Dual Higgs Model with Dual Dirac Strings

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Abstract

We investigate a dual Higgs model with dual Dirac strings invariant under $U(1)$ gauge transformations, where $U(1)$ is the Abelian gauge group of dual-vector potential transformations spontaneously broken by the Higgs mechanism. Non-perturbative phenomena of Compact QED are analysed. The action of a dual Dirac string calculated in leading order in the large Higgs-field mass expansion coincides with a dual version of that obtained by Nambu [2]. For the example of the string tension at the classical tree-level approach we develop a procedure for the computation of contributions of string-string interactions.

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Introduction

The main open question of QCD is the mechanism of quark confinement. The most fruitful ideas concerning the nature of the quark confinement are concentrated around the string picture of hadrons [1,2]. These ideas are based on the assumption of Dirac monopoles [3]. The string picture of hadrons suggests a linearly rising interquark potential due to which quarks are confined [4,5]. Such a potential involves only one phenomenological parameter which is the string tension $\sqrt{\sigma} = 450 \div 520$ MeV [2].

Compact Quantum Electrodynamics (CQED), defined for lattices as nonlinear $U(1)$ gauge theory, possesses a confined phase [6]. There is evidence [7] that the vacuum of CQED behaves like an effective dual superconductor with magnetic monopoles. The magnetic monopoles rearrange the electric fields so that there is a net flux tube between quarks which looks like a dual Dirac string. The energy per unit length of this flux is the string tension. Monopole condensation is able to realize the electric charge confinement in CQED. The admission that monopoles should be condensed in the confined phase has been shown in [8] by mapping CQED onto an Abelian Higgs model defined on a lattice.

Lattice calculations of interquark potentials, field and charge-density distributions, and chiral symmetry breaking show a very close similarity between CQED and QCD. However the mechanism of quark confinement realized by CQED and being most likely very similar to that of QCD could be carried over to the description of quark confinement in experimentally measured processes of low–energy interactions of hadrons only via the employment of a continuum model embodying the main non-perturbative properties of CQED.

The main hint at the kind of continuum field theory which one should apply as a continuum analogy of CQED, should be the result by Singh, Browne and Haymaker [7] concerning the nonperturbative vacuum of CQED behaving like an effective dual superconductor with monopoles. At the classical level the simplest theory of superconductivity has been suggested by London [9] within the framework of Maxwell’s Electrodynamics via the inclusion of London’s equation of superconductivity connecting a vector potential $\vec{A}(\vec{x},t)$ with the electric superconducting current. Therewith the coefficient of proportionality is the squared penetration depth $\lambda$ of the magnetic field inside the superconductor.

A relativistically covariant generalization of London’s theory of superconductivity has been given by Nambu [2] within Dirac’s extension of Maxwell’s Electrodynamics. In Nambu’s consideration the reverse power of $\lambda$ has been identified with the mass of the vector field $A_\mu(x)$. Dirac string has been included by hands in terms of an external magnetic field induced by the Dirac string and depending on the string shape. The inclusion has been carried out by a manner being in accordance with the “magnetic Gauss” law. Effectively Nambu’s approach is equivalent to the Higgs model with an external field in the symmetry–broken phase when the local Abelian $U(1)$ gauge symmetry is spontaneously broken. Thus we can conclude that the Higgs model in the symmetry–broken phase restores the simplest variant of superconductivity suggested by London.

Therefore if we like to describe a dual version of London’s theory of superconductivity we have to turn to a dual version of the Higgs model with a dual–vector field $C_\mu$ and dual Dirac strings included as external electric field depending on the string shape and saturating the “electric Gauss ” law. The $C_\mu$–field has to acquire a mass due to the Higgs mechanism and should induce thereby London’s equation of dual superconductivity. In
the symmetry-broken phase of the dual Higgs model the effective action of the massive dual-vector field should be converted into the effective action of a dual Dirac string giving rise to a linearly rising interquark potential. The string tension can be determined in terms of parameters of the dual Higgs model.

The starting Lagrangian of the dual Higgs model with a dual Dirac string should read (see Appendix)

\[
\mathcal{L}(x) = \frac{1}{4} F_{\mu\nu}(x) F^{\mu\nu}(x) + (\partial_{\mu} + ig C_{\mu}(x)) \Phi(x)^{*} \Phi(x) - \kappa^2 \left( v^2 - \Phi^{*}(x) \Phi(x) \right)^2 + \mathcal{L}_{\text{free quark}}(x)
\]

where \( \mathcal{L}_{\text{free quark}}(x) \) is a kinetic term for quark and antiquark [10]

\[
\mathcal{L}_{\text{free quark}}(x) = - \sum_{i=q,\bar{q}} m_i \int d\tau \left( \frac{dX_i^\mu(\tau)}{d\tau} \frac{dX_i^\nu(\tau)}{d\tau} g_{\mu\nu} \right)^{1/2} \delta^{(4)}(x - X_i(\tau)).
\]

Quark and antiquark are considered as classical particles with masses \( m_q \) and \( m_{\bar{q}} \), charges \( Q_q \) and \( Q_{\bar{q}} \) such as \( Q_q = -Q_{\bar{q}} = Q \), and trajectories \( X_q^\mu(\tau) \) and \( X_{\bar{q}}^\mu(\tau) \), respectively. The electric quark current \( J^\nu(x) \) is given by

\[
J^\nu(x) = \sum_{i=q,\bar{q}} Q_i \int d\tau \frac{dX_i^\nu(\tau)}{d\tau} \delta^{(4)}(x - X_i(\tau)).
\]

Then \( \Phi(x) \) is a complex Higgs field with the vacuum expectation value \( \nu \), i.e. \( < \Phi > = \nu \), and \( g \) and \( \kappa \) are the coupling constants. The field strength \( F^{\mu\nu}(x) \) is defined [11]

\[
F^{\mu\nu}(x) = \mathcal{E}^{\mu\nu}(x) - * \left( dC(x) \right)^{\mu\nu}
\]

where \( \left( dC(x) \right)^{\mu\nu} = \partial^\mu C^\nu(x) - \partial^\nu C^\mu(x) \), and \( * \left( dC(x) \right)^{\mu\nu} \) is a dual version

\[
* \left( dC(x) \right)^{\mu\nu} = \frac{1}{2} \varepsilon^{\mu\nu\alpha\beta} \left( dC(x) \right)_{\alpha\beta} \quad (\varepsilon^{0123} = 1).
\]

The electric strength field \( \mathcal{E}^{\mu\nu}(x) \), induced by a dual Dirac string, we determine following [3]

\[
\mathcal{E}^{\mu\nu}(x) = Q \int d^2 v \delta^{(4)}(x - X) \sigma^{\mu\nu}(X)
\]

where we have denoted \( d^2 v = d\tau d\sigma \) and

\[
\sigma^{\mu\nu}(X) = \frac{\partial X^\mu}{\partial \tau} \frac{\partial X^\nu}{\partial \sigma} - \frac{\partial X^\nu}{\partial \tau} \frac{\partial X^\mu}{\partial \sigma}.
\]

Here \( X^\mu = X^\mu(\tau, \sigma) \) represents the position of a point on the world sheet swept by the string. The sheet is parametrized by the internal coordinates \(-\infty < \tau < \infty \) and \( 0 \leq \sigma \leq \pi \), so that \( X^\mu(\tau, 0) = X^\mu_Q(\tau) \) and \( X^\mu(\tau, \pi) = X^\mu_{\bar{Q}}(\tau) \) represent the world lines of an antiquark and a quark [2,3]. Within the definition (5) the tensor field \( \mathcal{E}^{\mu\nu}(x) \) satisfies identically the equation of motion

\[
\partial_{\mu} F^{\mu\nu}(x) = J^\nu(x),
\]
that is the first pair of equations of motion of Dirac’s extension of Maxwell’s Electrodynamics. This means that the inclusion of a dual Dirac string in terms of $\mathcal{E}^{\mu\nu}(x)$ defined by (5) saturates completely the electric Gauss law.

The Lagrangian (1) is invariant under local $U(1)$ gauge transformations where $U(1)$ is the Abelian gauge group of dual–vector potential transformations. The Higgs mechanism breaks spontaneously a local $U(1)$ symmetry. That leads to the appearance masses of both the $C_{\mu}$-field and the Higgs field.

By varying the Lagrangian (1) with respect to $C_{\mu}(x)$ and $\Phi^\ast(x)$ we get the set of equations of motion

$$\partial_\mu F^{\mu\nu}(x) = \i g \left( \Phi^\ast(x) \partial^\nu \Phi(x) - \Phi(x) \partial^\nu \Phi^\ast(x) \right) + 2g^2 \Phi^\ast(x) \Phi(x) C^\nu(x),$$

(8)

$$\left( \partial_\mu - \i g C_\mu(x) \right) \left( \partial^\mu - \i g C^\mu(x) \right) \Phi(x) = -2\kappa^2 \Phi(x) \left( \partial^2 - \Phi^\ast(x) \Phi(x) \right).$$

(9)

The r.h.s. of eq.(8) should be identified with the magnetic current $J^\nu(x)$

$$J^\nu(x) = \i g \left( \Phi^\ast(x) \partial^\nu \Phi(x) - \Phi(x) \partial^\nu \Phi^\ast(x) \right) + 2g^2 \Phi^\ast(x) \Phi(x) C^\nu(x).$$

(10)

The magnetic current $J^\nu(x)$ can be obtained in a standard way by applying a local infinitesimal gauge transformation of the $C_{\mu}$-field

$$C_\mu(x) \rightarrow C_\mu'(x) = C_\mu(x) - \partial_\mu \alpha(x)$$

(11)

and defining the magnetic current as a derivative of the Lagrangian (1) with respect to $\partial_\mu \alpha(x)$

$$\mathcal{J}_\mu(x) = - \frac{\delta \mathcal{L}(x)}{\delta \partial^\mu \alpha(x)}.$$  

(12)

The infinitesimal change of the Lagrangian (1) caused by the gauge transformation (11) reads

$$\delta \mathcal{L}(x) = - \partial^\nu \alpha(x) \left[ \i g \left( \Phi^\ast(x) \partial_\mu \Phi(x) - \Phi(x) \partial_\mu \Phi^\ast(x) \right) + 2g^2 \Phi^\ast(x) \Phi(x) C_\mu(x) \right].$$

(13)

This gives the expression (10). Thus the equation of motion (8) should be valued as the second pair of equations of motion of Dirac’s extension of Maxwell’s Electrodynamics.

