Localization of Macroscopic Object Induced by the Factorization of Internal Adiabatic Motion

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Localization of Macroscopic Object Induced by the Factorization of Internal Adiabatic Motion

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Abstract

To account for the phenomenon of quantum decoherence of a macroscopic object, such as the localization and disappearance of interference, we invoke the adiabatic quantum entanglement between its collective states (such as that of the center-of-mass (C.M)) and its inner states based on our recent investigation. Under the adiabatic limit that motion of C.M dose not excite the transition of inner states, it is shown that the wave function of the macroscopic object can be written as an entangled state with correlation between adiabatic inner states and quasi-classical motion configurations of the C.M. Since the adiabatic inner states are factorized with respect to each parts composing the macroscopic object, this adiabatic separation can induce the quantum decoherence. This observation thus provides us with a possible solution to the Schroedinger cat paradox.

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I. INTRODUCTION

It is common sense that a macroscopic object should be localized in certain spatial domain. However, problem will appear if one directly use quantum mechanics to describe the motion of a free macroscopic object with spatial localization. This issue originated from the correspondence between Einstein and Born [1]. They observed that, in a spatially-localized state, generally a macroscopic object can only be described by a time-dependent localized wave packet, which is a coherent superposition of the eigen-states of the center-of-mass Hamiltonian $H_0 = \frac{p^2}{2M}$. If the macroscopic object is regarded as a heavy particle of a large mass $M$, its initial state $|\varphi\rangle$ should be a very narrow wave packet of width $a$. Since the wave packet spreads in evolution by the law

$$w(t) = a\sqrt{1 + \frac{t^2}{4M^2a^4}},$$

where $w(t)$ stands for the width of the wave packet, the spreading of an initially well localized wave packet can be reasonably ignored for very large mass. This seems to give a solution to the localization problem of the macroscopic object. But Einstein argued that the superposition of two narrow wave packets is no longer narrow with respect to the macro-coordinate, and on the other hand, it is still a possible state of the macroscopic object. So a contradiction to the superposition principle arises because of the requirement that the wave packet of a macroscopic object should be narrow [1].

To solve this problem, Wigner [2], Joos and Zeh [3] propose the so called scattering-induced-decoherence mechanism (or WJZ mechanism): scattering of photons and atoms off a macroscopic object records the information of its position to form a quantum measurement about the position. Then the interference terms between different paths of the macroscopic object are destroyed by the generalized "which-way" detection in association with scattering. In fact, in quantum measurement process, wave packet collapse (WPC, also called von Neumann’s projection) physically resembles the disappearance of interference pattern in Young’s two-slit experiment in the presence of a “which-way” detector. Associated with the wave-particle duality, this phenomenon of losing quantum coherence is referred to as quantum decoherence [4]: before a measurement to observe “which-way”
the particle actually takes, the quantum particle seems to move from one point to another along several different ways simultaneously. This just reflects the wave feature of a quantum particle. The detection of "which-way" means a probe for the particle feature, which leads to the disappearance of wave feature or quantum decoherence. Indeed, based on the Bragg's reflection of cold atoms and the electronic Aharonov-Bohm interference with a quantum point contact, the most recent experiments [5,6] shows that Schroedinger's concept of entangled state, rather than the unavoidable measurement distribution, is crucial for the wave-particle duality in this "which-way" detection. It is also pointed out that, similar gedenken experiments using photon and neutron have been considered before [7,8].

With these experiments and theory, it seems to be concluded that there does not exist the coherent superposition of states of a macroscopic object due to the quantum decoherence resulting from its coupling to an external environment as a generalized detector. But a natural question arises: If a macroscopic object, such as the famous Schroedinger cat, is completely isolated from any external environment, can its quantum coherence be maintained to realize macroscopic superposition states? To answer this question, one must consider the influences of the inner particles as the so-called "internal environment"[9] constituting the macroscopic object. Most recently, as a new way round the quantum coherence of macroscopic object, a novel experiment was presented to observe the matter wave interference of C_{60} molecules by diffraction at a material absorption grating [10]. For the purpose of observing quantum coherence this molecule is more massive than anything else that has been studied in this way before. As a large molecule at a high temperature, C_{60} contains atoms in continual motion which remains coherent while the molecule is passing through the slits. However, based on the above mentioned experiment, it might be possible to set up decoherence experiments so long as one can find a new way to effectively record the "which-way " information of C_{60}, which is manifested by the its radiated infrared photons. Actually, there still appears the coherent superposition of the macroscopic states in certain extreme cases such as in superconductivity and Bose-Einstein condensation [11], but these are not in our case since these macroscopically-quantum phenomenon must require each part of the macroscopic object has a same phase in evolution.
Motivated by these achievements both in theoretical and experimental aspects, we will show that the conception of adiabatic quantum entanglement (most recently proposed in refs.[12] based on the Born-Oppenheimer (BO) approximation) is mainly responsible for the decoherence phenomenon of the macroscopic object, such as the localization and disappearance of interference. This kind of quantum entanglement occurs between the states of the center-of-mass (C.M) of the macroscopic object and its inner states. In fact, when the motion of C.M does not excite the transition of inner states, the wave function of the macroscopic object can be adiabatically factorized with correlation between adiabatic inner states and quasi-classical motion configurations of the C.M. By this correlation or entanglement, the spatial localization of a macroscopic object can be explained and the dilemma of the Schroedinger cat can be resolved in a natural way.

II. ADIABATIC ENTANGLEMENT AND WJZ MECHANISM

In this section, based on the idea of the adiabatic entanglement, we incorporate the WJZ mechanism and the relevant studies in the dynamic theories of quantum measurement [13-16]developed by many people, including one (CPS) of the authors.

In the WJZ mechanism, the "which-way" information of the macroscopic object is recorded through the quantum entanglement formed by the scattering of atoms or photons forming so-called environment. Let $x$ and $q$ be, respectively, the collective position (C.M) of a macroscopic object and environment variables. The collective Hamiltonian is $H_s = \frac{p^2}{2M}$ and the canonical commutation relations are $[x, p] = i, [x, q] = 0$. To study how different positions of the macroscopic object entangle with environment (scattering atoms, photons, et.al.) , we suppose that the total system is initially in a product state

$$\Psi_x(t = 0) = |x\rangle \otimes |\phi\rangle \quad (2)$$

where the first component $|x\rangle$ is the eigen-state of the collective position operator $x$ while $|\phi\rangle$ is an arbitrary pure state of the environment. Usually, the collective motion acts on the environment in certain ways and the back-action of the environment can not be neglected physically for a large macroscopic object. So this generic interaction can not
produce an ideal entanglement between the collective position of the macroscopic object and the states of environment. By an argument by Joos and Zeh [4], one observes that only when the back-action is negligibly small, can the interaction between the collective and environment states realize a "measurement-like process":

$$|x\rangle \otimes |\phi\rangle \rightarrow U(t)|x\rangle \otimes |\phi\rangle = |x(t)\rangle \otimes S(x; t)|\phi\rangle$$  \hspace{1cm} (3)

Here, $U(t)$ is the total evolution matrix, $|x(t)\rangle = U_0(t)|x\rangle$ represents the free evolution in the absence of the coupling to the environment; and $S(x, t)$, acting on the environment states, denotes the effective $S - \text{matrix}$ parameterized by the collective position $x$ of the macroscopic object. If the collective motion is initially described by a wave packet $|\varphi\rangle = \int \varphi(x)|x\rangle dx$, then the continuous quantum entangling state

$$|\Phi\rangle = \int \varphi(x)|x(t)\rangle \otimes S(x; t)|\phi\rangle dx$$

defines the reduced density matrix

$$\rho(x, x', t) = \varphi(x, t) \varphi^*(x', t) \langle \phi|S^\dagger(x'; t)S(x; t)|\phi\rangle$$  \hspace{1cm} (4)

of the macroscopic object. Considering the translational invariance of the scattering process, Joos and Zeh showed that, the off-diagonal terms take the following form

$$F(x, x') = \langle \phi|S^\dagger(x'; t)S(x; t)|\phi\rangle \sim \exp(-\Lambda t|x - x'|^2).$$  \hspace{1cm} (5)

in $x - \text{representation}$. This means the decoherence factor is a damping function with the localization rate $\Lambda$, which depends on the total cross section.

In general, a quantum entangled state for a macroscopic object coupling to an external environment reads $|\Psi\rangle = \sum_n C_n|M_n\rangle \otimes |E_n\rangle$. It involves a correlation between the states $|M_n\rangle$ of the macroscopic object and the states $|E_n\rangle$ of the environment. The interference pattern

$$p(x) = \sum_m |\langle E_m, x |\Psi\rangle|^2 = \sum_n |C_n|^2 |M_n(x)|^2$$

$$+ \sum_{n \neq m} C^*_m C_n M^*_m(x)M_n(x)\langle E_m |E_n \rangle$$  \hspace{1cm} (6)
can be obtained from the total wave function $|\Psi\rangle$ by “summing over” all possible states of the environment. Here, $M_n(x) = \langle x|M_n \rangle$ is the state of the macroscopic object in the position representation. The second term on the r.h.s of the above equation is responsible for the interference pattern of the macroscopic object. It is easy to see that the interference fringes completely vanish when the states of the environment are orthogonal to one another, i.e., when $\langle E_m|E_n \rangle = \delta_{m,n}$. In this situation, an ideal “which-way” detection results from the ideal entanglement, in which one can distinguish the states of environment very well. Our previous works on quantum measurement theory [15,16] also show that an ideal entanglement may appear in the macroscopic limit that the number $N$ of particles making up the environment approaches infinity. It was found that the factorization structure in the overlap integral $F_{m,n} = \langle E_m|E_n \rangle \equiv \prod_{i=1}^{N} \langle E_{m,i}^{[i]}|E_{n,i}^{[i]} \rangle$, where $F_{m,n}$ are the overlapping of environment states, or the decoherence factor, plays the main part in the quantum decoherence. Here, $|E_{n,i}^{[i]}\rangle$ are the single states of those blocks constituting the environment. Since each factor $\langle E_{m,i}^{[i]}|E_{n,i}^{[i]} \rangle$ in $F_{m,n}$ has a norm less than unity, the product of infinite such factors may approach zero. This investigation was developed based on the Hepp-Coleman model[13].

Now we consider a macroscopic object with collective and internal variables, say $x$ and $q$. Applying the above discussion one easily sees that the interaction between these two kinds of variables may lead to an ideal quantum entanglement between the collective and internal states, when the collective states are free of the back-action. This conception is initiated by Onnes with the so-called ”internal environment” naming the system of internal variables [9]. But the question is whether the negligibility of the back-action is the unique cause for the appearance of the above mentioned "measurement-like process". If not, what are the other causes beyond it? To resolve this problem, we invoke the BO approximation to adiabatically separate the collective and internal variables. Assume that the total Hamiltonian is $H = \frac{q^2}{2M} + h(q, x)$, where the Hamiltonian $h(q, x)$ describes the motion of the internal variables $q$ coupling to the collective variable $x$. For a fixed value of the slow variable $x$, the eigen-state $|n[x]\rangle$ and the corresponding eigen-values $V_n[x]$ are
determined by the eigen-equation

$$h(q, x)|n[x]\rangle = V_n(x)|n[x]\rangle. \quad (7)$$

Regarding $x$ and $q$ as the slow and fast variables respectively in the BO adiabatic approach, we approximately obtain the complete set \{\langle n|n, \alpha \rangle \equiv \phi_{n, \alpha}(x)|n[x]\rangle\} of eigenstates of the total system, where $\phi_{n, \alpha}(x)$ come from the eigen-equation

$$H_n \phi_{n, \alpha}(x) = E_{n, \alpha} \phi_{n, \alpha}(x) \quad (8)$$

and

$$H_n = \frac{p^2}{M} + V_n[x] \quad (9)$$

is the effective Hamiltonian associated with the internal state $|n[x]\rangle$. Here, we do not consider the induced gauge potential connected with Berry phase factor through the quantum adiabatic method \cite{17,18}. Then, we can see how the “measurement-like process” naturally appears as a result of the adiabatic dynamic evolution.

In fact, under the BO approximation, we can expand the factorized initial state $|\Psi(0)\rangle = |x\rangle \otimes |\phi\rangle$ in terms of the adiabatic basis $\{|n, \alpha\rangle\}$ and then we obtain the total wave function\cite{12}

$$|\Psi(t)\rangle = \sum_n \langle n|x\rangle \langle x| \phi \rangle \int dx' K(x', x, t) |x'\rangle \otimes |n[x']\rangle \quad (10)$$

where we have used the completeness relations for the full eigen-functions expressed in $x$ - representation and $K(x', x, t) = \langle x'| e^{-iH_n t} |x\rangle$. Generally, the propagator $K(x', x, t)$ is not diagonal for $|x\rangle$ is not an eigen-state of $H_n$ and then $|\Psi(t)\rangle$ can defines an ideal entanglement state. However, for the large mass $M$, we can prove that, to the first order approximation, $K(x', x, t)$ takes a diagonal form proportional to a $\delta$ - function. Actually, in the large limit, the kinetic term $p^2/2M$ can be regarded as a perturbation in comparison with the effective potential $V_n(x)$. Using Dyson expansion to the first order of $\frac{1}{M}$, we have

$$e^{-iH_n t} = e^{-iV_n t} \left(1 - i \int_0^t e^{iV_n t'} \frac{p^2}{2M} e^{-iV_n t'} dt' + \cdots \right)$$
\[ e^{-iV_n t} \left( 1 - \frac{i p^2 t^2}{2M} + i \frac{t^2}{4M} (p \frac{\partial}{\partial x} V_n + [\partial_x V_n] p) - \frac{i t^3 \partial_x V_n^2}{6M} + \ldots \right) \]  

(11)

Since

\[ \int \langle x' | P^n | x \rangle f(x') dx = 0 \]

for \( n=1,2,\ldots \), we conclude that

\[ K(x', x, t) = e^{-iV_n [x]'} \delta(x - x') + \frac{i}{2M} \int_0^t d\tau e^{-iV_n [x]' \tau} \frac{\partial^2}{\partial x'^2} \delta(x - x') e^{iV_n (x) \tau}. \]  

(12)

Then, we show that if is approximately diagonalized: \( K(x', x, t) = e^{-iH_n (x)' \delta(x - x')} \), the adiabatic wave-function can lead to an ideal entanglement

\[ |\Psi(t)\rangle = \sum_n \langle n [x] | \phi \rangle e^{-iH_n (x)' \delta(x - x')} \otimes |n [x]\rangle \]  

(13)

We call this entanglement adiabatic entanglement.

In conclusion, up to the first order approximation of \( \frac{1}{M} \), we have the quantum entanglement in adiabatic evolution. The Born- Oppenheimer adiabatic approximation has provided us with a novel mechanism to produce the quantum entanglement between the macroscopic object and its internal variables.

### III. LOCALIZATION INDUCED BY FACTORIZED INTERNAL MOTION

We notice the above simple result has the following physical explanation: the evolution state of a heavy particle for very large \( M \), which is almost steady, is approximately an eigenstate of the position operator if it is initially in a state with a fixed position. Then, it follows from eq.(13) that, in the large-mass limit, the wave function \( |\Psi(t)\rangle \) can be factorized approximately: \( |\Psi(t)\rangle = |x\rangle \otimes S(x,t) |\phi\rangle \) where the entangling \( S - matrices \)

\[ S(x, t) = \sum_n e^{-iV_n t} |n [x]\rangle \langle n [x]| \]  

(14)

are defined in terms of the adiabatic projection \( |n [x]\rangle \langle n [x]| \).

According to our previous argument about the factorized structure of \( S - matrix \) in the dynamic theory of quantum measurement \([11,12]\), if the internal degree of freedom has
many components, e.g., if \( q = (q_1, q_2, \ldots, q_N) \), then in their normal non-interaction modes, 
\( S(x; t) \) can be factorized as:

\[
S(x; t) = \prod_{j=1}^{N} S_j(x; t)
\]

(15)

with

\[
S_j(x; t) = e^{-i h_j(q_j, x)t}
\]

(16)

with \( h(q, x) = \sum_j h_j(q_j, x) \) and \( h_j(q_j, x) \) are the single particle Hamiltonians of the parts of 
the macroscopic object. Of course, in the derivation of the above factorized structure for 
the \( S \- \mat \), we have made some simplifications. Roughly speaking, we have assumed 
that the adiabatic effective potential takes the form of direct sum \( V_n = \sum_j V_{n_j}(q_j) \), and 
the eigenstate the form of direct product

\[
|n[x]\rangle = \prod_{j=1}^{N} \otimes |n_j[x]\rangle
\]

(17)

neglecting the higher order terms \( \approx O(\frac{1}{N}) \).

For the initial state \( |\phi\rangle = \prod_{j=1}^{N} \otimes |\phi_j\rangle \) factorized with respect to internal components, 
the reduced density matrix

\[
\rho(x, x', t) = \varphi(x)\varphi^*(x')F_N(x', x, t):
\]

(18)

can be re-written in terms of the so called decoherence factor

\[
F_N(x', x, t) = \prod_{j=1}^{N} F_{[j]}(x', x, t) \equiv \prod_{j=1}^{N} \langle \phi_j | S_{v_j}^+ (x'; t)S_{v_j}(x; t) | \phi_j \rangle.
\]

(19)

This factor is expressed as an \( N \)-multiple product of the single decoherring factors

\[
F_{[j]}(x, x') = \langle \phi_j | S_{v_j}^+ (x'; t)S_{v_j}(x; t) | \phi_j \rangle
\]

(20)

with norms less than unity. Thus in the macroscopic limit \( N \to \infty \), it is possible that 
\( F_N(x', x, t) \to 0 \), for \( x' \neq x \). In fact, this factor reflects almost all the dynamic features 
of the influence of the fast part on the slow part. Physically, an infinite \( N \) means that 
the object is macroscopic since it is made of infinite number of particles in that case. 
On the other hand, the happening of decoherence at infinite \( N \) manifests a transition of
the object from the quantum realm to the classical realm. Here, as expected, the physical picture is consistent.

As to the localization problem raised by Einstein and Born [1], we, based on the above argument, comment that one can formally write down the wave function of a macroscopic object as an narrow pure state wave packet, but it is not the whole of a real story. Actually, the statement that an object is macroscopic should physically imply that it contains many particles. So a physically correct description of its state must concern its internal motions coupling to the collective coordinates (e.g., its center-of-mass). Usually, one observe this collective coordinate to determine whether two spatially-localized wave packets can interfere with each other. If there does not exist such interference, one may say that, the superposition of two narrow wave packets for the macro-coordinate is no longer a possible pure state of the macroscopic object. Indeed, because the “which-way” information of the macro-coordinate is recorded by the internal motions of particles making up the macroscopic object, the induced decoherence must destruct the coherence in the original superposition so that the state of the macroscopic object is no longer pure.

The present argument also provides a possible solution for the Schröedinger cat paradox. If we consider the Schröedinger cat as a macroscopic object consisting of many internal particles, then we can never observe anything corresponding to the interference between the dead and the living cats. This is because the macroscopically-dead and the macroscopically-living states, \( |D \rangle \) and \( |L \rangle \), of the cat are correlated to the corresponding internal states, \( |d_j \rangle \) and \( |l_j \rangle \). The cat state

\[
|\text{Cat} \rangle = |L \rangle \otimes \prod_{j=1}^{N} |l_j \rangle + |D \rangle \otimes \prod_{j=1}^{N} |d_j \rangle
\]  

(21)

follows from the argument this section when \( |D \rangle \) and \( |L \rangle \) are regarded as the collective states while \( \prod_{j=1}^{N} |l_j \rangle \) and \( \prod_{j=1}^{N} |d_j \rangle \) describes the corresponding internal motion. It leads to a reduced density matrix with the off-diagonal elements proportional to \( \prod_{j=1}^{N} \langle d_j | l_j \rangle \). Once there is only one pair of inner states are orthogonal, the off-diagonal elements vanish and decoherence happen. Even though there does not exists any pair of inner states orthogonal with each other, for the norm of \( \langle d_j | l_j \rangle \) less than unity, it is also possible that
\[ \prod_{j=1}^{N} \langle d_j | l_j \rangle \to 0 \] in the macroscopic limit \( N \to \infty \). In this sense, we conclude that the Schrödinger cat paradox is not a paradox at all in practice. Rather, it essentially arises from overlooking the internal motions of a macroscopic cat or the multi-particle scattering off it.

Now, we have shown that the localization phenomena of a macroscopic object can boil down to an entanglement between its collective position (or C.M) and internal variables in the adiabatic evolution with the above mentioned factorization structure. Closely related to the Schrödinger cat phenomenon, this entanglement results from their adiabatic separation of collective and internal variables. With the point view from the above theoretical analysis, as for the \( C_{60} \) molecule interference experiment. To our surprise, it turns out that an elegant interference pattern appears in the experiment. But there is no contradiction here. Truly, at high temperature \( C_{60} \) would emit two or three infrared photons during its passage through the apparatus. But as the wavelength of the radiating photons from the internal motion of \( C_{60} \) is much greater than the distance between the neighboring slits, the photons carry no information about the route the molecule takes. In this sense, \( C_{60} \) can not well be considered as a macroscopic object since its internal variable is almost frizzed. Therefore the interference pattern is not affected. Similarly, though there exists interaction with the external air particles, the scattering rates on the macroscopic object are far too small to induce quantum decoherence. This explains the persistence of interference pattern in the experiment [10]. However, we can imagine that in such experiments, the internal motion (such as radiation of photons of various frequencies) produces an effective coupling with the collective motion of the C.M. Then, the configurations of internal motion can record the "which-way" information even through a single thermal photon so that the interference contrast should thus be completely destroyed. Moreover, the parameters (such as the internal temperature of the fullerene, the temperature of the environment, the intensity and frequency of external laser radiation) can be controlled continuously so that quantitative natures such as those described in this paper could be tested. The study in the present paper is not directly applicable to such "which-way" experiments because it is based on the assumption that the macroscopic
object is composed of two-level subsystems and does not concern the concrete structure of C_{60} fullerenes. Nevertheless, for the quantitative investigation of the dynamic details of decoherence process in such experiments, it can serve as a starting point.

IV. SIMPLE MODEL FOR MACROSCOPIC LOCALIZATION

To make a deeper elucidation of the above general arguments about the localization of a macroscopic object of mass $M$, we model the macroscopic object as consisting of $N$ two level particles, which are fixed at certain positions to form a whole without internal spatial motion. The collective position $x$ is taken to be its mass-center or any reference position in it while the internal variables are taken to be the quasi-spins associated with two level particles. Generally, if we assume that the back-action of the internal variables on the collective position is relatively small, the model Hamiltonian can be written as

$$H = \frac{p^2}{2M} + h(x):$$

$$h(x) = \sum_{j=1}^{N} (f_{j}(x)|e_j\rangle\langle g_j| + f_{j}^{*}(x)|g_j\rangle\langle e_j|) + \sum_{j=1}^{N} \omega_j(|e_j\rangle\langle e_j| - |g_j\rangle\langle g_j|)$$

(22)

where $|g_j\rangle$ and $|e_j\rangle$ are the ground and the excited states of the $j$’th particle and $f_{j}(x)$ denotes the position-dependent couplings of the collective variable to the internal variables. Let $l_j$ be the relative distance between the $j$’th particle and the reference position $x$. Further we assume $f_{j}(x) = f(x + l_j)$. Physically, we may think that these couplings are induced by an inhomogeneous external field, e.g., they may be the electric dipole couplings of two-level atoms in an inhomogeneous electric field.

We remark that the above model enjoys some universality under certain conditions, compared with various environment models inducing both dissipation and decoherence of quantum processes. In fact, Caldeira and Leggett [19] have pointed out that any environment weakly coupling to a system may be approximated by a bath of oscillators under the condition that “each environmental degree of freedom is only weakly perturbed by its interaction with the system”. We observe that any linear coupling only involves
transitions between the lowest two levels (ground state and the first excitation state) of each harmonic oscillator in the perturbation approach though it has many energy levels. Therefore in such a case we can also describe the environment as a combination of many two level subsystems without losing generality \cite{20}. To some extent, these arguments justify our choosing the two level subsystems to model the internal motion of the macroscopic object. We will soon see its advantage: the character of localization can be manifested naturally and clearly.

Now let us calculate the $S_j(x; t)$ for this concrete model. The single-particle Hamiltonian $h_j(x) = \omega_j |\langle \epsilon_j | - | g_j \rangle |\langle g_j | + (f_j(x)|\epsilon_j|\langle g_j | + h.c)$ has the $x$-dependent eigenvalues

$$V_{j x} = n \Omega_j(x) \equiv \pm \sqrt{|f_j(x)|^2 + \omega_j^2} \quad (n = \pm)$$

and the corresponding eigen-vectors $|n_j[x]|$ are

$$| + j [x] \rangle = \cos \frac{\theta_j}{2} |\epsilon_j \rangle + \sin \frac{\theta_j}{2} |g_j \rangle,$$  

$$| - j [x] \rangle = \sin \frac{\theta_j}{2} |\epsilon_j \rangle - \cos \frac{\theta_j}{2} |g_j \rangle,$$

where $\tan \theta_j = \frac{f_j(x)}{\omega_j}$. Explicitly, the corresponding single-particle $S - m a t r i x$

$$S_j(x; t) = \begin{pmatrix} \cos(\Omega_j t) - i \sin(\Omega_j t) \cos \theta_j, & i \sin(\Omega_j t) \sin \theta_j \\ i \sin(\Omega_j t) \sin \theta_j, & \cos(\Omega_j t) + i \sin(\Omega_j t) \cos \theta_j \end{pmatrix}$$

Here in the derivation we have used the formula

$$\exp[i \vec{\sigma} \cdot \vec{A}] = \cos A + i \vec{\sigma} \cdot \vec{n} \sin A$$

for a given vector $\vec{A}$ of norm $A$ along the direction $\vec{n}$. Having obtained the above analytic results about $S - m a t r i x$, we can further calculate the single-particle decoherence factors $F[\psi(x', x, t) \equiv \langle g_i | S_j(x; t) S_j(x; t) | g_i \rangle$ for a given initial state $\langle \phi \rangle = \prod_{j=1}^{N} \otimes |g_j \rangle$. For simplicity we use the notation $f(x') = f'$. We have

$$F[\psi(x', x, t) = \{ \sin(\Omega_j' t) \sin \theta_j' \sin(\Omega_j t) \sin \theta_j + \cos(\Omega_j' t) \cos(\Omega_j t) + \sin(\Omega_j' t) \cos \theta_j' \sin(\Omega_j t) \cos \theta_j \cos \theta_j$$

$$(28)$$
\[ +i \{ \cos(\Omega_j t) \sin(\Omega_j t) \cos \theta_j - \sin(\Omega_j t) \cos \theta_j \cos(\Omega_j t) \} \]

In the weakly coupling limit, \( g_j \ll \omega_j \) and the coupling \( f_j \approx g_j x \), thus we have \( \sin \theta_j \approx \theta_j \approx \frac{f_j}{\omega_j}, \cos \theta_j \approx 1 - \frac{1}{2} \theta_j^2 \) and \( \Omega_j \approx \omega_j \). Then, the decohering factors can be simplified as

\[ F[1](x', x, t) \approx 1 - (x - x')^2 \left\{ \frac{|g_j|^2}{2\omega_j^2} \sin^2(\omega_j t) \right\} + \left( \frac{1}{4\omega_j^2} (x^2 - x'^2) \sin(2\omega_j t) \right) \]

Consequently, the temporal behavior of the decoherence is determined by

\[ F(x', x, t) = |F(x', x, t)| \exp \left( \frac{1}{4\omega_j^2} (x^2 - x'^2) \sin(2\omega_j t) \right) \]

where

\[ |F(x', x, t)| = \exp \left( - (x - x')^2 \frac{|g_j|^2}{2\omega_j^2} \sin^2(\omega_j t) \right) \]

In the case of continuous spectrum, the sum

\[ R(t) = \sum_{j=1}^{N} \frac{g_j^2}{2\omega_j^2} \sin^2(\omega_j t) \]

can be re-expressed in terms of a spectrum distribution \( \rho(\omega_k) \) as

\[ R(t) = \int_0^{\infty} \frac{\rho(\omega_k) g_k^2}{2\omega_k^2} \sin^2(\omega_k) d\omega_k. \]

\[ \text{From some concrete spectrum distributions, interesting circumstances may arise. For instance, when } \rho(\omega_k) = \frac{1}{\pi} \gamma / g_k^2 \text{ the integral converges to a negative number proportional to time } t, \text{ precisely, } R(t) = \gamma t. \text{Therefore, our analysis recovers the result} \]

\[ \rho(x, x', t) = \varphi(x) \varphi^*(x') e^{-\gamma(x-x')^2} \exp[i \pi (x^2 - x'^2)s(t)] \]

for the reduced density matrix of the macroscopic object, which was obtained by Joos and Zeh [3] through the multi-particle external scattering mechanism and by Zurek separately through Markov master equation. Here,

\[ s(t) = \sum_{j=1}^{N} \sin(2\omega_j t) \frac{1}{4\pi \omega_j^2} \]

is a time-dependent periodic function. This shows that the norm of the decoherence factor is exponentially decaying and as \( t \to \infty \), the off-diagonal elements of the density matrix vanish simultaneously!
We will show that for quite general distribution $\rho(\omega)$ the off-diagonal elements of the reduced density matrix decline rather sharply with time $t$ if the particle number $N$ is large. Assume that all $g_j$ are equal: $g_j = g$. If the frequencies lie within an interval $[\omega_1, \omega_2]$ and the distribution is homogeneous, we have $\rho(\omega) = N/(\omega_2 - \omega_1)$. Then

$$R(t) = \int_{\omega_1}^{\omega_2} \frac{g^2}{2\omega^2} \sin^2 \omega t \rho(\omega) d\omega$$

$$= \frac{N}{(\omega_2 - \omega_1)} \frac{g^2}{2} \int_{\omega_1}^{\omega_2} \frac{1}{\omega^2} \sin^2 \omega t d\omega$$

$$\geq \frac{N}{(\omega_2 - \omega_1)} \frac{g^2}{2\omega_2^2} \int_{\omega_1}^{\omega_2} \sin^2 \omega t d\omega$$

$$= \frac{Ng^2}{4\omega_2^2} \left( 1 - \cos(\omega_2 + \omega_1) t \frac{\sin(\omega_2 - \omega_1) t}{(\omega_2 - \omega_1) t} \right)$$  \hspace{1cm} (35)$$

For a general $\rho(\omega)$ in the interval $[\omega_1, \omega_2]$, we have

$$\int_{\omega_1}^{\omega_2} \rho(\omega) d\omega = N.$$ 

Then there exists some $\bar{\omega}$ in $[\omega_1, \omega_2]$ such that

$$\rho(\bar{\omega}) = \frac{N}{\omega_2 - \omega_1}$$

If the frequency spectrum of the system is such that there exist $\omega_3$ and $\omega_4$ in the interval $[\omega_1, \omega_2]$ satisfying

$$\rho(\omega) \geq \frac{N}{\omega_2 - \omega_1} \text{ for } \omega_3 \leq \omega \leq \omega_4.$$  \hspace{1cm} (36)$$

From the derivation of (44) it then follows that

$$R(t) \geq \frac{Ng^2}{4\omega_4^2} \frac{\omega_4 - \omega_3}{\omega_2 - \omega_1} \left( 1 - \cos(\omega_4 + \omega_3) t \frac{\sin(\omega_4 - \omega_3) t}{(\omega_4 - \omega_3) t} \right)$$ \hspace{1cm} (37)$$

After a moment’s thought, one can easily convince oneself that the condition (36) is rather easy to satisfy. From the inequality (37) we observe that although in the weakly coupling limit, we should have $\frac{\omega_3^2}{\omega_4^2} \ll 1$, $R(t)$ can increase sharply with time $t$ if the particle number is large enough. This just means that the off-diagonal elements of the reduced density matrix will decline sharply with time $t$. In conclusion, despite the complexity of $\rho(x, x', t)$ due to the presence of the oscillating factor $s(t)$, in many cases it can well describe the decoherence of macroscopic object thanks to its simple decaying norm.
Let us now turn to consider an example similar to that studied by Joos and Zeh. We take a coherent superposition of two Gaussian wave packets of width \(d\)

\[
\varphi(x) = \frac{1}{\sqrt{8\pi d^2}} \left\{ \exp \left( -\frac{(x-a)^2}{4d^2} \right) + \exp \left( -\frac{(x+a)^2}{4d^2} \right) \right\}
\]

The norm of the corresponding reduced density matrix

\[
|\rho(x, x', t)| = \sum_{k,l=0}^1 P_{kl}(x, x', t)
\]

contains 4 peaks:

\[
P_{11}(x, x', t) = \frac{1}{\sqrt{8\pi d^2}} e^{-\gamma(x-x')^2} \exp[-\frac{(x-a)^2}{4d^2} - \frac{(x'-a)^2}{4d^2}],
\]

\[
P_{10}(x, x', t) = \frac{1}{\sqrt{8\pi d^2}} e^{-\gamma(x-x')^2} \exp[-\frac{(x-a)^2}{4d^2} - \frac{(x'+a)^2}{4d^2}],
\]

\[
P_{01}(x, x', t) = \frac{1}{\sqrt{8\pi d^2}} e^{-\gamma(x-x')^2} \exp[-\frac{(x+a)^2}{4d^2} - \frac{(x'-a)^2}{4d^2}],
\]

\[
P_{00}(x, x', t) = \frac{1}{\sqrt{8\pi d^2}} e^{-\gamma(x-x')^2} \exp[-\frac{(x+a)^2}{4d^2} - \frac{(x'+a)^2}{4d^2}],
\]

centering respectively around the points \((a, a), (a, -a), (-a, a)\) and \((-a, -a)\) in \(x - x'\)-plane. The heights are respectively \(1/\sqrt{8\pi d^2}, e^{-4\gamma a^2}/\sqrt{8\pi d^2}, e^{-4\gamma a^2}/\sqrt{8\pi d^2}\) and \(1/\sqrt{8\pi d^2}\). Obviously, two peaks with centers at \((a, -a)\) and \((-a, a)\) decay with time while the other two keep their heights constant. Fig.1. shows this time-dependent configuration at \(t=0\), and a finite \(t\). As \(t \to \infty\), two off-diagonal terms \(P_{10}\) and \(P_{01}\) decay to zero so that the interference of the two Gaussian wave packets are destroyed. In this sense, we say that the pure state \(\rho(x, x', t = 0) = \int dx \varphi(x) \varphi^* (x') |x \rangle \langle x'|\) becomes a mixed state

\[
\rho(t) = \int dx \varphi(x) \varphi^*(x) |x \rangle \langle x|
\]

in \(x\)-representation.

Interference of two plane waves of wave vector \(k_1, k_2\) provides us another simplest example. Without decoherence induced by its internal motions or the external scattering, their coherent superposition \(\varphi(x) = \sqrt{\frac{1}{4\pi}} [e^{ik_1 x} + e^{ik_2 x}]\) yields a spatial interference described by the reduced density matrix

\[
\rho_0(x, x', t) = \frac{1}{4\pi} \{e^{ik_1 (x-x')} + e^{ik_2 (x-x')} +
\]

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\[
\exp[i\left(\frac{k_2^2 t - k_2^2 l}{2m} + k_2 x - k_1 x'\right)] + \exp[i\left(\frac{k_2^2 t - k_1^2 l}{2m} + k_1 x - k_2 x'\right)]
\]

(42)

Under the influence of internal motions, it becomes

\[
\rho(x, x', t) \approx \rho_0(x, x', t)e^{-\gamma t (x-x')^2}
\]

for large mass. We see that the difference created by decoherence is only reflected in the off-diagonal elements, and the pure decoherence (without dissipation) does not destroy the interference pattern described by the diagonal term \(\rho(x, x, t) = \rho_0(x, x, t)\). This simple illustration tells us that the present quantum decoherence mechanism may not have to do with the interference pattern of the first order coherence, but it does destroy the higher
order quantum coherence: $\rho(x, x', t) \rightarrow 0$ as $t \rightarrow \infty$. In fact, due to the induced loss of energy, quantum dissipation is responsible for the disappearance of the interference pattern of the first order coherence. The influences of internal motions or external scattering on the decoherence of a macroscopic object may be very complicated. Intuitively, these dynamic effects should depend on the details of interaction between the collective variables and the internal and external degrees of freedom. Practically, we can classify these influences into two categories, namely, quantum dissipation and quantum decoherence, and then study them separately by different models.

V. CONCLUDING REMARKS

It is noticed that, so long as the “which-way” information of the collective motion of a macroscopic object already stored in the internal motion could be read out, the phenomenon caused by interference would be destroyed without any data being read out in practice [5,13-16]. In this sense the internal degrees of freedom interacting with the macroscopic object behaves as a detector to realize a “measurement-like” process. Thus, the internal motion configuration is imagined as an objective detector detecting the collective states. Provide the internal motion configuration couples with the collective motion and produce an ideal entanglement, the collective motion must lose its coherence. It is worthy to point out that this simple entanglement conserves the energy of the collective motion while destroying the quantum coherence.

In the case with no energy conservation, the quantum dissipation can also induce the localization of the macroscopic object. Based on the studies of quantum dissipation stimulated by Caldeira and Leggett [18], Yu and one (C.P.S) of the authors found a novel mechanism which sheds new light on the localization problem of macroscopic object [21]. They studied the quantum dynamics of a simplest dissipative system: one particle moving in a constant external field and interacting with a bath of harmonic oscillators with Ohmic spectral density. It was found that the wave function of the total system can be factorized
as a product of the system part and the bath part. When one ignores the effect of Brownian motion or the quantum fluctuation in the system caused by the bath, the product wave function becomes a direct product and the dissipative evolution of the system is governed by Caldirola-Kani (CK) Hamiltonian. Using this effective Hamiltonian, they discovered the following interesting result: the dissipation suppresses the wave packet spreading and cause the localization of the wave packet. Actually, it was shown that the breadth of the wave packet changes with time $t$ in the following way: $w(t) = a \sqrt{1 + \frac{\gamma_t}{4\gamma^2}}$. Here $a$ is the initial breadth of the wave packet and $t_\eta = \frac{M(1-e^{-\eta/M})}{\eta}$, where $\eta$ is the damping rate. Comparing this formula with the equation (1), we find that the effect of the influence of the bath is the replacement of $t$ by $t_\eta$ in (1). We have $t_\eta \to t$ when $\eta/M \to 0$. So one can regard $t_\eta$ as a deformation of time $t$ caused by dissipation. Notice that $t_\eta$ approaches the limit $\frac{M}{\eta}$ as $t \to \infty$. This means localization of the wave packet in the presence of dissipation. Indeed, we have the limit breadth: $a_{limit} = a \sqrt{1 + (1/2\gamma a^2)^2}$. This suppression of the wave packet spreading by dissipation possibly provides a useful mechanism for the localization of quantum particle. It is a little bit surprising that the limit width of the damped particle wave packet as $t \to \infty$ is exactly the same as the “uncertainty product” of the damped particle, established by Schuch et al. through nonlinear Schroedinger equation [22].

Summarily, the environment induced dissipation as well as decoherence can provide an important mechanism for the localization of a macroscopic object. Mentioning macroscopicness implies the requirement that the macroscopic object must contain a large number of internal blocks. Then the macroscopic object, coupled to the internal variables, should be described by collective variables subject to an interaction similar to that concerning the external scattering in WJZ mechanism and the quantum dissipation of a particle in a bath.
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