A Note on D–Branes in Group Manifolds:
Flux Quantisation and D0–Charge

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A NOTE ON D-BRANES IN GROUP MANIFOLDS: FLUX QUANTISATION AND D0-CHARGE

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Abstract. We show that a D-brane in a group manifold given by a (twisted) conjugacy class is characterised by a gauge invariant two-form field determined in terms of the matrix of gluing conditions. Using a quantisation argument based on the path integral one obtains the known quantisation condition for the corresponding D-branes. We find no evidence for the quantisation of the U(1) flux. We propose an expression for the D0 charge of such D-branes.

1. Introduction

Recently the issues of D0 charge and U(1) flux quantisation for a class of D-branes in the SU(2) group manifold have attracted a great deal of attention [1, 2, 3, 4, 5]. Our aim here is to show, using a somewhat different line of argument, that the same basic results can be obtained without any reference to a hypothetical quantisation of U(1) gauge field or, indeed, of its flux.

In this letter we show, using the formalism developed in [6], that a D-brane in a group manifold sitting on a (twisted) conjugacy class \(\mathcal{C}\), and described, in the framework of the boundary state approach, by the matrix \(R\) of gluing conditions is characterised by a gauge invariant two-form field \(\omega\) defined on the worldvolume of the D-brane whose components are determined by \(R\). By comparing the boundary conditions coming from the gluing conditions with the ones deduced from the classical sigma model action, we are able to identify this two-form field \(\omega\) with the gauge-invariant combination \(B + 2\pi \alpha' F\).

In order to write the boundary WZW action in terms of the three-form \(H\) and the two-form \(\omega\) one is forced to introduce, much as in the case of the standard WZW model, a certain field extension \(\tilde{g}\). The requirement that the quantum theory be independent of the choice of field extension imposes two quantisation conditions [7, 8]: the first one, imposed on \(H\) alone and similar to the closed string case, is an integrality condition on the cohomology class of \(H\) in \(H^3(\mathbf{G})\); whereas the second is that the periods of \(\langle H, \omega \rangle\) over cycles in the relative homology \(H_3(\mathbf{G}, \mathcal{C})\) take integer values. In the case of SU(2) at level \(k\), these conditions yield \([9]\) \(k + 1\) D-branes: two point-like D-branes.
situated at the two elements in the centre of SU(2), and \( k - 1 \) spherical D2-branes.

A closer look at this quantisation condition suggests a natural definition for the D0-brane charge of a given D-brane, which reads

\[
Q_0 = k T \left( \int_{\mathcal{C}} \tilde{g}^* \omega - \int_{\mathcal{B}} \tilde{g}^* H \right),
\]

where \( \mathcal{B} \) is a three-manifold such that \( \partial \mathcal{B} = \mathcal{C} \). This quantity is naturally gauge invariant and quantised (with integer values), independent of any assumption regarding the U(1) gauge field. In the particular case of the D2-branes in SU(2), the \( H \) field contribution is similar to the Poynting-type bulk contribution advocated in [3], which is valid also when \( H \) belongs to a nontrivial cohomology class. This quantity can be thought of as a generalisation of the U(1) flux in the case where the three-form field \( H \) belongs to a nontrivial cohomology class.

2. **Semi-classical analysis**

2.1. **Boundary conditions from the boundary state approach.**

We consider D-branes in a group \( \mathbf{G} \) which preserve conformal invariance and the infinite-dimensional symmetry of the current algebra of the bulk theory. They are described in terms of the following gluing conditions:

\[
J = R\bar{J},
\]

where the matrix of gluing conditions \( R : \mathfrak{g} \to \mathfrak{g} \) is a metric-preserving automorphism of the Lie algebra \( \mathfrak{g} \); that is,

\[
[R(T_a), R(T_b)] = R([T_a, T_b]),
\]

and

\[
R^T \Omega T = \Omega,
\]

in the obvious notation. This type of gluing conditions describe [9, 10, 6] D-branes whose worldvolumes lie on twisted conjugacy classes. More precisely, D-branes in a WZW model with group \( \mathbf{G} \) come in several types, classified [11] by the group \( \text{Out}_c(\mathbf{G}) \) of metric-preserving outer automorphisms of \( \mathbf{G} \), which is defined as the quotient \( \text{Out}_c(\mathbf{G})/\text{Inn}_c(\mathbf{G}) \) of the group of metric-preserving automorphisms by the invariant subgroup of inner automorphisms.

In [6] it was shown that the above gluing conditions give rise, at a given point \( g \) in \( \mathbf{G} \), to the following boundary conditions

\[
\partial g = R(g) \bar{\partial} g,
\]

where the map \( R(g) : T_g \mathbf{G} \to T_g \mathbf{G} \) is defined as

\[
R(g) = -(\rho_g)_* \circ R \circ (\lambda_g)^{-1}.
\]

For the purposes of this paper it will be convenient to write the above boundary conditions in a different form. We therefore parametrise \( \mathbf{G} \)
by introducing the coordinates $X^\mu$, with $\mu = 1, \ldots, \dim G$; we also introduce the left- and right-invariant vielbeins
\[ g^{-1} dg = \varepsilon^a\mu \, dX^\mu T_a \quad \text{and} \quad dg g^{-1} = \varepsilon^a\mu \, dX^\mu T_a. \]
These vielbeins are related by $\varepsilon^a\mu = \varepsilon^b\mu \, A^a_b$, where $A$ denotes the adjoint action of the group: $g T_a g^{-1} = A^a_b T_b$. Using this set-up, one can easily see that the gluing conditions (2) give rise to the following boundary conditions for the component fields $X^\mu$:
\[ \partial X^\mu = \tilde{R}(g)^\mu_\nu \partial X^\nu , \]
where the matrix of boundary conditions $\tilde{R}(g)$ is given by
\[ \tilde{R}(g) = -\varepsilon^{-1} R e . \]  
A Dirichlet direction is determined by an eigenvector of $\tilde{R}(g)$ with eigenvalue $-1$, whereas all the other eigenvectors correspond to Neumann directions, that is, directions tangent to the worldvolume of the D-brane.

If we parametrise the worldsheet of the string by $(\sigma, \tau)$ we can rewrite the above boundary conditions in the following form
\[ \imath (1 + \tilde{R}) \partial_\sigma X = (1 - \tilde{R}) \partial_\tau X, \]  
where $\partial, \bar{\partial} = \partial_\tau \mp i \partial_\sigma$. We know that in this case the worldvolume of a D-brane passing through $g$ and being described by (2) is given by the twisted conjugacy class $C g$. We therefore consider the following split
\[ T_g G = T_g G^\parallel \oplus T_g G^\perp, \]  
of the tangent space of $G$ at the point $g$, where $T_g G^\parallel$ is the tangent space to the twisted conjugacy class, and $T_g G^\perp$ is its orthogonal complement. Using this, one can split the boundary conditions (6) into two sets of conditions:
\[ \imath (1 + \tilde{R}) \partial_\sigma X^\parallel = (1 - \tilde{R}) \partial_\tau X^\parallel , \]
\[ \imath (1 + \tilde{R}) \partial_\sigma X^\perp = (1 - \tilde{R}) \partial_\tau X^\perp , \]
in the obvious notation. Since $\tilde{R} \big|_{T_g G^\perp} = -1$, from the second equation above we obtain the Dirichlet boundary conditions
\[ \partial_\tau X^\perp = 0. \]
On the other hand, by using the fact that $(1 + \tilde{R}) \big|_{T_g G^\parallel}$ is invertible, we obtain the Neumann boundary conditions
\[ \partial_\sigma X^\parallel + \imath \frac{1 - \tilde{R}(g)}{1 + \tilde{R}(g)} \partial_\tau X^\parallel = 0 , \]
We will now show that the matrix which defines the above Neumann boundary conditions coincides with the one defining the two-form $\omega$ on the worldvolume of the D-brane.

2.2. Boundary conditions from the sigma model. In the next section we will briefly review the definition of the boundary WZW model. In particular we will see that the action (10) of a generic WZW model on a 2-space with a disc topology (and with an additional interaction $A$ at the boundary) is specified in terms of the three-form field $H$, familiar from the standard case (without boundary), and a two-form field $\omega$ defined on the worldvolume $\mathcal{C}$ of the D-brane, and satisfying $d\omega = H\big|_e$.

The infinitesimal variation of the boundary WZW action contains a bulk term (yielding the same equations of motion as in the closed string case) and a boundary term which reads

$$\int_{\partial \Sigma} d\tau (g^{-1} \delta g)^{\alpha} \left[ G_{\alpha \beta} (g^{-1} \partial_{\sigma} g)^{\beta} - i \omega_{\alpha \beta} (g^{-1} \partial_{\tau} g)^{\beta} \right]_{\sigma = 0}^{\sigma = \pi}$$

where we have denoted by $G$ is the bi-invariant metric on the group manifold.

Here we are interested in D-branes described by (twisted) conjugacy classes. Thus, in order to separate the Neumann and Dirichlet boundary conditions encoded in the boundary term above, we must make use of the specific form of a conjugacy class. We recall (for details see, e.g., [6]) that this is defined as

$$\mathcal{C}_R(g_0) = \{ g = r(h)g_0h^{-1} \mid h \in \mathbf{G} \} ,$$

where the map $r : \mathbf{G} \to \mathbf{G}$ is defined by

$$r (e^{iT}) = e^{iR(T)} ,$$

for $t$ small enough and $T$ any element in the Lie algebra.

Hence in this case $g$ maps the boundary of the worldsheet $\partial \Sigma$ into the conjugacy class $\mathcal{C}_R(g_0)$ that is, $g(\partial \Sigma) \subset \mathcal{C}_R(g_0)$, and therefore

$$g^{-1} \delta g|_{e^{R(0)}} = (\text{Ad}_{g^{-1} R - 1}) \delta hh^{-1} .$$

Assuming that the metric restricts nondegenerately to the worldvolume of the D-brane (this is only a restriction in pseudo-riemannian signature), then the infinitesimal variation $g^{-1} \delta g$ can be written as the sum of two terms given by

$$(g^{-1} \delta g)^{||} = (\text{Ad}_{g^{-1} R - 1})^{||} \delta hh^{-1} ,$$

$$(g^{-1} \delta g)^{\perp} = (\text{Ad}_{g^{-1} R - 1})^{\perp} \delta hh^{-1} .$$

Since $(\text{Ad}_{g^{-1} R - 1})^{\perp} = 0$, the second equation above yields the Dirichlet boundary conditions

$$(g^{-1} \delta g)^{\perp} = 0 ,$$
whereas the boundary term in the infinitesimal variation of the action becomes
\[
\int_{\partial \Sigma} (\text{Ad}_{g^{-1}} R - 1) \| (\delta \varepsilon^{-1}) [G(g^{-1} \partial_y g) - i \omega(g^{-1} \partial_y g)] \|.
\]
Taking into account that \((\text{Ad}_{g^{-1}} R - 1) \|\) is nondegenerate, we obtain the Neumann boundary conditions
\[
(g^{-1} \partial_y g)^\| - i G^{-1} \omega(g^{-1} \partial_y g)^\| = 0.
\]
If we now consider the field \( g \) to be parametrised as in the previous paragraph, we can rewrite the Dirichlet and Neumann boundary conditions in the following form
\[
\delta X^\perp = 0,
\]
\[
\partial_y X^\| - i \tilde{G}^{-1} \tilde{\omega} \partial_y X^\| = 0,
\]
where \( \tilde{G} = e^T G e, \tilde{\omega} = e^T \omega e. \)

2.3. The two-form field \( \omega \). By identifying now the Neumann boundary conditions obtained from the boundary state approach with the Neumann conditions obtained from the classical sigma model, we can deduce that the two-form \( \omega \) is uniquely determined by the matrix of gluing conditions \( R \). Indeed we first deduce that
\[
\tilde{\omega} = -\frac{1}{2} \langle dX, \frac{1 - \tilde{R}(g)}{1 + \tilde{R}(g)} dX \rangle,
\]
from where we finally obtain that
\[
\omega = -\frac{1}{2} \langle g^{-1} d g, \frac{1 + \text{Ad}_{g^{-1}} R}{1 - \text{Ad}_{g^{-1}} R} g^{-1} d g \rangle.
\]
Notice that this form is well defined on \( C_R(g_0) \), as \( 1 + \tilde{R}(g_0) \) is invertible on \( T_{g_0} G^\| \). We know that the basic property that this field must satisfy is
\[
d \omega = H|_{C_R(g_0)},
\]
where \( H \) is the WZW three-form. In order to verify that our \( \omega \) given by (8) does indeed satisfy this property, we use the fact that the left-invariant Maurer-Cartan form evaluated on \( C_R(g_0) \) reads
\[
g^{-1} dg|_{C_R(g_0)} = (\text{Ad}_{g^{-1}} R - 1) d hh^{-1}.
\]
This allows us to evaluate \( H \) on the conjugacy class
\[
H|_{C_R(g_0)} = -d \langle d hh^{-1}, \text{Ad}_{g^{-1}} R(d hh^{-1}) \rangle,
\]
whereas for \( \omega \) itself we obtain
\[
\omega = -\langle d hh^{-1}, \text{Ad}_{g^{-1}} R(d hh^{-1}) \rangle.
\]
We thus conclude that a D-brane configuration which is given by a (twisted) conjugacy class \( C_R \) in a group manifold \( G \) is endowed with a
two-form field $\omega$ which is uniquely determined in terms of the matrix of gluing conditions $R$. This implies, in particular, that if one makes a certain gauge choice for the $B$ field in the bulk, then the $U(1)$ field $F$ on a given D-brane is uniquely determined in terms of $\omega$ and the pull-back on the worldvolume of the D-brane of that $B$ field.

3. QUANTUM CONSIDERATIONS

We recall that the WZW model is defined by the action

$$\langle \beta i \rangle^{-1} I[\tilde{g}] = \int_{\Sigma} \langle g^{-1} \partial g, g^{-1} \tilde{\partial} g \rangle + \int_{B} \tilde{g}^* H,$$

where $G_{ab} = \langle T_a, T_b \rangle$ denotes a bi-invariant metric on the group manifold, $\Sigma$ is Riemann surface without boundary, and $\mathcal{B}$ is a three-manifold with boundary $\partial \mathcal{B} = \Sigma$. As is well known, the WZ term in this case is a nonlocal term, defined in terms of an extension $\tilde{g} : \mathcal{B} \to G$ of the map $g$, such that $\tilde{g}|_{\partial \mathcal{B} = \Sigma} = g$, and given by

$$\tilde{g}^* H = -\frac{1}{3} \langle \tilde{g}^{-1} d \tilde{g}, d(\tilde{g}^{-1} d \tilde{g}) \rangle. \quad (9)$$

Thus the WZ term depends on the choice of extension $\tilde{g}$, which introduces in the action $I[\tilde{g}]$ an ambiguity proportional to the periods of $H$ over the integer homology $H_3(G)$. At the classical level these discrete contributions are not relevant, as they do not affect the equations of motion. However at the quantum level, the requirement that the path integral be independent of the choice of extension $\tilde{g}$ will in general quantise $\beta$:

$$\beta i = \frac{k}{4 \pi i},$$

with $k$ a positive integer (the level).

The boundary WZW model was analysed in some detail in [7, 8]; here we review a few aspects of particular interest for our discussion. The classical theory can be defined by an action

$$(\langle \beta i \rangle) S = \int_{\Sigma} \langle g^{-1} \partial g, g^{-1} \tilde{\partial} g \rangle + \int_{\Sigma} \tilde{g}^* B + \int_{\partial \Sigma} \tilde{g}^* A. \quad (10)$$

In this case the worldsheet $\Sigma$ is a two-dimensional manifold with boundary $\partial \Sigma$, and $B$ represents a particular choice for the antisymmetric tensor field, such that $dB = H$. A D-brane configuration is characterised in this setting by a two-form $\omega$ defined on its worldvolume $\mathcal{C}$, and satisfying $d \omega = dB|_{\mathcal{C}} = H|_{\mathcal{C}}$. Since $d(B - \omega)|_{\mathcal{C}} = 0$, one can define locally the one-form potential $A$ such that $dA = B - \omega$.

One can rewrite the boundary WZW action a manifestly gauge invariant form, by using the three-form $H$ and the two-form $\omega$. Let us assume, for simplicity, that we have one D-brane sitting on a (twisted) conjugacy class $\mathcal{C}$ in $G$. In this case the worldsheet $\Sigma$ can be represented in terms of a closed surface $\Sigma'$, where $\Sigma = \Sigma' \setminus D$, and $D$ is a (unit) disk
embedded in $\Sigma'$. One can then extend $g$ to a map $g' : \Sigma' \to G$ such that $g'(D) \subset \mathcal{C}$; $g'$ can be further extended to a map $\tilde{g}' : B' \to G$, with $B'$ a three-dimensional manifold such that $\partial B' = \Sigma'$. This allows us to write the WZ term in a more familiar form
\[
(\beta i) S = \int_{\Sigma} \langle g^{-1} \partial g, g^{-1} \tilde{g} \rangle + \int_{B'} \tilde{g}'^* H - \int_{D} \tilde{g}'^* \omega .
\] (11)

Thus, in this case, the WZ term has a bulk component and a boundary component (defined on the worldvolume of the D-brane). Similarly to the standard case, the WZ term depends on the extension $\tilde{g}'$, which introduces an ambiguity in the action
\[
(\beta i) \left( \int_{\mathcal{B}} \tilde{g}'^* H - \int_{S^2} \tilde{g}'^* \omega \right) ,
\] (12)

where $\mathcal{B}$ is a three-dimensional manifold with $\partial \mathcal{B} = S^2$ and $\tilde{g} : \mathcal{B} \to G$ such that $\tilde{g}(S^2) \subset \mathcal{C}$. As shown in [7, 8] these are proportional to the periods of $(H, \omega)$ over the cycles of the relative homology $H_3(G, \mathcal{C})$.

In order to evaluate this ambiguity and compare it to the standard case without boundary it is convenient to "fill" $\mathcal{B}$ with the unit ball $\mathbb{B}$ (whose boundary is $S^2$), ending up with a three-dimensional manifold $\mathbb{B}$ without boundary; if we also extend $\tilde{g}$ to a map $\tilde{g} : \mathbb{B} \to G$, we can rewrite (12) as the sum of two terms, where the first one
\[
(\beta i) \int_{\mathbb{B}} \tilde{g}'^* H ,
\] (13)

has the same form as the ambiguity appearing in the standard WZW action. This means that the same quantisation condition for $\beta$ as in the previous case ensures that (13) induces no dependence on our field extensions at the level of the path integral. This leaves us with the second term
\[
\frac{k}{4\pi i} \left( \int_{S^2} \tilde{g}'^* \omega - \int_{\mathbb{B}} \tilde{g}'^* H \right) .
\] (14)

which is characteristic to the boundary WZW model. Hence if we want that the path integral be independent of $\tilde{g}$, this term must take values in $2\pi i \mathbb{Z}$. This can be thought of as a generalisation of the Dirac quantisation condition.

4. The SU(2) case

4.1. (Semi-)classical analysis. Let us now apply the above discussion to the particular case of D-brane configurations $g$ given by conjugacy classes in SU(2). We use the following parametrisation [1]:
\[
g = e^{i(\psi_1 \sigma_1 + \psi_2 \sigma_2 + \psi_3 \sigma_3)} ,
\]
where $(\psi_1, \psi_2, \psi_3)$ forms a vector of length $\psi$ pointing in the direction $(\theta, \phi)$, and $(\sigma_1, \sigma_2, \sigma_3)$ are the Pauli matrices. This parametrisation, whose spacetime fields are $(\psi, \theta, \phi)$, has the advantage that one of the
coordinates, namely $\psi$, corresponds to the Dirichlet direction, as we will see explicitly in a moment. We can now compute, as usual, the invariant vielbeins $e$ and $\tilde{e}$, the sigma model metric $G$, and the Wess-Zumino three-form $H$ thus obtaining

$$G = d\psi^2 + \sin^2 \psi (d\theta^2 + \sin^2 \theta d\phi^2),$$

$$H = 4 \sin \theta \sin^2 \psi \theta \wedge d\theta \wedge d\phi.$$ 

Furthermore, by using (5), we obtain the matrix of boundary conditions

$$\hat{R}(\theta) = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -\cos 2\psi & -\sin \theta \sin 2\psi \\ 0 & \sin 2\psi \csc \theta & -\cos 2\psi \end{pmatrix}.$$ 

From the form of this matrix we can immediately read off that we always have a Dirichlet boundary condition along the 'radial' coordinate $\psi$. In other words, the D-branes described by these gluing conditions are normal to the $\psi$ direction at any given point. On the other hand, we know that these D-branes are conjugacy classes in SU(2)—hence every such conjugacy class $\mathcal{C} = \mathcal{C}(\psi)$ is a two-sphere centred around the identity. In particular, for $\psi = 0, \pi$ we get the zero-dimensional D-branes since $\hat{R} = -1$.

According to the discussion in Section 2, we can now calculate the two-form field $\omega$ associated to a given D-brane $\mathcal{C}(\psi)$ obtaining

$$\tilde{\omega} = -\sin 2\psi \sin \theta d\theta \wedge d\phi.$$ 

Using the explicit knowledge of this field, we can evaluate the energy of such a configuration from the Born-Infeld action

$$E(\psi) = kT \int_{\mathcal{C}(\psi)} \sqrt{\det(\tilde{g} + \tilde{\omega})}$$

$$= 8\pi kT \sin \psi,$$ 

where we denoted by $T$ the D-brane tension, and by $\tilde{g}$ the metric induced on the worldvolume of the D-brane. From this expression we can immediately see that the energy of such a D-brane configuration reaches its minimum for $\psi = 0, \pi$, i.e., for the zero-dimensional D-branes. Hence, from a classical point of view, it is only the two D0-branes that give rise to stable configurations.

Let us compare our result with the one obtained in [1]. The main difference between the two approaches lies in the way one determines the $B + 2\pi \alpha' F$ field. In [1] some gauge choices were involved. Here, this field was determined uniquely, imposing consistency between the gluing conditions and the classical sigma model boundary conditions, and thus the expression for the energy (15) holds independent of any particular gauge choice. One could argue that the only necessary conditions are that $dB = H$ and $dF = 0$. However, as we showed in Section 2, consistency between the gluing conditions and the classical sigma model...
boundary conditions constrains $B + 2\pi \alpha' F$ to be equal to the two-form field $\omega$. One might also expect that the energy minimisation procedure itself selects the right combination, but the different results obtained in the two approaches indicate that this is not the case. It is perhaps useful to remark that minimising the energy basically fixes the “radius” of the D-brane, whereas the gluing conditions fix its shape (spherical in this case). Moreover, there is an infinite number of D2-branes which, despite the fact that they satisfy the gluing conditions, do not minimise the Born-Infeld action.

4.2. Quantum analysis. Let us now apply the quantum considerations of the previous section to our D-branes $\mathcal{C}(\psi)$. To this end, let us compute the period of $(H, \omega)$ over the cycle in $H_3(G, \mathcal{C})$. In this case $\mathcal{B} = \mathcal{B}(\psi)$ is a three-ball bounded by $\mathcal{C}(\psi)$ and we calculate

$$\frac{k}{4\pi i} \left( \int_{\mathcal{C}(\psi)} \tilde{g}^* \omega - \int_{\mathcal{B}(\psi)} \tilde{g}^* H \right) = i2k\psi .$$

Hence, in order for the path integral to be independent of our field extensions, $\psi$ must be quantised as follows

$$\psi_n = \frac{n\pi}{k} , \quad n = 0,1,\ldots,k .$$

This result, which agrees with the analysis of [9] (for a detailed exposition see also [8]), allows us to conclude that the $k + 1$ D-branes singled out in this fashion are stable, and their masses, evaluated from the Born-Infeld action, are given by

$$M_n = 8\pi kT \sin \left( \frac{n\pi}{k} \right) , \quad n = 0,1,\ldots,k ,$$

which, as pointed out in [1], agrees with the CFT calculations.

Such a quantisation condition appears to be a non-local condition imposed on $\psi$, and concerns about its physical meaningfulness are well founded\(^1\). However the weakest link in the argument which leads to this result seems to be the very starting point that is, the gluing conditions that give rise to this type of D-brane configurations.

4.3. D0 charge. These results prompt us to propose the following definition for the D0-charge of such D-branes:

$$Q_0 = kT \left( \int_{\bar{g}_B} \tilde{g}^* \omega - \int_{\mathcal{B}} \tilde{g}^* H \right) \pmod k \quad (16)$$

In our particular case of a D-brane given by $\mathcal{C}(\psi)$ one obtains

$$Q_0(\psi) = 8\pi kT \psi \pmod k ,$$

which takes integer values for the $k + 1$ D-branes obtained before.

It has to be remarked here that there seems to be a discrepancy between this result for $Q_0$ and the CFT calculations. However if one

\(^1\)I thank Michael Douglas for raising this point.
computes the flux of $\omega$ alone, one obtains the same result as in [1], which moreover agrees with the CFT results. This seems to indicate that the path integral and the boundary state approach compute two distinct quantities, and it would be nice to shed further light on their physical meaning and relation.

This definition for the D6-brane charge has the following virtues: it is manifestly gauge invariant, just as the one introduced in [1], but unlike the one based on the flux of the U(1) field [3, 5]. It is naturally quantised with integer values, as is natural to expect of a RR charge. Moreover, it includes a contribution coming from the bulk field $H$, similar to the one advocated in [3]. Notice however that this bulk term does not cancel the $B$ field contribution included in the flux of $\omega$. It is perhaps useful to discuss this point also in the framework used in [3], where the Poynting-type contribution to the D6-brane charge reads

$$\frac{1}{6} \int G^{(4)}_{ij} F^{ijk},$$

with obvious notation. In evaluating this contribution we must take into account that, in this case, $H$ belongs to a nontrivial cohomology class and hence there is no globally defined gauge invariant $B$ field\(^2\). Therefore this results in a bulk contribution which agrees with the bulk term in our definition of $Q_0$ in (16).

Last but not least, notice that, although our $Q_0$ is not the same as the flux of the gauge field $F$, it is nevertheless a natural generalisation of the U(1) flux in the case of a D-brane in a WZW background, since it reduces to this in the particular case where $H$ is an exact form, as one can verify by using the Stokes theorem in the first term in (16).

5. Discussion

In this letter we have analysed a certain class of D-brane configurations in group manifolds. This type of D-branes is characterised by gluing conditions that preserve the maximum amount of symmetry of the bulk theory, namely, the current algebra of the WZW theory. We know that the gluing conditions generally fix the “shape” of a D-brane; in particular, this type of gluing conditions give rise to D-branes described by (twisted) conjugacy classes. Here we have shown that consistency between the gluing conditions and the sigma model boundary conditions also fixes the gauge invariant field $B + 2\pi \alpha' F$. Using this fact, one can estimate the energy of such a D-brane configuration, from the Born-Infeld action, independent of any particular gauge choice for either $B$ or $F$. One thus obtains that in the SU(2) case, at the classical level, it is only the two D6-branes that are stable. We have then used a quantisation argument based on the path integral which requires that the periods of $(H, \omega)$ over the cycles of the relative homology $H_3(G, C)$

\(^2\)A similar observation was made independently by A Tseytlin.
take integer values; in the SU(2) case this produces a discrete set of allowed D-brane configurations, whose mass spectrum agrees with the CFT calculations.

This quantisation argument also prompted us to make an alternate proposal for the D0-brane charge of such D-branes, which differs from the one introduced in [1] by a bulk term, similar, yet not identical, to the one advocated in [3]. We believe this to be a natural definition for a number of reasons. First of all it is manifestly gauge invariant, and we consider this to be an important feature, as the both the boundary WZW model and the Born-Infeld action of a D-brane in such a background are manifestly gauge invariant. This D0 charge is also naturally quantised, taking integer values, and this is clearly a desirable feature for a RR charge. Finally, this definition constitutes a natural generalisation of the U(1) flux.

It must be said, however, that this definition also raises a number of questions. One of them is the apparent discrepancy with the CFT results for the D0 charge. More precisely, it seems that what the boundary state calculates is the flux of $\omega$ through the D-brane, without the $H$ bulk contribution. This seems to suggest that both these quantities have a natural physical interpretation. One problematic aspect of this approach is the fact that the quantisation argument seems to impose a non-local condition on the spacetime field $\psi$. Since the evaluation of the D0 charge was made for a specific class of D-branes, described by a very special type of gluing conditions, one might put in question the very physicality of these D-brane configurations.

Finally, one of the most puzzling conclusions of this analysis is that there appears to be no evidence for the quantisation of the $\mathbb{U}(1)$ flux in this particular context. This clearly challenges our intuition about the gauge theories which should describe the worldvolume theory of these D-branes, although one cannot logically exclude such a possibility.

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