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Workshop on

"Nonlinear Water Waves - an Interdisciplinary Interface"

November 27 – December 7, 2017

organized by

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Week 2

December 4 – December 7, 2017

- **Monday, December 4, 2017**

  09:00 – 09:30 **Registration**

  09:30 – 10:30 **Boris Buffoni**
  Steady three-dimensional rotational flows: variational structure and Nash-Moser iteration
  The stationary flow of an inviscid and incompressible fluid of constant density is considered in the region $D = (0, L) \times \mathbb{R}^2$. The flow is assumed periodic in the second and third variables and to have prescribed flux through each point of the boundary ∂D. The values of the "Bernoulli function" $H = \frac{1}{2}|v|^2 + p$ are also prescribed on ∂D, where v is the velocity and p the pressure. Since H is allowed to be non constant on ∂D, three-dimensional flows with non-vanishing vorticity are included. I will present an existence theory near a given constant solution that relies on writing the velocity field in the form $v = \nabla f \times \nabla g$ and on solving a nonlinear system by a Nash-Moser method. This is a joint work with Erik Wahlén.

  10:30 – 11:00 *Coffee / Tea Break*

  11:00 – 12:00 **Mark Groves**
  Fully localised solitary gravity-capillary water waves
  Joint work with B. Buffoni and E. Wahlén. We consider the classical gravity-capillary water-wave problem in its usual formulation as a three-dimensional free-boundary problem for the Euler equations for a perfect fluid. A solitary wave is a solution representing a wave which moves in a fixed direction with constant speed and without change of shape; it is fully localised if its profile decays to the undisturbed state of the water in every horizontal direction. The existence of fully localised solitary waves has been predicted on the basis of simpler model equations, namely the Kadomtsev-Petviashvili (KP) equation in the case of strong surface tension and the Davey-Stewartson (DS) system in the case of weak surface tension. In this talk we confirm the existence of such waves as solutions to the full water-wave problem and give rigorous justification for the use of the model equations.

  12:00 – 14:00 *Lunch Break*

  14:00 – 15:00 **Mariana Haragus**
  Formation of Kerr optical frequency combs : a water waves bifurcation scenario
  Frequency combs are optical signals consisting of a superposition of modes with equally spaced frequencies. The corresponding mathematical model is the Lugiato-Lefever equation, a cubic nonlinear Schrdinger equation with damping, detuning and driving force. Using a spatial dynamics approach, we study the existence of steady waves. It turns out that the resulting bifurcation diagram is very similar to the one found in the two-dimensional steady water-wave problem. Relying upon tools from bifurcation

theory and the results obtained for the water-wave problem, we prove the existence of various types of steady solutions, including spatially localized and periodic solutions. We conclude with a discussion of the stability properties of periodic waves.

15:00 – 15:30 *Break*

15:30 – 16:30 **David Lannes**
Survey lecture: *The water waves equations and asymptotics*
The water waves equation govern the evolution of the free surface of an incompressible, non viscous, fluid under the influence of gravity. They are given by the incompressible Euler equations in the domain occupied by the fluid, complemented by appropriate kinematic and dynamic boundary conditions.
Assuming further that the flow is irrotationnal, there exist several reductions that allow to recast the problem as a set of evolution equations on the horizontal domain only. For instance, the Euler equations can be vertically integrated to eliminate the vertical variable; as a result, one obtains a system coupling the surface parametrization to the vertically averaged horizontal velocity. These equations are however quite complicated due to the nonlinearities: the average of a square is not in general the square of the average!
In shallow water, i.e. when the horizontal scale is much larger than the depth, we will show that it is possible to relate quite precisely and explicitly these two quantities, leading to reduced asymptotic models such as the nonlinear shallow water equations, the Boussinesq equations, the Serre-Green-Naghdi equations, etc. A brief review of these systems and of some related open problems will be proposed. We will also comment on the equations obtained under the further assumption that the waves are unidirectional at leading order, such as the KdV, BBM, Camassa-Holm, and Whitham equations...
Finally, we will comment on recent developments where the irrotationality assumption is relaxed. Relating the average of a square to the square of the averageïs then much more delicate, and requires the introduction of a (finite) cascade of equations reminiscent of the scenario of turbulence for compressible gas dynamics.

- **Tuesday, December 5, 2017**

09:30 – 10:30 **Gareth Thomas**
Prediction of the free-surface elevation for rotational water waves using the recovery of pressure at the bed
In this talk we consider the pressure-streamfunction relationship for a train of regular water waves propagating on a steady current, which may possess an arbitrary distribution of vorticity, in two dimensions. The application of such work is to both nearshore and offshore environments, and in particular for linear waves we provide a description of the role which the pressure function on the sea-bed plays in determining the free-surface profile elevation.
Our approach is shown to provide a good approximation for a range of current conditions, leading to the derivation of expressions for the pressure transfer function, and the related pressure amplification factor, which generalise the well-known formulae for irrotational waves. An implementation of the moderate current approximation renders these expressions more tractable, leading to quite elegant and explicit formulae. This is joint work with David Henry.

10:30 – 11:00 *Coffee / Tea Break*

11:00 – 12:00 **Ton van den Bremer**
Experimental validations of various aspects of the wave-induced mean flow for surface gravity wave groups
For sufficiently directionally spread surface gravity wave groups, the set-down of the wave-averaged free surface, first described by Longuet-Higgins & Stewart (1962, JFM), can turn into a set-up. Using a multiple-scales expansion for two crossing wave groups, we examine the structure and magnitude of this wave-averaged set-up, which is part of a crossing wave pattern that behaves as a modulated partial standing wave: in space, it consists of a rapidly varying standing-wave pattern slowly modulated by the product of the envelopes of the two groups; in time, it grows and decays on the slow time scale associated with the translation of the groups. Whether this crossing wave pattern actually enhances the

surface elevation at the point of focus depends on the phases of the linear wave groups, unlike the set-down, which is always negative and inherits the spatial structure of the underlying envelope(s).
We present detailed laboratory measurements of the wave-averaged free surface, examining both single wave groups, varying the degree of spreading from small to very large, and the interaction between two wave groups, varying both the degree of spreading and the crossing angle between the groups. In both cases, we find good agreement between experiments, our simple expressions for the set-down and set-up and existing second-order theory based on the component-by-component interaction of individual waves with different frequencies and directions. We predict and observe a set-up for wave groups with a Gaussian angular amplitude distribution with standard deviations of above 30-40 deg. (21-28 deg. for energy spectra), which is relatively large for realistic sea states, and for crossing sea states with angles of separation of 50-70 deg. and above, which are known to occur in the ocean.

12:00 – 14:00 *Lunch Break*

14:00 – 15:00 **Raphael Stuhlmeier**
Evolution of statistically inhomogeneous degenerate water wave quartets
Nonlinear interaction, along with wind input and dissipation, is one of the three mechanisms which drive wave evolution, and is included in every modern waveforecast model. The mechanism behind the nonlinear interaction terms in such models is based on the kinetic equation for wave spectra derived by Hasselmann. This does not allow, for example, for statistically inhomogeneous wave fields, nor for the modulational instability which depends on such inhomogeneity, and which has been implicated in the appearance of exceptionally high rogue waves.
We derive a discretized equation for the evolution of random, inhomogeneous surface wave fields on deep water from Zakharov's equation, along lines first laid out by Crawford, Saffman, and Yuen. This allows for a general treatment of the stability and long-time behaviour of broad-banded sea states. It is investigated for the simple case of degenerate four-wave interaction, and the instability of statistically homogeneous states to small inhomogeneous disturbances is demonstrated. Furthermore, the long-time evolution is studied for several cases and shown to lead to a complex spatio-temporal energy distribution. The possible impact of this evolution on the statistics of rogue wave occurrence is explored within the framework of this simplified example.

15:00 – 15:30 *Break*

15:30 – 16:30 **Vera Hur**
Stokes waves with constant vorticity: numerical computation

- **Wednesday, December 6, 2017**

09:30 – 10:30 **Guido Schneider**
The KdV and the NLS equation on periodic metric graphs and relations to the water wave problem

10:30 – 11:00 *Coffee / Tea Break*

11:00 – 12:00 **Mats Ehrnström**
Enhanced existence time in fractional KdV equations
The fractional KdV equation $u_t + uu_x - |D|^\alpha u_x = 0$ contains the KdV ($\alpha = 2$), the Benjamin–Ono ($\alpha = 1$), the Burgers (an exceptional case, ($\alpha = 0$)), the Burgers–Hilbert ($\alpha = -1$) and the reduced Ostrovskii equation ($\alpha = -2$), and is known as family of dispersive model equations suitable to the study of the balance between dispersive and nonlinear effects. Classical solutions exist globally in time for large values of $\alpha > 0$, whereas wave-breaking comes into play as the dispersion gets weaker (α negative). Hunter and Ifrim showed that, for $\alpha = -1$, the time of existence of classical solutions in the Burgers-Hilbert equation can be extended from $1/\varepsilon$ to $1/\varepsilon^2$ when the initial data is of order ε. Using a normal-form transformation inspired by theirs, but working almost completely in Fourier space, we extend their result to all $\alpha \in (-1, 0) \cup (0, 1/2)$ (the result is true for all $\alpha \geq -1$, $\alpha \neq 0$). This is joint work with Yuexun Wang, NTNU.

12:00 – 14:00 *Lunch Break*

14:00 – 15:00 **Konstantinos Kalimeris**
Non-local formulations for water waves
We discuss a non-local formulation for the classical equations of rotational water waves, which is based on the so-called *unified transform* or the Fokas method, which provides a novel approach for the analysis of linear and integrable nonlinear boundary value problems. After reviewing some of the basic aspects of this approach we analyse inviscid, irrotational, two dimensional water waves in a bounded domain, and in particular we study the generation of waves by a moving bottom, as it occurs in tsunamis. We show that this problem is characterised by two equations which involve only first order derivatives. Under the assumption of "small amplitude" waves, these equations yield a new generalisation of the Boussinesq system which is valid without the long wave approximation.

15:00 – 15:30 *Break*

15:30 – 16:30 **Ronald Quirchmayr**
Singular traveling waves of a highly nonlinear shallow water equation
We consider a shallow water equation which has recently been derived from the classical water wave problem for unidirectional water waves, where the divergence-free flow beneath the free water surface over a flat bed is governed by the incompressible Euler equations, with gravity acting as the only external force. This model equation describes the evolution of the horizontal velocity component of the flow field evaluated at a specific depth in a shallow water regime, allowing for waves of large amplitude. After a brief discussion of the derivation and local well-posedness results, we will focus on traveling wave solutions of this highly nonlinear partial differential equation with an emphasis on singular traveling waves. By applying qualitative methods from the theory of dynamical systems, in particular tools from integrable planar systems, we establish a full classification of all traveling waves. Thereby we discover completely new types of traveling waves such as peaked solitary waves with compact support and periodic traveling waves with peaked crests and troughs, which do not appear as solutions of shallow water equations for waves of moderate amplitude like the well-known Camassa-Holm equation.

- **Thursday, December 7, 2017**

09:30 – 10:30 **Eugen Varvaruca**
Large-amplitude steady water waves with constant vorticity

10:30 – 11:00 *Coffee / Tea Break*

11:00 – 12:00 **Sam Walsh**
Existence, nonexistence, and asymptotics of deep water solitary waves with localized vorticity
In this talk, we discuss some recent work on the qualitative theory of two- or three-dimensional solitary water waves with localized vorticity in infinite depth. Our results apply to both capillary-gravity and gravity waves and allow the free surface to be overhanging in the near-field. We assume that the vorticity is localized in the sense that it satisfies certain moment conditions, and we permit there to be finitely many point vortices in the bulk of the fluid in two dimensions. We also consider a two-fluid model with a vortex sheet. Under mild decay assumptions, we obtain precise asymptotics for the velocity field and free surface, and relate this to global properties of the wave. For instance, we rule out the existence of waves whose free surface elevations have a single sign and of vortex sheets with finite angular momentum. On the other hand, building on previous work with Shatah and Zeng, we prove the existence of families of two-dimensional capillary-gravity waves with compactly supported vorticity that have (optimal) algebraic decay rates for the free surface and velocity. This is joint work with Miles Wheeler and Ming Chen.

12:00 – 14:00 *Lunch Break*

14:00 – 15:00 **Delia Ionescu-Kruse**
On the short-wavelength stabilities of some geophysical flows
This talk is a survey of the short-wavelength stability method for rotating flows. This method is applied to the specific study of some exact geophysical flows. For Gerstner-like geophysical flows one can identify perturbations in certain directions as a source of instabilities with an exponentially growing amplitude, the growth rate of the instabilities depending on the steepness of the travelling wave profile. On the

other hand, for certain physically realistic velocity profiles, steady flows moving only in the azimuthal direction, with no variation in this direction, are locally stable to the short-wavelength perturbations.

15:00 – 15:30 *Break*

15:30 – 16:30 **Anna Geyer**
Spectral stability of periodic waves in the generalized reduced Ostrovsky equation
This talk is about stability of periodic travelling wave solutions of the generalized reduced Ostrovsky equation with respect to co-periodic perturbations. We give a simple argument based on the energy convexity which proves spectral stability of all smooth periodic travelling waves independent of the nonlinearity power. This is joint work with D. Pelinovsky.

All talks take place at ESI, Boltzmann Lecture Hall!