The present paper is organized as follows. In Sect. 1 we consider the Higgs mechanism of spontaneous breaking of $U(1)$ symmetry in the dual Higgs model and discuss the problem of the imaginary mass of the $C_{\mu}$-field induced by duality. In Sec. 2 we analyse the problem of the condensation of the magnetic current. We connect the magnetic current condensation with the condensation of the $C_{\mu}$-field. In the strong $\kappa$-coupling limit we integrate over the $\sigma$-field and evaluate the effective potential of the $C_{\mu}$-field up to one-loop contributions of the $\sigma$-field exchange. The account of two-loop contributions and so on goes beyond the Gaussian approximation of the path integral over the $\sigma$-field. The corresponding terms in the action, responsible on these contributions, are of order $O(1/\kappa)$ and seem suppressed in the strong $\kappa$-coupling limit. After the integration over the $\sigma$-field the dual Higgs model reduces to the effective theory of self–interacting dual–vector field $C_{\mu}$. We show that the $C_{\mu}$-field cannot be condensed in the tree $C_{\mu}$-field exchange.
approximation. The magnetic current induced within this effective theory of the $C_\mu$-field also cannot be condensed in the tree $C_\mu$-field exchange approximation. Therewith we have shown that the magnetic current condensation would not occur in the tree approximation even if the $C_\mu$-field could be condensed in the tree level. Thus the condensation of the magnetic current is the matter of one-loop contributions of the $C_\mu$-field exchange. In Sect. 3 we show that due to the one-loop contributions of the $\sigma$-field exchange the $C_\mu$-field acquires a real mass. As a result the classical solution of the equation of motion of the $C_\mu$-field, induced by a dual Dirac string, has the shape of a dual Abrikosov flux line in a type II dual superconductor [2]. In Sect. 4 we derive London's equation of dual superconductivity and show that in the symmetry-broken phase the equations of motion describing the dual Higgs model coincide fully with the equations of motion of Dirac's extension of Maxwell's Electrodynamics. In Sect. 5 we evaluate the action of a dual Dirac string. We show that in leading order in the large $M_s$ expansion the string action coincides fully with that obtained by Nambu [2]. In Sect. 6 we calculate the string tension in leading order in the large $M_s$ expansion. In Sect. 7 we take into account next-to-leading order corrections in the large $M_s$ expansion to the string tension. In the Conclusion we resume the obtained results. In the Appendix we discuss the Lagrangian formulation of different approaches to Dirac's extension of Maxwell's Electrodynamics without Dirac strings and including them. We extend these approaches to Higgs models. We show that the Higgs mechanism does not distinguish a vector field from a dual-vector one. The imaginary mass of the dual-vector field is inspired by the kinetic term $\frac{1}{4} (dC(x))_{\mu\nu} (dC(x))^{\mu\nu}$ that enters in the Lagrangian with a positive sign instead of having a negative like the vector field. Since the positive sign of the kinetic term is a peculiarity of the dual-vector field in Dirac's extension of Maxwell's Electrodynamics, so that we argue that the imaginary mass of the $C_\mu$-field is the manifestation of duality but not a Higgs mechanism.

1. Higgs mechanism and duality

In order to proceed to the symmetry-broken phase riched by non-pertubative phenomena it is convenient to employ a polar representation of the Higgs field $\Phi$ that is

$$\Phi(x) = \rho(x) e^{i\theta(x)}. \quad (14)$$

In the polar representation of the Higgs field (14) the Lagrangian (1) transforms itself to the form

$$\mathcal{L}(x) = -\frac{1}{4} F_{\mu\nu}(x) F^{\mu\nu}(x) +$$

$$+ (\partial_\mu + i g \tilde{C}_\mu(x)) \rho(x) (\partial^\mu - i g \tilde{C}^\mu(x)) \rho(x) - \kappa^2 (v^2 - \rho^2(x))^2 \quad (15)$$

where we have denoted

$$\tilde{C}_\mu(x) = C_\mu(x) - \frac{1}{g} \partial_\mu \theta(x). \quad (16)$$

Below we omit the tilde over the $C_\mu$-field.

As has been shown by Nielsen and Olesen [12] for infinitely-long Dirac strings directed along the $z$-axis, the electric Gauss law should be saturated by the phase field $\theta$ containing
a multi-valued component. We have saturated the electric Gauss law by the electric tensor field $\mathbf{E}^{\mu\nu}(x)$ induced by a dual Dirac string. This should imply that the multi-valued component of the phase field $\theta$ has been picked up by the $\mathbf{E}^{\mu\nu}$-field, and the remainder can be considered as a trivial component that due to the Higgs mechanism contributes to the longitudinal part of the massive $C_\mu$-field.

In the polar representation of the Higgs field (14) the transition to the symmetry-broken phase can be gained by the shift of the $\rho$-field

$$\rho(x) = v + \frac{1}{\sqrt{2}} \sigma(x).$$

After the shift (17) the Lagrangian (15) acquires the form

$$\mathcal{L}(x) = -\frac{1}{4} F_{\mu\nu}(x) F^{\mu\nu}(x) +$$

$$+ \frac{1}{2} \sigma \sigma \sigma(x) \sigma(x) + g M C \sigma(x) \left[ 1 + \frac{\kappa}{\sqrt{2}} \frac{\sigma(x)}{M_{\sigma}} \right] C_\mu(x) C^\mu(x)$$

$$+ \frac{1}{2} \partial \mu \sigma(x) \partial \nu \sigma(x) - \frac{1}{2} M_{\sigma}^2 \sigma^2(x) \left[ 1 + \frac{\kappa}{\sqrt{2}} \frac{\sigma(x)}{M_{\sigma}} \right]^2$$

$$+ \mathcal{L}_{\text{free quark}}(x)$$

where

$$M_C^2 = 2 g^2 v^2 \quad , \quad M_{\sigma}^2 = 4 \kappa^2 v^2$$

such as $M_{\sigma}$ has the meaning of the mass of the $\sigma$-field. By varying the Lagrangian (18) with respect to the fields $C_\nu(x)$ and $\sigma(x)$ we get the following equations of motion

$$\partial \mu \partial \mu \sigma(x) = \frac{1}{\sqrt{2}} \frac{\sigma(x)}{M_{\sigma}} \left[ 1 + \frac{\kappa}{\sqrt{2}} \frac{\sigma(x)}{M_{\sigma}} \right] C_\mu(x) C^\mu(x),$$

$$\Delta \sigma(x) = - \frac{1}{\sigma^2} \sigma(x) \left[ 1 + \frac{\kappa}{\sqrt{2}} \frac{\sigma(x)}{M_{\sigma}} \right] \left[ 1 + \frac{\kappa}{\sqrt{2}} \frac{\sigma(x)}{M_{\sigma}} \right] +$$

$$+ g M C \left[ 1 + \frac{\kappa}{\sqrt{2}} \frac{\sigma(x)}{M_{\sigma}} \right] C_\mu(x) C^\mu(x).$$

Now let us consider the equation of motion of the $C_\mu$-field (20). Substituting (3) in (20) we obtain a negative sign in the mass term of the $C_\mu$-field instead of a positive one. This implies that the $C_\mu$-field has an imaginary mass. The imaginary mass is inspired by the kinetic term $\frac{1}{4} (dC(x)_{\mu\nu} (dC(x))^{\mu\nu}$ that enters in the Lagrangian (18) with a positive sign instead of having a negative like a vector field. Thus the appearance of the dual-vector field with an imaginary mass one cannot count as new peculiarity of the Higgs mechanism. This is merely a consequence of duality, i.e. the dual character of the $C_\mu$-field. In the Appendix we follow the comparison of our result with that obtained in the standard Higgs model with a vector field.
2. Magnetic current and dual–vector field condensation

In the confined phase of CQED the condensation of the magnetic current $\mathcal{J}^\mu(x)$ has been observed by DeGrand and Toussaint [13]. Since we investigate the affinity between CQED and the dual Higgs model, we have to analyse the phenomenon of magnetic current condensation within the dual Higgs model. For this aim we have to consider the vacuum expectation value of the product $\mathcal{J}_\mu(x)\mathcal{J}^\nu(x)$. The rough estimate obtained by applying eq.(10) gives

$$< \mathcal{J}_\mu(x)\mathcal{J}^\nu(x)> = 2g^2v^2 < C_\mu(x)C^\nu(x) > .$$

Thus the problem of the magnetic current condensation can be reduced to the problem of the condensation of the dual–vector field $C_\mu$.

No doubts, that the $C_\mu$–field condensation can come off via interactions of $C_\mu$ and $\sigma$ fields. Thereby in order to analyse the possibility of the $C_\mu$–field condensation we suggest to treat that part of the Lagrangian (18) that involves the corresponding interactions, i.e.

$$\mathcal{L}'(x) = \frac{1}{2}\partial_\mu\sigma(x)\partial^\mu\sigma(x) - \frac{1}{2}M^2\sigma^2(x)\left[1 + \frac{\kappa}{\sqrt{2}}\frac{\sigma(x)}{M}\right]^2 +$$

$$+ g M C \sigma(x)\left[1 + \frac{\kappa}{\sqrt{2}}\frac{\sigma(x)}{M}\right]C_\mu(x)C^\mu(x).$$

(22)

Following Nambu [2] we assume that the $\sigma$–field is rather heavy, i.e. $M_\sigma \gg M_C$. From eq.(19) there follows that the restriction $M_\sigma \gg M_C$ implies the strong $\kappa$–coupling limit, i.e. $\kappa \gg g$ and $\kappa \gg 1$. In this limit the $\rho$–field should be close to $v$, and the $\sigma$–field is small compared with $v$. Therewith the fluctuations of $|\sigma(x)/v|$ should be of order $O(1/\kappa)$. Thereby we can expand $\mathcal{L}'(x)$ in powers of $|\sigma(x)/v|$, restricting by lowest powers, and integrate over the $\sigma$–field by applying the path–integral method. In the limit $|\sigma(x)/v| < 1$ the Lagrangian (22) reads

$$\mathcal{L}'(x) = \frac{1}{2}\sigma(x)(\Box + M^2_\sigma - g^2 C_\mu(x)C^\mu(x))\sigma(x) +$$

$$+ g M C C_\mu(x)C^\mu(x)\sigma(x) + O(\sigma^3(x)).$$

(23)

Here we have used the relations (19). The strong $\kappa$–coupling limit supression of terms of order $O(\sigma^3(x))$, responsible for two–loop contributions and so on, implies the dominance of tree and one–loop contributions of the $\sigma$–field exchange.

The integration over the $\sigma$–field gives the following effective Lagrangian

$$\mathcal{L}_{\text{eff}}[C_\mu(x)] = \frac{1}{2}i \left\langle x|\ln\left(1 - \frac{1}{\Box + M^2_\sigma}g^2C_\alpha C^\alpha\right)|x\right\rangle +$$

$$+ \frac{1}{2}g^2 M^2 C_\mu(x)C^\mu(x)(\Box + M^2_\sigma - g^2 C_\alpha(x)C^\alpha(x))^{-1} C_\nu(x)C^\nu(x).$$

(24)

The second term in (24) can be represented by the infinite series of tree diagrams with $\sigma$–field exchanges. Since the transferred momenta in these diagrams are of order $O(M_C)$,
so that applying the restriction $M_\sigma \gg M_C$ we can neglect the transferred momenta in comparison with $M_\sigma$ that entails the neglect of the contributions of the quarka, that is we neglect derivatives of $C_\mu(x) C^\mu(x)$. Effectively this means that the scale of fluctuations of $\partial^\nu \left( C_\mu(x) C^\nu C^\mu(x) \right)$ is much smaller than $M_\sigma$. This approximation brings up the effective Lagrangian (24) to the form

$$
\mathcal{L}'_{\text{eff}}[C_\mu(x)] = \frac{1}{2} i \left\langle x \left| \ln \left( 1 - \frac{1}{\Box + M_\sigma^2} g^2 C_\alpha C^\alpha \right) \right| x \right\rangle + \\
+ \frac{1}{2} g^2 \frac{\dot{M}_\sigma^2}{M_\sigma^2} \left[ C_\mu(x) C^\mu(x) \right]^2 \left[ 1 - g^2 \frac{C_\mu(x) C^\mu(x)}{M_\sigma^2} \right]^{-1}
$$

(25)

Thus after the integration over the $\sigma$–field we have gained the following total effective Lagrangian of the $C_\mu$–field

$$
\mathcal{L}_{\text{eff}}(x) = -\frac{1}{4} F_{\mu\nu}(x) F^{\mu\nu}(x) + \frac{1}{2} M_\sigma^2 C_\mu(x) C^\mu(x) \left[ 1 - g^2 \frac{C_\mu(x) C^\mu(x)}{M_\sigma^2} \right]^{-1} \\
+ \frac{1}{2} i \left\langle x \left| \ln \left( 1 - \frac{1}{\Box + M_\sigma^2} g^2 C_\mu C^\mu \right) \right| x \right\rangle + \mathcal{L}_{\text{free quark}}(x).
$$

(26)

Now we proceed to the evaluation of one–loop contributions that are described by the functional logarithm. For the evaluation of the functional logarithm it is convenient to represent it in the form of infinite series

$$
\frac{1}{2} i \left\langle x \left| \ln \left( 1 - \frac{1}{\Box + M_\sigma^2} g^2 C_\mu C^\mu \right) \right| x \right\rangle = -\frac{1}{2} \sum_{n=1}^\infty \frac{i}{n} \left\langle x \left| \left( \frac{1}{\Box + M_\sigma^2} g^2 C_\mu C^\mu \right)^n \right| x \right\rangle.
$$

(27)

By applying the same approximation that has been used above, i.e. neglecting the contributions of derivatives like $\partial^\nu \left( C_\mu(x) C^\nu C^\mu(x) \right)$, being valid due to the restriction $M_\sigma \gg M_C$, we get the following expression

$$
\frac{1}{2} i \left\langle x \left| \ln \left( 1 - \frac{1}{\Box + M_\sigma^2} g^2 C_\mu C^\mu \right) \right| x \right\rangle = \\
= \frac{1}{2} \left[ \frac{g^2 C_\mu(x) C^\mu(x)}{M_\sigma^2} \right] \int \frac{d^4 k}{(2\pi)^4} \frac{M_\sigma^2}{k^2} + \\
+ \frac{1}{4} \left[ \frac{g^2 C_\mu(x) C^\mu(x)}{M_\sigma^2} \right]^2 \int \frac{d^4 k}{(2\pi)^4} \frac{M_\sigma^4}{k^2} + \\
+ \frac{1}{2} \sum_{n=3}^\infty \left[ \frac{g^2 C_\mu(x) C^\mu(x)}{M_\sigma^2} \right]^n \frac{1}{n} \int \frac{d^4 k}{(2\pi)^4} \frac{M_\sigma^{2n}}{(M_\sigma^2 - k^2)^n}.
$$

(28)

The integration over $k$ gives

$$
\frac{1}{2} i \left\langle x \left| \ln \left( 1 - \frac{1}{\Box + M_\sigma^2} g^2 C_\mu C^\mu \right) \right| x \right\rangle = \\
= \frac{1}{2} \Delta_1(M_\sigma^2) \left[ \frac{g^2 C_\mu(x) C^\mu(x)}{M_\sigma^2} \right] + \frac{1}{4} \Delta_2(M_\sigma^2) \left[ \frac{g^2 C_\mu(x) C^\mu(x)}{M_\sigma^2} \right]^2 + \\
+ \frac{M_\sigma^4}{32 \pi^2} \sum_{n=3}^\infty \frac{1}{n(n-1)(n-2)} \left[ \frac{g^2 C_\mu(x) C^\mu(x)}{M_\sigma^2} \right]^n
$$

(29)
where $\Delta_1(M_\sigma^2)$ and $\Delta_2(M_\sigma^2)$ are divergent integrals

\[
\Delta_1(M_\sigma^2) = \int \frac{d^4k}{(2\pi)^4} i \frac{M_\sigma^2}{M_\sigma^2 - k^2},
\]

\[
\Delta_2(M_\sigma^2) = \int \frac{d^4k}{(2\pi)^4} i \frac{M_\sigma^4}{(M_\sigma^2 - k^2)^2}.
\]

For the evaluation of these integrals we apply dimensional regularization. The use of dimensional regularization should be justified by the following reasoning. Through the model we keep both divergent and convergent contributions in momentum space. For the evaluation of the convergent contributions we integrate over momenta from zero to infinity. The same should be done for the computation of the divergent contributions. The self-consistent integration in momentum space over an infinite region, i.e. $0 \leq k < \infty$, can be carried out by applying dimensional regularization [10]. Within the framework of the dimensional regularization we represent the result of the evaluation in terms of the parameter $M$ where $M > M_\sigma$. The parameter $M$ embodies all uncertainties of the manipulation with divergent integrals within this regularization scheme. As a result the quantities $\Delta_1(M_\sigma^2)$ and $\Delta_2(M_\sigma^2)$ differ in sign and are given by

\[
\Delta_2(M_\sigma^2) = - \Delta_1(M_\sigma^2) = \frac{M_\sigma^4}{8\pi^2} \ln \left( \frac{M}{M_\sigma} \right).
\]

The infinite series can be summed up to the form

\[
\sum_{n=3}^{\infty} \frac{1}{n(n-1)(n-2)} \left[ \frac{g^2 C_\mu(x) C^{\mu(x)}}{M_\sigma^2} \right]^n = - \frac{1}{2} \left[ \frac{g^2 C_\mu(x) C^{\mu(x)}}{M_\sigma^2} \right] \left[ 1 - \frac{3}{2} \frac{g^2 C_\mu(x) C^{\mu(x)}}{M_\sigma^2} \right] - \frac{1}{2} \left[ 1 - \frac{g^2 C_\mu(x) C^{\mu(x)}}{M_\sigma^2} \right]^2 \ln \left[ 1 - \frac{g^2 C_\mu(x) C^{\mu(x)}}{M_\sigma^2} \right].
\]

Thus the total contribution of the one-loop corrections is given by

\[
\frac{1}{2} \left\langle x \left\{ \ln \left( 1 - \frac{1}{\Box + M_\sigma^2} g^2 C_\mu C^{\mu} \right) \right\} x \right\rangle = - \frac{M_\sigma^4}{16\pi^2} \ln \left( \frac{M}{M_\sigma} \right) \left[ \frac{g^2 C_\mu(x) C^{\mu(x)}}{M_\sigma^2} \right] \left[ 1 - \frac{1}{2} \frac{g^2 C_\mu(x) C^{\mu(x)}}{M_\sigma^2} \right] - \frac{M_\sigma^4}{64\pi^2} \left[ \frac{g^2 C_\mu(x) C^{\mu(x)}}{M_\sigma^2} \right] \left[ 1 - \frac{3}{2} \frac{g^2 C_\mu(x) C^{\mu(x)}}{M_\sigma^2} \right] - \frac{M_\sigma^4}{64\pi^2} \left[ 1 - \frac{g^2 C_\mu(x) C^{\mu(x)}}{M_\sigma^2} \right]^2 \ln \left[ 1 - \frac{g^2 C_\mu(x) C^{\mu(x)}}{M_\sigma^2} \right].
\]

Substituting (33) in (26) we obtain the total effective Lagrangian of the $C_\mu$-field, containing tree and one-loop contributions caused by $\sigma$-field exchanges

\[
\mathcal{L}_{\text{eff}}(x) = - \frac{1}{4} F_{\mu\nu}(x) F^{\mu\nu}(x) - V [C_\mu(x)],
\]
where \( V[C_\mu(x)] \) is the effective potential of the \( C_\mu \)-field

\[
V[C_\mu(x)] = -\frac{1}{2} M_\sigma^2 C_\mu(x) C_\mu(x) \left[ 1 - g^2 \frac{C_\mu(x) C_\mu(x)}{M_\sigma^2} \right]^{-1} + \]

\[
+ \frac{M_\sigma^4}{16 \pi^2} \ell_n \left( \frac{M}{M_\sigma} \right) \left[ g^2 C_\mu(x) C_\mu(x) \right] \left[ 1 - \frac{1}{2} g^2 \frac{C_\mu(x) C_\mu(x)}{M_\sigma^2} \right] + 
\]

\[
+ \frac{M_\sigma^4}{64 \pi^2} \left[ g^2 C_\mu(x) C_\mu(x) \right] \left[ 1 - \frac{3}{2} g^2 C_\mu(x) C_\mu(x) \right] + 
\]

\[
+ \frac{M_\sigma^4}{64 \pi^2} \left[ 1 - \frac{g^2 C_\mu(x) C_\mu(x)}{M_\sigma^2} \right]^2 \ell_n \left[ 1 - \frac{g^2 C_\mu(x) C_\mu(x)}{M_\sigma^2} \right].
\]

(35)

In order to investigate the problem of the \( C_\mu \)-field condensation we have to find the minimum of the effective potential \( V[C_\mu(x)] \). For this aim it is convenient to introduce a variable \( \omega(x) = -g^2 C_\mu(x) C_\mu(x)/M_\sigma^2 \), so that \( \bar{\omega} \) is the vacuum expectation value, i.e., \( \bar{\omega} = -g^2 < C_\mu(x) C_\mu(x) > /M_\sigma^2 \). In terms of \( \bar{\omega} \) the vacuum expectation value of the effective potential calculated in the tree \( C_\mu \)-field exchange approximation reads

\[
\frac{4}{M_\sigma^4} V(\bar{\omega}) = \frac{1}{\kappa^2} \frac{\bar{\omega}}{1 + \bar{\omega}} - \frac{1}{8 \pi^2} \ell_n \left( \frac{M}{M_\sigma} \right) \bar{\omega} (2 + \bar{\omega}) - 
\]

\[
- \frac{1}{16 \pi^2} [\bar{\omega} \left( 1 + \frac{3}{2} \bar{\omega} \right) - (1 + \bar{\omega})^2 \ell_n (1 + \bar{\omega})].
\]

(36)

The minimum of \( V(\bar{\omega}) \) is defined by the condition

\[
\frac{2}{M_\sigma^4} \frac{dV(\bar{\omega})}{d\bar{\omega}} = \frac{1}{2 \kappa^2} (1 + \bar{\omega})^2 - \frac{1}{8 \pi^2} \ell_n \left( \frac{M}{M_\sigma} \right) (1 + \bar{\omega}) - 
\]

\[
- \frac{1}{16 \pi^2} [\bar{\omega} - (1 + \bar{\omega}) \ell_n (1 + \bar{\omega})] = 0.
\]

(37)

We have to search the solution of eq. (37) satisfying the constraint \( \bar{\omega} \ll 1 \). The small value of \( \bar{\omega} \) is connected with the small value of the scale characterizing the \( C_\mu \)-field fluctuations. The fluctuations of the \( C_\mu \)-field are induced by the effective potential (34) that have been obtained due to the \( \sigma \)-field interactions at the restriction \( \sigma(x) \ll v \). Therefore the scale of the \( C_\mu \)-field fluctuations should be also restricted by \( |C_\mu(x)| \ll v \). In the approximation \( \bar{\omega} \ll 1 \) eq. (37) does not have solutions.

Thus the vacuum expectation value of the effective potential calculated in the tree \( C_\mu \)-field exchange approximation does not acquire a minimum if \( \bar{\omega} \ll 1 \). This means that in the strong \( \kappa \)-coupling limit the \( C_\mu \)-field cannot be condensed. One can show that the minimum of the effective potential appears only at \( \bar{\omega} \gg 1 \). However this limit goes beyond our approximation admitting only small scale fluctuations of the \( C_\mu \)-field.

Therefore, the \( C_\mu \)-field cannot be condensed in the strong \( \kappa \)-coupling constant limit where small fluctuations of the \( C_\mu \)-field are allowed.

Now let us turn to the problem of magnetic current condensation? First we have to evaluate the magnetic current. For this aim we should perform an infinitesimal local gauge transformation of the \( C_\mu \)-field (11) and define the magnetic current as a derivative of the
Lagrangian (34) with respect to $\partial_{\mu} \alpha(x)$ (12). The infinitesimal change of the Lagrangian (34) caused by the gauge transformation (11) reads

$$\delta \mathcal{L}(x) = -2 \frac{\delta V[C_\nu(x)]}{\delta \omega(x)} C_\mu(x) \partial^\mu \alpha(x)$$  \hspace{1cm} (38)

This gives the expression of the magnetic current in terms of the $C_\mu$-field

$$\mathcal{J}_\mu(x) = 2 C_\mu(x) \frac{\delta V[C_\nu(x)]}{\delta \omega(x)}.$$  \hspace{1cm} (39)

The vacuum expectation value of the product $\mathcal{J}_\mu(x) \mathcal{J}^\mu(x)$ reads

$$<\mathcal{J}_\mu(x) \mathcal{J}^\mu(x)> = \left< C_\mu(x) C^\mu(x) \left[ \frac{\delta V[C_\nu(x)]}{\delta \omega(x)} \right]^2 \right>.$$  \hspace{1cm} (40)

In the tree $C_\mu$-field exchange approximation the r.h.s. of eq.(40) can be defined in terms of $\bar{\omega}$

$$<\mathcal{J}_\mu(x) \mathcal{J}^\mu(x)> = -\frac{M^2}{g^2} \bar{\omega} \left[ \frac{dV(\bar{\omega})}{d\bar{\omega}} \right]^2.$$  \hspace{1cm} (41)

It is seen that even if the $C_\mu$-field could be condensed the magnetic current condensation could not appear in the tree $C_\mu$-field exchange approximation. The former is due to eq.(37). Thus, the magnetic current condensation should be the matter of one-loop contributions of the $C_\mu$-field exchange.

Thus in the strong $\kappa$-coupling limit the dual Higgs model does not describe the magnetic current condensation at the classical level. This should be the matter of the quantum level.

### 3. Dual–vector field with a real mass

Now let us proceed to the discussion of the problem of the appearance of a real mass of the $C_\mu$-field due to the effective potential $V[C_\nu(x)]$ given by (35).

Recall that for $\omega(x) = -g^2 C_\mu(x) C^\nu(x) / M^2$ small compared with unity we can expand $V[C_\nu(x)]$ in powers of $\omega(x)$ picking up the terms proportional to $\omega(x)$ and $\omega^2(x)$ only. In this approximation and in the strong $\kappa$-coupling limit ($\kappa \gg 1$) we get

$$V[C_\nu(x)] = \frac{1}{2} M^2 \left[ M \frac{\kappa^2}{4 \pi^2} \ln \left( \frac{M}{M_\sigma} \right) \right] C_\mu(x) C^\mu(x) -$$

$$- \frac{g^4}{8 \pi^2} \ln \left( \frac{M}{M_\sigma} \right) \left[ C_\mu(x) C^\mu(x) \right]^2 + \ldots.$$  \hspace{1cm} (42)

The quantity

$$\bar{M}^2 = M^2 \left[ \frac{\kappa^2}{4 \pi^2} \ln \left( \frac{M}{M_\sigma} \right) \right]$$  \hspace{1cm} (43)
should be identified with the mass squared of the $C_\mu$-field. It is seen that the mass $\hat{M}_C$ is a real now.

Thus in the strong $\kappa$-coupling limit ($\kappa \gg 1$) the one-loop contributions of the $\sigma$-field exchange alter the sign of the $C_\mu$-field mass term. As a result the $C_\mu$-field acquires a real mass. In this case the classical solution of the $C_\mu$-field induced by a dual Dirac string, i.e. the electric field strength $E^{\mu\nu}(x)$, should have the shape of an Abrikosov flux line in a type II dual superconductor.

The effective Lagrangian describing the dual–vector field $C_\mu$ with real mass and self-interaction reads

$$L_{\text{eff}}(x) = -\frac{1}{4} F_{\mu\nu}(x) F^{\mu\nu}(x) -$$

$$-\frac{1}{2} \hat{M}_C^2 C_\mu(x) C^\mu(x) + \frac{1}{4} g^2 \frac{\hat{M}_C^2}{M_\sigma^2} \left[ C_\mu(x) C^\mu(x) \right]^2 +$$

$$+ L_{\text{free quark}}(x) + \ldots$$

(44)

Below we apply the effective Lagrangian (44) to the evaluation of a dual Dirac string action.

4. London’s equation of dual superconductivity

The magnetic current expressed in terms of the $C_\mu$-field can be obtained from the Lagrangian (44) by applying infinitesimal local gauge transformation to the $C_\mu$-field as this has been carried out above for the $C_\mu$-field (see eqs.(11) and (12))

$$J_\mu(x) = -\hat{M}_C^2 C_\mu(x) \left[ 1 - \frac{g^2}{M_\sigma^2} C^\nu(x) C_\nu(x) \right].$$

(45)

This should be valued as London’s equation of dual superconductivity expressing the magnetic current in terms of the $C_\mu$-field. Eq.(45) represents a nonlinear extension of London’s equation of dual superconductivity which should read [2,11]

$$J_\mu(x) = -\hat{M}_C^2 C_\mu(x).$$

(46)

In the strong $\kappa$-coupling limit eq.(45) is reducing to eq.(46).

By varying the Lagrangian (44) with respect to $C_\mu(x)$–field we get the equation of motion

$$\partial_\mu * F_{\mu\nu}(x) = -\hat{M}_C^2 C^\nu(x) + g^2 \frac{\hat{M}_C^2}{M_\sigma^2} \left[ C_\mu(x) C^\mu(x) \right] C^\nu(x).$$

(47)

Using the expression of the magnetic current $J_\mu(x)$ in term of the $C_\mu$–field (45) we represent the equation of motion (47) in the form

$$\partial_\mu * F_{\mu\nu}(x) = J^\nu(x),$$

(48)

that can be identified with the second pair of equations of motion of Dirac’s extension of Maxwell’s Electrodynamics [2,3,11].
By adding the equation of motion $\partial_\mu F^{\mu\nu}(x) = J^\nu(x)$, the first pair of equations of motion of Dirac's extension of Maxwell's Electrodynamics, that is satisfied identically due the $E^{\mu\nu}$-field, one can conclude that the equations of motion describing the symmetry-broken phase of the dual Higgs model can be reduced to the equations of motion of London's theory of dual superconductivity within Dirac's extension of Maxwell's Electrodynamics.

5. A dual string action

Now we can proceed to the evaluation of the action of a dual Dirac string connecting a pair of point-like electric charges $\pm Q$ that is a quark and an antiquark. In the large $M_\sigma$ expansion the classical solution of the $C^\mu$-field reads [11]

$$C^\nu(x) = -\int d^4 x' \Delta(x - x') \partial'_\mu \ast E^{\mu\nu}(x') + O(1/M_\sigma^2)$$

where $\Delta(x - x')$ is the Green function of the $C^\mu$-field obeying the equation

$$(\Box_x + M_\sigma^2) \Delta(x - x') = \delta^d(x - x').$$

Further following [11] we evaluate the string action defined by the effective Lagrangian (44) calculated at the solution (49)

$$S_{\text{string}} = \int d^4 x L_{\text{eff}}[E(x)] = S_N + S_{\text{free quark}} + \ldots$$

where $S_N$ is the string action of Nambu's form [2,11]. In terms of the string shape $S_N$ reads [2,11]

$$S_N = \frac{1}{4} Q^2 M^2 \int d^2 v d^2 v' \sigma^\mu v \Delta(X) \Delta(X - X') \sigma_{\mu\nu}(X') -$$

$$- \sum_{i,j} \frac{1}{2} Q_i Q_j \int d\tau d\tau' \frac{dX^\mu_i(\tau)}{d\tau} \frac{dX^\nu_j(\tau')}{d\tau'} \Delta(X_i(\tau) - X'_j(\tau')).$$

Then $S_{\text{free quark}} = \int d^4 x L_{\text{free quark}}(x)$ is the action of the free quark and antiquark. The ellipses in (50) denote the contribution of nonlinear and nonlocal string-string interactions of order of $O(1/M_\sigma^2)$. The contributions of these interactions are taken into account in Sect. 8 for the calculation of the string tension.

6. The string tension in leading order in the large $M_\sigma$ expansion

Thus we have shown that the dual Higgs model in the symmetry-broken phase can be reduced to a model, describing a self-interacting massive $C^\mu$-field. The equations of motion of the $C^\mu$-field can be solved in terms of a strength field $E^{\mu\nu}(x)$ induced by a dual Dirac string and depending on the string shape. The interquark potential produced
by a dual Dirac string should rise linearly at large relative distances and only depend on the string tension \( \sqrt{\sigma} = 450 \div 520 \text{ MeV} [1] \).

In this Section we calculate the string tension by keeping to leading terms in the large \( M_\sigma \) expansion. In turn in Sect. 8 we give the analysis of the influence of the string-string interaction, induced by the self-interaction of the \( C_\mu \)-field, on the string tension and calculate the string tension up to next-to-leading order in the large \( M_\sigma \) expansion.

For the calculation of the string tension we need to define the energy of a string. If the string action is defined by eq.\((51)\)

\[
S_{\text{string}} = \int d^4 x \, \mathcal{L}_{\text{eff}}[\mathcal{E}(x)],
\]

where \( \mathcal{L}_{\text{eff}}[\mathcal{E}(x)] \) is given by \((44)\) for the \( C_\mu \)-field expressed in terms of \( \mathcal{E}^{\mu \nu}(x) \), so the energy-momentum tensor of a dual Dirac string reads \([11]\)

\[
\Theta^{\mu \nu}_{\text{string}}(x) = - g^{\mu \nu} \mathcal{L}_{\text{eff}}[\mathcal{E}(x)].
\]  

This leads to the string energy defined by the integral

\[
W_{\text{string}} = \int d^3 x \, \Theta^{00}_{\text{string}}(x) = - \int d^3 x \, \mathcal{L}_{\text{eff}}[\mathcal{E}(x)].
\]  

For the calculation of the string tension it is sufficient to consider the static string. In the static case the nonzero components of the tensor \( \mathcal{E}^{\mu \nu}(x) \) are given by \([11]\)

\[
\mathcal{E}(\vec{x}) = Q \int_{\vec{X}_Q}^{\vec{X}_{-Q}} d\vec{X} \, \delta^{(3)}(\vec{x} - \vec{X}).
\]

where the integration has to be carried out along a dual Dirac string. The radius-vectors \( \vec{X}_Q \) and \( \vec{X}_{-Q} \) define the positions of quark and antiquark at the string ends.

In order to simplify the problem of the calculation of the string tension we suggest to consider an infinitely-long dual Dirac string directed along the \( z \)-axis. In the case of the infinitely-long string the string tension can be defined as the string energy per unit of length, i.e.

\[
\sigma = - \int d^2 r \, \mathcal{L}_{\text{eff}}[\vec{\mathcal{E}}(\vec{r})]
\]

where \( \vec{r} \) is the radius-vector in the plane perpendicular the \( z \)-axis, and

\[
\vec{\mathcal{E}}(\vec{r}) = \vec{e}_z Q \, \delta^{(2)}(\vec{r}).
\]

The unit vector \( \vec{e}_z \) is directed along the \( z \)-axis.

The classical solution \( C^{(0)\mu}(x) \) of the equation of motion \((48)\), obtained in leading order in the large \( M_\sigma \) expansion and expressed in terms of the tensor \( \mathcal{E}^{\mu \nu}(x) \), is given by the formula

\[
C^{(0)\mu}(x) = - \int d^4 x' \, \Delta(x - x') \, \partial_{\nu}^{*} \mathcal{E}^{\mu \nu}(x').
\]

Eq.\((57)\) differs from eq.\((50)\) by the change \( C^{\nu}(x) \rightarrow C^{(0)\nu}(x) \). Reflecting the fact that we are keeping to the leading order in the large \( M_\sigma \) expansion.
Substituting (56) in (57) we obtain the dual-vector potential \( \vec{C}^{(0)}(\vec{r}) \)
\[
\vec{C}^{(0)}(\vec{r}) = \frac{\vec{r} \times \vec{e}_z}{r} C^{(0)}_\alpha (r),
\]
(58)
The azimuthal component \( C^{(0)}_\alpha (r) \) is given by
\[
C^{(0)}_\alpha (r) = \frac{Q}{2 \pi} \frac{d \varphi (r)}{d r} = - \frac{Q \tilde{M}_C}{2 \pi} K_1(\tilde{M}_C r)
\]
(59)
where \( K_1(\tilde{M}_C r) \) is the McDonald function. The scalar function \( \varphi (r) \) is defined by
\[
\varphi (r) = \int \frac{d^2 k}{2 \pi} \frac{e^{i \vec{k} \cdot \vec{r}}}{\tilde{M}_C^2 + k^2} = \int_0^\infty \frac{d k k J_0(k r)}{\tilde{M}_C^2 + k^2} = K_0(\tilde{M}_C r)
\]
(60)
where \( J_0(k r) \) and \( K_0(\tilde{M}_C r) \) are Bessel and McDonald functions, respectively, and \( K_0'(\tilde{M}_C r) = - K_1(\tilde{M}_C r) \). Here and below primes imply derivatives with respect to the argument.

Now let us proceed to the calculation of the string tension \( \sigma \). It is convenient to represent \( \sigma \) as a decomposition
\[
\sigma = \sigma^{(0)} + \sigma^{(1)}
\]
(61)
where \( \sigma^{(0)} \) defines the string tension in leading order in the large \( M_\sigma \) expansion. In turn \( \sigma^{(1)} \) is of order \( O(1/M_\sigma^2) \) and describes the contribution to the string tension, caused by the string-string interactions calculated in next-to-leading order in the large \( M_\sigma \) expansion.

In this Section we calculate \( \sigma^{(0)} \) only. From formula (55) the string tension \( \sigma^{(0)} \) should be given by
\[
\sigma^{(0)} = - \int d^2 r \mathcal{L}_{\text{eff}}^{(0)} [\vec{E} (\vec{r})].
\]
(62)
The effective Lagrangian \( \mathcal{L}_{\text{eff}}^{(0)} [\vec{E} (\vec{r})] \) one obtains from (44) in leading order in the large \( M_\sigma \) expansion. It reads
\[
\mathcal{L}_{\text{eff}}^{(0)} [\vec{E} (\vec{r})] = \frac{1}{4} \tilde{M}_C^2 \int d^4 x' E_{\mu \nu} (x) \Delta (x - x') E^{\mu \nu} (x') =
\]
\[
= - \frac{1}{2} \tilde{M}_C^2 \int d^2 r' d^2 z' \vec{E} (\vec{r'}) \cdot \vec{E} (\vec{r'}) \int \frac{d^2 k k_z e^{i \vec{k} \cdot (\vec{r} - \vec{r'})} + i k_z (z - z')}{(2 \pi)^3 \tilde{M}_C^2 + k^2 + k_z^2} =
\]
\[
= - \frac{1}{2} Q^2 \tilde{M}_C^2 \int \frac{d^2 k}{(2 \pi)^2} \frac{e^{i \vec{k} \cdot \vec{r}}}{\tilde{M}_C^2 + k^2} = - \frac{Q^2 \tilde{M}_C^2}{4 \pi} \delta^{(2)}(\vec{r}) K_0(\tilde{M}_C r),
\]
(63)
Substituting (63) in (62) we obtain
\[
\sigma^{(0)} = \frac{Q^2 \tilde{M}_C^2}{4 \pi} \int d^2 r \delta^{(2)}(\vec{r}) K_0(\tilde{M}_C r).
\]
(64)
In the polar coordinates the \( \delta^{(2)}(\vec{r}) \)-function reads [14]
\[
\delta^{(2)}(\vec{r}) = \frac{1}{r} \delta (r) \delta (\alpha).
\]
(65)
The integration over $r$ and azimuthal angle $\alpha$ results
\[
\sigma^{(0)} = \frac{Q^2 \bar{M}_C^2}{4\pi} K_0(0).
\] (66)

The quantity $K_0(0)$ diverges logarithmically. Therefore it should be regularized
\[
K_0(0) \rightarrow \left[ K_0(0) \right]_R = \lim_{r \rightarrow 0} \int_0^\Lambda \frac{d k k J_{\alpha}(k r)}{M_C^2 + k^2} = \int_0^\Lambda \frac{d k k}{M_C^2 + k^2} = \frac{1}{2} \ln \left( 1 + \frac{\Lambda^2}{M_C^2} \right).
\] (67)

The regularized expression of the string tension $\sigma^{(0)}$ is given by
\[
\sigma^{(0)} = \frac{Q^2 \bar{M}_C^2}{4\pi} \left[ K_0(0) \right]_R = \frac{Q^2 \bar{M}_C^2}{8\pi} \ln \left( 1 + \frac{\Lambda^2}{M_C^2} \right).
\] (68)

The string tension given by (68) coincides fully with that calculated by Nambu [2]. Then $\Lambda$ is the cut-off in the direction perpendicular to the world sheet of a dual Dirac string. Following Nambu one can identify $\Lambda$ with the mass of the Higgs field, i.e. $\Lambda = M_\sigma$.

7. The string tension in next-to-leading order in the large $M_\sigma$ expansion

The string tension $\sigma^{(1)}$, describing the contributions of next-to-leading terms in the large $M_\sigma$ expansion, should be defined by
\[
\sigma^{(1)} = - \int d^2 r \mathcal{L}^{(1)}_{\text{eff}}[\vec{E}(\vec{r})].
\] (69)

The effective Lagrangian $\mathcal{L}^{(1)}_{\text{eff}}[\vec{E}(\vec{r})]$ reads
\[
\mathcal{L}^{(1)}_{\text{eff}}[\vec{E}(\vec{r})] = \frac{1}{2} \left( d C^{(1)} (x) \right)_{\mu \nu} \mathcal{E}^{\mu \nu}(x) - \frac{1}{4} g^2 \frac{\bar{M}_C^2}{M_\sigma^2} \left[ C^{(0)}(x) C^{(0) \mu}(x) \right]^2 = - \vec{E}(\vec{r}) \cdot \text{rot} \mathcal{C}^{(1)}(\vec{r}) - \frac{1}{4} g^2 \frac{\bar{M}_C^2}{M_\sigma^2} \left[ \mathcal{C}^{(0)}(\vec{r}) \right]^4 = - Q \delta^{(2)}(\vec{r}) \frac{1}{r} \frac{d}{d r} (r \mathcal{C}^{(0)}(r)) - \frac{1}{4} g^2 \frac{\bar{M}_C^2}{M_\sigma^2} \left[ \mathcal{C}^{(0)}(r) \right]^4.
\] (70)

$C^{(1)}_\alpha(r)$, the contribution of order $O(1/M_\sigma^2)$ to the dual-vector potential, is given by eq.(48)
\[
C^{(1)}_\alpha(r) = - g^2 \frac{\bar{M}_C^2}{M_\sigma^2} \int_0^\infty \frac{d k k J_{\alpha}(k r) J_{\alpha}(k r')}{M_C^2 + k^2} \left[ \mathcal{C}^{(0)}(r) \right]^3 = g^2 \frac{\bar{M}_C^2}{M_\sigma^2} \frac{Q^3 \bar{M}_C^5}{(2\pi)^3} \int_0^\infty d r' r' K^3_1 (\bar{M}_C r') \int_0^\infty \frac{d k k J_{\alpha}(k r) J_{\alpha}(k r')}{M_C^2 + k^2} = \frac{Q}{2\pi} \frac{\bar{M}_C^5}{M_\sigma^2} \int_0^\infty d r' r' K^3_1 (\bar{M}_C r') \int_0^\infty \frac{d k k J_{\alpha}(k r) J_{\alpha}(k r')}{M_C^2 + k^2}.
\] (71)
Here we have applied the charge quantization condition [3]

\[ g Q = 2\pi \]  

(72)

The contribution of \( C_\alpha^{(1)}(r) \) to the effective Lagrangian \( \mathcal{L}_{\text{eff}}^{(1)}[\vec{\xi}(\vec{r})] \) is given by

\[
\frac{1}{r} \frac{d}{dr} \left( r C_\alpha^{(1)}(r) \right) = \frac{Q}{2\pi} \frac{\bar{M}_C^5}{M_\sigma^2} \frac{1}{r} \int_0^\infty dr' r' K_1^3(\bar{M}_C r') \times \\
\times \left\{ \int_0^\infty \frac{d k k J_0(k r) J_0(k r')}{\bar{M}_C^2 + k^2} - r \int_0^\infty \frac{d k k^2 J_1(k r) J_0(k r')}{\bar{M}_C^2 + k^2} \right\} 
\]  

(73)

The integrals over \( k \) can be evaluated explicitly [15]

\[
\int_0^\infty \frac{d k k J_0(k r) J_0(k r')}{\bar{M}_C^2 + k^2} = \begin{cases} 
K_0(\bar{M}_C r) I_0(\bar{M}_C r') & , \ r > r' \\
I_0(\bar{M}_C r) K_0(\bar{M}_C r') & , \ r' > r 
\end{cases} 
\]  

(74)

\[
\int_0^\infty \frac{d k k^2 J_1(k r) J_0(k r')}{\bar{M}_C^2 + k^2} = \begin{cases} 
\bar{M}_C K_1(\bar{M}_C r) I_0(\bar{M}_C r') & , \ r > r' \\
-\bar{M}_C I_1(\bar{M}_C r) K_0(\bar{M}_C r') & , \ r' > r 
\end{cases} 
\]  

(75)

where \( I_\nu(\bar{M}_C r) \) (\( \nu = 0, 1 \)) are modified Bessel functions. Eq.(75) can be derived from eq.(74) by a direct differentiation over \( r \) and taking into account that \( K'_0(\bar{M}_C r) = -K_1(\bar{M}_C r) \) and \( I'_0(\bar{M}_C r) = I_1(\bar{M}_C r) \). Keeping to (74) and (75) the r.h.s. of (73) can be reduced to the form

\[
\frac{1}{r} \frac{d}{dr} \left( r C_\alpha^{(1)}(r) \right) = \frac{Q}{2\pi} \frac{\bar{M}_C^5}{M_\sigma^2} \frac{1}{r} \times \\
\times \left[ K_0(\bar{M}_C r) \int_0^r dr' r' I_0(\bar{M}_C r') K_1^3(\bar{M}_C r') + \\
+ I_0(\bar{M}_C r) \int_r^\infty dr' r' K_0(\bar{M}_C r') K_1^3(\bar{M}_C r') - \right. \\
- \bar{M}_C r K_1(\bar{M}_C r) \int_0^r dr' r' I_0(\bar{M}_C r') K_1^3(\bar{M}_C r') + \\
+ \bar{M}_C r I_1(\bar{M}_C r) \int_r^\infty dr' r' K_0(\bar{M}_C r') K_1^3(\bar{M}_C r') \right].
\]  

(76)

As a result the effective Lagrangian \( \mathcal{L}_{\text{eff}}^{(1)}[\vec{\xi}(\vec{r})] \) reads

\[
\mathcal{L}_{\text{eff}}^{(1)}[\vec{\xi}(\vec{r})] = -\frac{Q^2}{16\pi^2} \frac{\bar{M}_C^6}{M_\sigma^2} K_4(\bar{M}_C r) - \delta^{(2)}(\vec{r}) \frac{Q^2}{2\pi} \frac{\bar{M}_C^5}{M_\sigma^2} \frac{1}{r} \times \\
\times \left[ K_0(\bar{M}_C r) \int_0^r dr' r' I_0(\bar{M}_C r') K_1^3(\bar{M}_C r') + \\
+ I_0(\bar{M}_C r) \int_r^\infty dr' r' K_0(\bar{M}_C r') K_1^3(\bar{M}_C r') - \right. \\
- \bar{M}_C r K_1(\bar{M}_C r) \int_0^r dr' r' I_0(\bar{M}_C r') K_1^3(\bar{M}_C r') + \\
+ \bar{M}_C r I_1(\bar{M}_C r) \int_r^\infty dr' r' K_0(\bar{M}_C r') K_1^3(\bar{M}_C r') \right].
\]  

(77)
It is seen that the effective Lagrangian \( \mathcal{L}^{(1)}_{\text{eff}}[\bar{\mathcal{E}}(\bar{r})] \) is very singular at \( r \to 0 \). Thereby any application of this Lagrangian demands very careful handling.

First we proceed to the calculation of the contribution to the string tension, entering from the first term in (77). We denote

\[
\sigma_1^{(1)} = \frac{Q^2}{16\pi^2} \frac{\hat{M}_C^2}{\hat{M}_\sigma^2} \int d^2r \ K_1^4(\hat{M}_C r).
\]

The integral over \( \bar{r} \) diverges quadratically. By applying the cut-off regularization we get

\[
\int d^2r \ K_1^4(\hat{M}_C r) = \pi \frac{\Lambda^2}{\hat{M}_C^2} + \frac{1}{\hat{M}_C} O\left( \log\left( \frac{\Lambda}{\hat{M}_C} \right) \right) + \ldots
\]

where ellipses denote the contribution independent on \( \Lambda \). As a result \( \sigma_1^{(1)} \) reads

\[
\sigma_1^{(1)} = \frac{Q^2 \hat{M}_C^2}{8\pi} \left[ \frac{1}{2} \frac{\Lambda^2}{\hat{M}_C^2} + \frac{\hat{M}_C^2}{\hat{M}_\sigma^2} O\left( \log\left( \frac{\Lambda}{\hat{M}_C} \right) \right) \right].
\]

At \( M_\sigma \gg \hat{M}_C \) the main contribution comes from the quadratically divergent term, i.e.,

\[
\sigma_1^{(1)} = \frac{Q^2 \hat{M}_C^2}{8\pi} \left[ \frac{1}{2} \frac{\Lambda^2}{\hat{M}_C^2} \right].
\]

Thus the next-to-leading correction in the large \( M_\sigma \) expansion has turned out to be quadratically divergent. Thereby in the case of an arbitrary large cut-off \( \Lambda \), i.e. \( \Lambda \gg M_\sigma \), this correction could give a substantial contribution with respect to \( \sigma^{(0)} \). However we can avoid this problem putting \( \Lambda = M_\sigma \ [2] \). As a result we get

\[
\sigma_1^{(1)} = \frac{1}{2} \frac{Q^2 \hat{M}_C^2}{8\pi}.
\]

For \( M_\sigma \gg \hat{M}_C \) the magnitude of \( \sigma_1^{(1)} \) defined by (82) is logarithmically small compared with respect to \( \sigma^{(0)} \).

Now we proceed to the estimate of the contribution to the string tension from the term proportional to the \( \delta^{(2)}(\bar{r}) \)-function. If \( \delta^{(2)}(\bar{r}) \) is a standard Dirac \( \delta \)-function, so the contribution of this term is described by the integrand at \( r = 0 \). Due to the singular behaviour of the integrand at \( r \to 0 \) this contribution is fully undefined. Thereby one should conclude that standard Dirac \( \delta \)-functions are not good defined to be adjusted for the calculations of parameters induced by such singular objects as strings. In order to make these calculations much more definite it should be convenient to apply the \( \delta \)-functions introduced by Infeld and Plebański [16]. They denoted them as \( \hat{\delta} \) and called "good" \( \hat{\delta} \)-functions. The \( \hat{\delta} \)-functions possess all properties of the standard Dirac \( \delta \)-functions with the additional one

\[
\int d^2r \frac{\hat{\delta}^{(2)}(\bar{r})}{r p} = 0
\]
p \geq 1$. This allows to carry out integrations with integrands singular at $r \to 0$.

Thus to define the contribution under consideration we have to change everywhere
\[
\delta^{(2)}(\vec{r}) \to \hat{\delta}^{(2)}(\vec{r}) = \frac{1}{r} \hat{\delta}(r) \hat{\delta}(\alpha).
\]

This does not destroy the results of the calculations having been performed above but is of use for the description of singular contributions.

The correction to the string tension from the term proportional to the $\hat{\delta}$-function term in the effective Lagrangian $L_{\text{eff}}[\tilde{E}(\vec{r})]$ (77) is given by
\[
\sigma_2^{(1)} = \frac{Q^2}{2\pi} \frac{\tilde{M}_G^5}{M_c^2} \int_0^\infty \frac{d\hat{\delta}(r)}{r} \times \\
\times \left[ K_0(\tilde{M}_G r) \int_0^\infty dr' r' I_0(\tilde{M}_G r') K_1^3(\tilde{M}_G r') + \\
+ I_0(\tilde{M}_G r) \int_0^\infty dr' r' K_0(\tilde{M}_G r') K_1^3(\tilde{M}_G r') - \\
- \tilde{M}_G r K_1(\tilde{M}_G r) \int_r^\infty dr' r' I_0(\tilde{M}_G r') K_1^3(\tilde{M}_G r') + \\
+ \tilde{M}_G r I_1(\tilde{M}_G r) \int_r^\infty dr' r' K_0(\tilde{M}_G r') K_1^3(\tilde{M}_G r') \right].
\]

By applying the property of the $\hat{\delta}$-function (83) one should obtain that the integral over $r$ equals zero. This gives one
\[
\sigma_2^{(1)} = 0.
\]

Thus the correction to the string tension, caused by the string-string interaction determined in next-to-leading order in the large $M_c$ expansion, is fully defined by $\sigma_1^{(1)}$. As a result the total string tension is given by
\[
\sigma = \frac{Q^2 \tilde{M}_G^2}{8\pi} \left[ \ln \left( 1 + \frac{M_c^2}{\tilde{M}_G^2} \right) + \frac{1}{2} \right].
\]

Here following Nambu [2] we have identified the cut-off $\Lambda$ with the mass of the Higgs field, i.e., $\Lambda = M_c$.

Due to the inequality $M_c \gg \tilde{M}_G$ the contribution of the string-string interaction calculated in next-to-leading order in the large $M_c$ expansion is rather small and does not change essentially the result obtained in leading order in the large $M_c$ expansion. Therefore one can conclude that the string-string interactions induced by the self-interaction of the dual-vector field cannot influence substantially the magnitudes of physical parameters such as the string tension.

**Conclusion**

We have analysed the dual Higgs model with dual Dirac strings. We have shown that in the large Higgs field mass expansion, corresponding to the strong Higgs field coupling constant $\kappa$ limit, the dual Higgs model can be reduced to the effective theory of a massive
We have calculated the effective potential of the $C_{\mu}$-field including one-loop contributions of the Higgs field exchange. We have found that due to one-loop contributions to the effective potential the $C_{\mu}$-field acquires a real mass. The mass of the $C_{\mu}$-field has been found imaginary in the tree Higgs field exchange approximation.

We have shown that the effective theory of the $C_{\mu}$-field can be described by the equations of motion that can be identified with the equations of motion of Dirac's extension of Maxwell's Electrodynamics supplemented by London's equation of dual superconductivity. We have found a nonlinear extension of London's equation of dual superconductivity reducing to the linear one in leading order in the large $M_\sigma$ expansion that corresponds the strong $\kappa$-coupling limit. In the symmetry-broken phase the massive $C_{\mu}$-field has the form of a dual Abrikosov flux line in type II superconductivity, and the reverse power of the $C_{\mu}$-field mass $1/M_C$ compares with the penetration depth $\lambda$ of the electric field in a dual superconductor.

The estimate of contributions of the self-interaction of the $C_{\mu}$-field has been carried out for the evaluation of the string tension. We have found that the corrections can get small if the cut-off $\Lambda$, regularizing divergences in the direction perpendicular to the world sheet of a dual Dirac string, is identified with the mass of the Higgs field, i.e. $\Lambda = M_\sigma$. Therewith the contribution to the string tension, coinciding with that calculated by Nambu [2], should be dominant for $M_\sigma \gg M_C$ and $\ell n(M_\sigma/M_C) \gg 1$.

We have analysed the magnetic current condensation. Since the magnetic current should be expressed in terms of the $C_{\mu}$-field, the problem of the magnetic current condensation has been reduced to the problem of the $C_{\mu}$-field condensation. We have found that the $C_{\mu}$-field cannot be condensed in the tree $C_{\mu}$-field exchange approximation. Then we have shown that the magnetic current cannot be condensed in the $C_{\mu}$-field exchange approximation even if the $C_{\mu}$-field would be condensed. Thus, the condensation of the magnetic current should be the matter of the one-loop contributions of the $C_{\mu}$-field exchange. On this way the simplest expression of $<\mathcal{J}_\mu(x)\mathcal{J}^\mu(x)>$ one can obtain by applying London's equation of dual superconductivity (45). Since the nonlinear contributions are not important, so that the lowest approximation gives $<\mathcal{J}_\mu(x)\mathcal{J}^\mu(x)> = M_C^4 <C_{\mu}(x)C^{\mu}(x)>$. In the one-loop approximation the r.h.s. of this relation is the Green function of the $C_{\mu}$-field. By analogy with $<\mathcal{J}_\mu(x)\mathcal{J}^\mu(x)>$ one can evaluate the vacuum expectation value of any order derivative of the magnetic current, i.e. $<\partial_{\mu_1}\ldots\partial_{\mu_n}\mathcal{J}_\mu(x)\partial^{\nu_1}\ldots\partial^{\nu_n}\mathcal{J}^\nu(x)>$. The obtained results should testify the affinity between CQED and the dual Higgs model.

In the succession of papers [17] Baker, Ball and Zachariazen have been investigating the problems of dual QCD. Within this approach dual gluons acquire masses due to Higgs mechanism. This mass is real that has been inspired by the kinetic term of dual gluons included in the dual QCD Lagrangian with a negative sign. This has been made, since "one does not in practice know how to express the Yang–Mills Lagrangian in terms of dual potentials. In other words, "dual Yang–Mills" theory (i.e. the Yang–Mills Lagrangian as a function of the dual variables $C_{\mu}$) cannot be explicitly written down." As a result the problem of the imaginary mass of dual gluons does not emerge in a dual QCD within the formulation suggested by Baker, Ball and Zachariazen.

In the conclusion we would like to tell a few words concerning the application of the strong $\kappa$-coupling limit in the standard Higgs model with a vector field. As we have shown the Higgs mechanism does not distinguish vector and dual-vector fields. Thereby,
the inclusion of one-loop contributions of the $\sigma$-field exchange calculated in the strong $\kappa$-coupling constant limit should change the sign of the mass term of a vector field as this has occurred in the case of a dual-vector one. The former is rather crucial for stability of the vector field. Thus the application of the strong $\kappa$-coupling limit within the standard Higgs model becomes a sensitive procedure. We are inclined to deem that most likely this limit does not exist in the standard Higgs model. Therefore our approach cannot be extended on the standard Higgs model.

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Appendix

In this Appendix we outline the procedure of the construction of the Lagrangian (1) that fixes the sign of the kinetic term of a dual-vector field. In order to show that the appearance of the imaginary mass of the dual-vector field $C_{\mu}$ is fully due to its dual properties we consider the Higgs mechanism of the acquirement of the mass for an arbitrary vector field. We show that the Higgs mechanism does not distinguish vector fields from dual-vector ones. Thus the imaginary mass of the $C_{\mu}$-field is inspired by the kinetic term $\frac{1}{4}(dC(x))^{\mu\nu}(dC(x))^\mu\nu$ entering to the Lagrangian (1) with a positive sign instead of having a negative like a vector field.

Now let us commence from the discussion of the Lagrange formulation of Dirac’s extension of Maxwell’s Electrodynamics without strings. If one would like to describe the system of electric and magnetic charges without Dirac strings, one should write the Lagrangian that is given by

$$\mathcal{L}(x) = -\frac{1}{4} F_{\mu\nu}(x) F^{\mu\nu}(x) - A_{\mu}(x) J^\mu(x) + C_{\mu}(x) \mathcal{J}^\mu(x),$$  \hspace{1cm} (88)

where $A_{\mu}(x)$ and $C_{\mu}(x)$ are vector and dual-vector potentials, $J_{\mu}(x)$ and $\mathcal{J}_{\mu}(x)$ are external electric and magnetic currents, and

$$F^{\mu\nu}(x) = (dA(x))^{\mu\nu} - *(dC(x))^{\mu\nu}$$ \hspace{1cm} (89)

is the field strength tensor. Varying the Lagrangian (88) with respect to $A_{\mu}(x)$ and $C_{\mu}(x)$ we obtain the equations of motion

$$\partial_\mu F^{\mu\nu}(x) = J^\nu(x),$$
$$\partial_\mu *F^{\mu\nu}(x) = \mathcal{J}^\nu(x).$$ \hspace{1cm} (90)

Thus the Lagrangian (88) and the equations of motion (90) describe the interactions of electric and magnetic charges. Eqs.(90) are responsible on the electric and magnetic Gauss law, respectively.

Now let us include Dirac strings. If this is magnetic Dirac strings with point-like magnetic charges at the ends, the Lagrangian (88) should undergo the following changes

$$\mathcal{L}(x) = -\frac{1}{4} F_{\mu\nu}(x) F^{\mu\nu}(x) - A_{\mu}(x) J^\mu(x)$$ \hspace{1cm} (91)
where

\[ F^{\mu \nu}(x) = (dA(x))^{\mu \nu} - \ast G^{\mu \nu}(x), \]
\[ G^{\mu \nu}(x) = g \int d^2 z \delta^{(4)}(x - X) \sigma^{\mu \nu}(X) \]  
(92)

where \( g \) is the magnetic charge. The equation of motion \( \partial_\mu \ast F^{\mu \nu}(x) = \mathcal{J}^{\nu}(x) \), describing the magnetic Gauss law, is satisfied identically. The magnetic current \( \mathcal{J}^{\nu}(x) \) is given by eq.(2) with the change \( Q \to g \).

In the case of dual Dirac strings with point–like electric charges ending the string, the Lagrangian (88) should be changed as follows

\[ \mathcal{L}(x) = -\frac{1}{4} F_{\mu \nu}(x) F^{\mu \nu}(x) + C_\mu(x) \mathcal{J}^{\mu}(x) \]  
(93)

where

\[ F^{\mu \nu}(x) = \mathcal{E}^{\mu \nu}(x) - \ast (dC(x))^{\mu \nu}, \]
\[ \mathcal{E}^{\mu \nu}(x) = Q \int d^2 z \delta^{(4)}(x - X) \sigma^{\mu \nu}(X) \]  
(94)

where \( Q \) is the electric charge. The equation of motion \( \partial_\mu F^{\mu \nu}(x) = J^{\nu}(x) \), describing the electric Gauss law, is satisfied identically. The electric current \( J^{\nu}(x) \) is given by eq.(2).

Now if we want to sink our system of electric and magnetic charges with Dirac strings in a superconducting medium, we should include the source of the superconducting medium. If we assume that the source of the superconducting medium can be described by London equation of superconductivity, so that we have to turn to a Higgs model inducing this equation in the symmetry–broken phase.

The Higgs sector of the total Lagrangian invariant under local \( U(1) \) gauge group, i.e. describing the symmetric phase, reads

\[ \mathcal{L}_H(x) = (\partial_\mu + i g_V V_\mu(x)) \Phi^* (x) (\partial^\mu - i g_V V^\mu(x)) \Phi(x) - \kappa^2 (v^2 - \Phi^* (x) \Phi(x))^2. \]  
(95)

Here \( V_\mu(x) \) stands by for \( A_\mu(x) \) or \( C_\mu(x) \), and \( g_V \) should be \( g \) and \( Q \), respectively. We keep the \( V_\mu \)-field unfixed in order to show that the Higgs mechanism does not distinguish vector fields from dual–vector ones. Then \( \Phi(x) \) is a complex Higgs field with a vacuum expectation value \( \langle \Phi(x) \rangle = v \).

Applying the polar representation

\[ \Phi(x) = \rho(x) e^{i \vartheta(x)} \]  
(96)

we reduce the Lagrangian (95) to the form

\[ \mathcal{L}_H(x) = (\partial_\mu + i g_V V_\mu(x)) \rho(x) (\partial^\mu - i g_V V^\mu(x)) \rho(x) - \kappa^2 (v^2 - \rho^2(x))^2 \]  
(97)

where \( V_\mu(x) = V^\mu(x) - (1/g_V) \partial_\mu \vartheta(x) \). Below we omit tilde over the \( V_\mu \)-field.

In order to pass to the symmetry–broken phase we have to shift the \( \rho(x) \)-field

\[ \rho(x) = v + \frac{1}{\sqrt{2}} \sigma(x). \]  
(98)
This brings up the Lagrangian (97) to the form

\[
\mathcal{L}_H(x) = \frac{1}{2} M_V^2 V_\mu(x) V^\mu(x) + g M_V \sigma(x) \left[ 1 + \frac{\kappa}{\sqrt{2} M_\sigma^2} \right] V_\mu(x) V^\mu(x) \\
+ \frac{1}{2} \partial_\mu \sigma(x) \partial^\mu \sigma(x) - \frac{1}{2} M_\sigma^2 \sigma^2(x) \left[ 1 + \frac{\kappa}{\sqrt{2} M_\sigma} \right]^2.
\]

(99)

where

\[
M_V^2 = 2 g^2 v^2, \quad M_\sigma^2 = 4 \kappa^2 v^2.
\]

(100)

The quantity \(M_V^2\) should be identified with the mass squared of the \(V_\mu\)-field, and \(M_\sigma\) is the mass of the Higgs field.

It should be emphasized that we do not specify which \(V_\mu\)-field we use, i.e. \(A_\mu\) or \(C_\mu\). This should imply that the Higgs mechanism does not distinguish vector and dual-vector fields. Thus the Higgs sector in the symmetry-broken phase gives the same contribution to the total Lagrangian for vector and dual-vector fields. The distinction between vector and dual-vector fields should appear after the mergence of the Lagrangians (91) and (93) with a Higgs sector, i.e. the Lagrangian (99).

First let us consider the standard Higgs model producing ordinary superconductivity. In this case \(V_\mu = A_\mu\) and \(g_V = g\). The total Lagrangian in the symmetry-broken phase reads

\[
\mathcal{L}_A(x) = -\frac{1}{4} F_{\mu\nu}(x) F^{\mu\nu}(x) + \\
\frac{1}{2} M_A^2 A_\mu(x) A^\mu(x) + g M_A \sigma(x) \left[ 1 + \frac{\kappa}{\sqrt{2} M_\sigma} \right] A_\mu(x) A^\mu(x) \\
+ \frac{1}{2} \partial_\mu \sigma(x) \partial^\mu \sigma(x) - \frac{1}{2} M_\sigma^2 \sigma^2(x) \left[ 1 + \frac{\kappa}{\sqrt{2} M_\sigma} \right]^2.
\]

(101)

The term \(A_\mu(x) J^\mu(x)\) should be omitted, for the electric current is induced now by a Higgs field.

Substituting \(F_{\mu\nu}(x)\) in the form of (92) and using the constraint \(\partial_\mu A^\mu = 0\) we get

\[
\mathcal{L}_A(x) = \frac{1}{4} G_{\mu\nu}(x) G^{\mu\nu}(x) - A_\nu(x) \partial_\mu * G^{\mu\nu}(x) + \\
\frac{1}{2} A_\mu(x) \left( \Box + M_A^2 \right) A^\mu(x) + \\
+ g M_A \sigma(x) \left[ 1 + \frac{\kappa}{\sqrt{2} M_\sigma} \right] A_\mu(x) A^\mu(x) + \\
+ \frac{1}{2} \partial_\mu \sigma(x) \partial^\mu \sigma(x) - \frac{1}{2} M_\sigma^2 \sigma^2(x) \left[ 1 + \frac{\kappa}{\sqrt{2} M_\sigma} \right]^2
\]

(102)

where we have used the identity \(* G_{\mu\nu}(x) * G^{\mu\nu}(x) = - G_{\mu\nu}(x) G^{\mu\nu}(x)\). Varing the Lagrangian (102) with respect to \(A_\mu(x)\) we get the equation of motion of the \(A_\mu\)-field

\[
(\Box + M_A^2) A^\nu(x) = \partial_\nu * G^{\mu\nu}(x) - 2 g M_C \sigma(x) \left[ 1 + \frac{\kappa}{\sqrt{2} M_\sigma} \right] A^\nu(x).
\]

(103)
It is seen that the l.h.s. of eq.(103) describes the vector field with a real mass $M_A$ [10].

Now let us turn to a dual superconductivity. The total Lagrangian invariant under local $U(1)$ gauge transformations should be described by the sum of the Lagrangian (93), where the term $C_\mu(x)J^\mu(x)$ is omitted for the magnetic current should be induced by the Higgs field, and the Lagrangian (95) at $V_\mu = C_\mu$ and $g_V = Q$ (see (1)). In the symmetry–broken phase the total Lagrangian reads

$$\mathcal{L}_C(x) = -\frac{1}{4} F_{\mu\nu}(x) F^{\mu\nu}(x) +$$
$$+ \frac{1}{2} M_A^2 C_\mu(x) C^\mu(x) + g M_C \sigma(x) \left[ 1 + \frac{\kappa}{\sqrt{2} M_\sigma} \right] C_\mu(x) C^\mu(x) \quad (104)$$

$$+ \frac{1}{2} \partial_\mu \sigma(x) \partial^\mu \sigma(x) - \frac{1}{2} M_\sigma^2 \sigma^2(x) \left[ 1 + \frac{\kappa}{\sqrt{2} M_\sigma} \right]^2.$$

Substituting $F^{\mu\nu}(x)$ in the form of (94) and using the constraint $\partial_\mu C^\mu = 0$ we get

$$\mathcal{L}_C(x) = -\frac{1}{4} \mathcal{E}_{\mu\nu}(x) \mathcal{E}^{\mu\nu}(x) - C_\mu(x) \partial_\nu \mathcal{E}^{\mu\nu}(x) +$$
$$+ \frac{1}{2} C_\mu(x)(-\Box + M_C^2)C^\mu(x) +$$
$$+ g M_C \sigma(x) \left[ 1 + \frac{\kappa}{\sqrt{2} M_\sigma} \right] C_\mu(x) C^\mu(x) +$$
$$+ \frac{1}{2} \partial_\mu \sigma(x) \partial^\mu \sigma(x) - \frac{1}{2} M_\sigma^2 \sigma^2(x) \left[ 1 + \frac{\kappa}{\sqrt{2} M_\sigma} \right]^2 \quad (105)$$

where we have used the identity $*(dC(x))_{\mu\nu} * (dC(x))^{\mu\nu} = -(dC(x))_{\mu\nu} (dC(x))^{\mu\nu}$. Varying the Lagrangian (105) with respect to $C_\mu(x)$ we get the equation of motion of the $A_\mu$-field

$$(-\Box + M_C^2)C^\nu(x) = \partial_\nu \mathcal{E}^{\mu\nu}(x) - 2 g M_C \sigma(x) \left[ 1 + \frac{\kappa}{\sqrt{2} M_\sigma} \right] C^\nu(x). \quad (106)$$

It is seen that the quabla enters with a negative sign. