Some philosophical aspects of the renormalization of gauge theories

John Dougherty
john.dougherty@lrz.uni-muenchen.de

Munich Center for Mathematical Philosophy

18 November 2021

Outline

1. The contemporary picture of applicability
2. This picture's problems with gauge theories
3. Changing the picture

The early modern applicability problem

Combine

1. Methodology: proofs, not experiments
2. Applicability: mathematical

A new strategy

Mathematics is about features of us: our capacities for producing scientific representations

Ups and downs

- ↘ Turn towards naturalism: science isn't about us
- ↗ Try again: math is about our concepts
- ↘ Set-theoretic paradoxes: math isn't about concepts
- ↗ Try again: math is about our linguistic framework
- ↘ Formal semantics is successful: math is about math

Today

Most important idea

Mathematics is about mathematical objects, which are Tarski models (sets equipped with structures) satisfying certain axioms

Newtonian electromagnetism

Classical action

$$S_A(x) = \int dt \left(\frac{1}{2} m\dot{x}^2 + q\dot{x}^\mu A_\mu \right)$$

Aharonov–Bohm

Semiclassical theory

$$\psi(x,t) = \int dx_0 \int \mathcal{D}x \, e^{\frac{i}{\hbar} S_A(x)} \psi(x_0, t_0)$$

Main problem

What is the configuration space?

1. Ω^1 is too spooky
2. What else could it be?

Constraints on $\mathcal{C}$

1. Must be a set equipped with some structure
2. Should factor $\Omega^1 \to \mathcal{C} \to \Omega^2$
3. Can't be a sheaf (AB effect)
4. Must work for QFT

Quantum Yang–Mills

Naively

$$Z = \int_{\Omega^1_{\mathfrak{g}}} \mathcal{D}A_\mu \, \exp\left(-\frac{i}{\hbar}\int d^4x \, \frac{1}{4}(F^a_{\mu\nu})^2\right) \sim \frac{1}{\sqrt{|\mathrm{Hess}\, S|}}$$

Faddeev–Popov ghosts

New fields

$$\mathcal{L} = -\frac{1}{4}(F^a_{\mu\nu})^2 - \frac{1}{2}(\partial^\mu A^a_\mu)^2 - \overline{c}_a \partial^\mu D_\mu c^a + \frac{1}{2}\left(b_a + \partial^\mu A^a_\mu\right)^2$$

An interpretation

Stacks

- Integrate over the moduli *stack* of connections
- Ghost "fields" are coordinates on the "arrow directions"

Derived geometry

- Use stationary phase to replace with integral over *derived* critical locus of the action
- "Antifields" are coordinates on the shifted cotangent space

Putting these together

A homotopical interpretation

- Stacks (homotopy sheaves) enable quotient by gauge transformations
- Derived critical locus is homotopic correction of critical locus

Problem

Derived stacks cannot be interpreted as Tarskian models in the right way.

Conclusion

1. Standard story about applicability: mathematical objects have a certain linguistic structure and are applied using this structure
2. Gauge theories are in tension with this story
3. Recent work applying homotopical methods to the renormalization of gauge theories can be used to argue against the standard picture.