lambda_{4d} = \frac{M_p^2}{\left\langle \frac{1}{g^{3/8}} \int_0^{\infty} dS \exp\left(-\frac{S}{g^{1/2}}\right)\left(\frac{1}{1 - AS^2}\right)\right\rangle^{3/4}_{P.V}}$$

You may ask whether this is positive or not.

Yes, no matter what sign A takes!

Is this, or can this be made, small?

It appears so but we are not sure yet. This is a work in progress.

How small? Can this be $10^{-120} M_p^2$?

Absolutely no idea!

Please don't be alarmed, we haven't solved the CC problem! This is just a scalar field model. The actual analysis will no doubt be much harder to deal with.

However our analysis suggests that maybe dark energy is a cumulative effect coming from the summation of all quantum terms and is completely invisible at order-by-order computations. It is also an emergent effect, so it is unlikely that any vacuum configuration could reproduce it. As they say, the whole is greater than the sum of its parts.

Our analysis might also provide hints on precise reconciliation of GR with QM: All non-trivial GR configurations are like Glauber-Sudarshan and not coherent states in the quantum world. The GR EOMs would then be the Schwinger-Dyson equations for these emergent states.

This could probably make Einstein happy!

Conclusion

The summary of my talk may be tabulated in the following way

Vacuum configuration	**Replaced by**
Type II, I and Heterotic theories	M-theory and duality sequences
de Sitter space-time	Glauber-Sudarshan state $\vert\sigma\rangle = \mathbb{D}(\sigma)\vert\Omega\rangle, \mathbb{D}^\dagger(\sigma) \neq \mathbb{D}^{-1}(\sigma)$
Fluctuations over de Sitter space-time	Agarwal-Tara state
Trans-Planckian bound	$g_s < 1 \implies -\frac{1}{\sqrt{\Lambda}} < t < 0$
Black-hole in de Sitter space	Another Glauber-Sudarshan state $\vert\sigma'\rangle$
Equations of motion	Schwinger-Dyson equations
On-shell states	Off-shell states
Off-shell states	Off-shell states and non-local quantum terms
Feynman diagrams	Nodal diagrams

Conclusion

The summary of my talk may be tabulated in the following way

Space-time metric $g_{\mu\nu}(x,y)$	$\langle \mathbf{g}_{\mu\nu}\rangle_\sigma = \frac{\int [\mathcal{D}g_{\rm MN}][\mathcal{D}{\rm C}_{\rm MNP}][\mathcal{D}\overline{\Psi}_{\rm M}][\mathcal{D}\Psi_{\rm N}]\ e^{-\mathbf{S}_{\rm tot}}\ \mathbb{D}^\dagger(\sigma) g_{\mu\nu}(x,y)\mathbb{D}(\sigma)}{\int [\mathcal{D}g_{\rm MN}][\mathcal{D}{\rm C}_{\rm MNP}][\mathcal{D}\overline{\Psi}_{\rm M}][\mathcal{D}\Psi_{\rm N}]\ e^{-\mathbf{S}_{\rm tot}}\ \mathbb{D}^\dagger(\sigma)\mathbb{D}(\sigma)}$
Conformal time t, $-\infty < t < 0$	$-\frac{1}{\sqrt{\Lambda}} < t = t(g_s) < 0$
Internal metric $g_{mn}(y,t)$	$\langle \mathbf{g}_{mn}\rangle_\sigma = \frac{\int [\mathcal{D}g_{\rm MN}][\mathcal{D}{\rm C}_{\rm MNP}][\mathcal{D}\overline{\Psi}_{\rm M}][\mathcal{D}\Psi_{\rm N}]\ e^{-\mathbf{S}_{\rm tot}}\ \mathbb{D}^\dagger(\sigma) g_{mn}(y,t)\mathbb{D}(\sigma)}{\int [\mathcal{D}g_{\rm MN}][\mathcal{D}{\rm C}_{\rm MNP}][\mathcal{D}\overline{\Psi}_{\rm M}][\mathcal{D}\Psi_{\rm N}]\ e^{-\mathbf{S}_{\rm tot}}\ \mathbb{D}^\dagger(\sigma)\mathbb{D}(\sigma)}$
Positive cosmological constant Λ	$0 < \Lambda = \lim_{c_{(\mathbf{s})}\to 0} \frac{{\rm M}_p^2}{\left[\sum_{\{\mathbf{s}\}} \frac{\mathbb{F}_{(\mathbf{s})}}{c_{(\mathbf{s})}} \int_0^\infty du_{(\mathbf{s})}\ \frac{\exp\left(-u_{(\mathbf{s})}/c_{(\mathbf{s})}\right)}{1-u^l_{(\mathbf{s})}}\right]^{3/4}_{\rm P.V}} << {\rm M}_p^2$
Existence of EFT	$\frac{\partial g_s}{\partial t} = g_s^{+ive} \implies$ non-violation of NEC
Quantum tunneling	Real and complex instantons
Non-perturbative effects	Borel resummation of Gevrey series
Open quantum field theories	Wilsonian EFT or Exact Renormalization Group
Contributions from zero-point energies	Cancelled Zero-point energies
Non-susy vacuum configuration	Spontaneously broken supersymmetry by GS state $\vert\sigma\rangle$

Conclusion

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Merci beaucoup pour votre attention!