(More on) Gauge Networks & towards 'gauge foams'

C. Pérez Sánchez (Heidelberg + ESI-Guest) MiniTalk NCG meets TR

INTRO & MOTIVATION

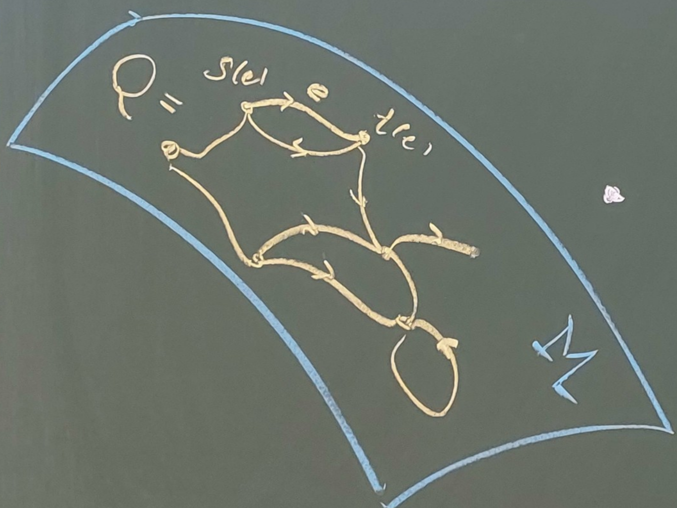
IN MATHEMATICS:

- Quiver Representations are <u>rich</u> theory: combinatorics, algebra, geometry ...
- Want to change the target category.

↓ why? ↓

IN PHYSICS:

- Quivers ⊆ Manifolds ($Q \subseteq M$) are used in 'spin network' approach to quantum gravity.
- Changing the target cat. allows to couple to gauge fields.



Lie gp $\dashrightarrow G \hookrightarrow P \dashleftarrow$ Ppal G-bundle

$$P \downarrow M$$

[Baez, '94]

SPIN NETWORKS

- Gauge theory on a graph $\Gamma \subseteq M$ described by

* $$\mathcal{A}(\Gamma) := \{\text{connections on } \Gamma \subseteq M\}$$
$$\cong G^{\Gamma_1} \qquad (\Gamma_1 = \mathrm{Edges}(\Gamma))$$

* $$\mathcal{G}(\Gamma) := \{\text{gauge transformations on } \Gamma\}$$
$$\cong G^{\Gamma_0} \qquad (\Gamma_0 = V(\Gamma) \text{, vertices})$$

* $$L^2(\mathcal{A}(\Gamma))^{\mathcal{G}(\Gamma)} = L^2(\mathcal{G}\text{-inv. connections}) \underset{\text{Weyl}}{\overset{\text{Peter}}{=}} \cdots$$
$$\cdots = \mathrm{span}\{\text{spin networks}\}$$

[Marcolli – van Suijlekom, '13] coupled these to matter gauge fields

NCG

• $\mathrm{SpTr}^{0} = \{ (A, \mathcal{H}, \cancel{D}^{0}) :$ unital $*$-alg A of $\dim A < \infty$, $A \overset{\lambda}{\curvearrowright} \mathcal{H}$, and $D : \mathcal{H} \to \mathcal{H}$ s.a. $\}$

assume λ faithful

• SpTr can be a category as follows:

$$\mathrm{Mor}_{\mathrm{SpTr}}(\xi_s, \xi_t) = \left\{ (\phi, L) : \begin{array}{l} *\text{-alg unital map} \\ \phi : A_s \to A_t, \\ L \in \mathcal{U}(\mathcal{H}_s, \mathcal{H}_t) \\ \text{and such that for all } a \in A \\ \mathrm{Ad}\, L \circ \lambda_s(a) = \lambda_t(\phi(a)) \end{array} \right\}$$

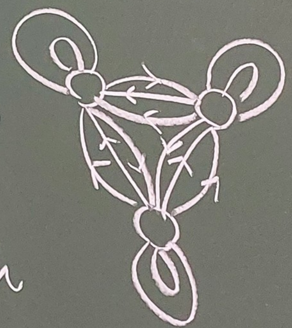
objects in SpTr

QUIVER REPS.

• Quiver Q is (Q_0, Q_1) with $Q_1 \overset{s}{\underset{t}{\rightrightarrows}} Q_0$, e.g.

finite sets

Jordan,



• Traditionally $\mathcal{C} = \mathrm{Vect}_k$, but generally $\mathcal{C}$ any category. Given a quiver Q (whose objects are Q_0 and morphisms Q_1) a $\mathcal{C}$-representation is a functor $\rho : Q \to \mathcal{C}$

WANT spectral triples

QUIVER REPRESENTATIONS

* Interesting & traditional: $\mathcal{C} = \mathrm{Vect}_k$, given $\rho : Q \to \mathrm{Vect}_k$ repr.

$$d = (\dim(\rho_v))_{v \in Q_0} \in \mathbb{N}^{Q_0}$$

· look for fixed d at:

$$\mathrm{Rep}_d(Q) = \coprod_{e \in Q_1} \hom\left(k^{d(s(e))}, k^{d(t(e))}\right)$$

$$\mathrm{Aut}_d(Q) = \coprod_{v \in Q_0} GL_{d(v)}(k)$$

* [MvS'13] Say how the quotient $\Large\wedge\!\!\!\!\wedge$ (without fixed d) looks like for $\mathcal{C} = \mathrm{SpTr}$ & showed that $L^2(\text{such quotient}) = \mathrm{span}\{\text{"gauge networks"}\}$

"quanta of NCG"

GAUGE NETWORKS IN NCG

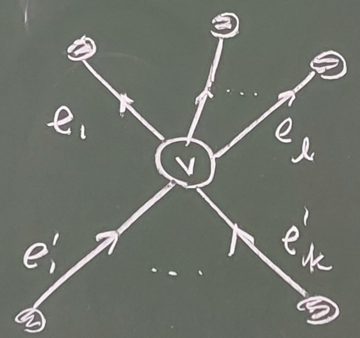
<u>DEF.</u> A gauge network $(A, \mathcal{H}, \cancel{D}^{\circ}, \iota \mid \phi, \rho)_Q$

consists of:

* a Quiver Q
* a map $Q_0 \xrightarrow[\xi]{} \mathrm{SpTr}^{\circ}$ & a map $Q_1 \xrightarrow{\phi} \hom_{*\text{-alg}}\left(A_{s(-)}, A_{t(-)}\right)$
* a map $Q_1 \xrightarrow[\rho]{} \mathrm{Rep}(G_\bullet)$, $\rho_e \in \mathrm{Rep}(G_e)$

  where $G_e := \{ u \in \mathcal{U}(\mathcal{H}_{t(e)}) : u A_{t(e)} u^* = A_{t(e)} \}$
* a map $\iota : Q_0 \to \mathrm{Inv}\left(\mathcal{U}(A_\bullet) \rtimes \mathrm{Sym}(A_\bullet)\right)$

  "act on blocks of $A_\bullet$ of identical size"
* $\iota_v : \bigotimes_{e' \in t^{-1}(v)} \rho_{e'} \to \bigotimes_{e \in s^{-1}(v)} \rho_e \circ \phi_e$



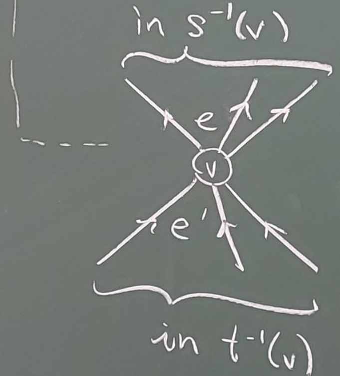
GAUGE NETWORK

Def ...... (for 5-min talk too long) Given a quiver Q

Heuristically: · put spectral triples on Q_0

· put repr. ρ_e of $G_e = \{ u \in \mathcal{U}(\mathcal{H}_{t(e)}) : u A_{t(e)} u^* = A_{t(e)} \}$ on edges $e \in Q_1$, as well as maps $A_{s(e)} \xrightarrow{\phi_e} A_{t(e)}$

· put intertwiners $\iota_v : \bigotimes_{e' \in t^{-1}(v)} \rho_{e'} \to \bigotimes_{e \in s^{-1}(v)} \rho_e \circ \phi_e$



I further develop gauge networks ...

GAUGE FOAMS?

... and (j.w. with L Glaser & S. Steinhaus) we elaborate on 'gauge foams' in NCG.

MORE TR-RELATED

... still NCG but more in context

- Path-int over (M, g) Riemannian manifold

$$\Downarrow$$

- Path-int over Dirac operators $(A, H, \cdot)$ fixed

$$Z_{\text{"Quantum gravity"}} = \int_{\text{Dirac}} e^{-\underbrace{\mathrm{Tr}[f(D)]}_{\text{spectral action w. } f(x)\to\infty \text{ as } |x|\to\infty}} \underbrace{dD}_{\text{makes sense if } D \text{ has finite rank}}$$

spectral action w. $f(x) \to \infty$ as $|x| \to \infty$

makes sense if D has finite rank

$\Downarrow$

multimatrix model of this type

QUESTION:

- TR for multimatrix models of wilder interactions ?

multitrace int.

$$\mathrm{Tr}_N(AABB) \times \mathrm{Tr}(ABAB)$$

e.g. two matrices $A, B \in (M_N^{\mathbb{C}})_{\text{s.a.}}$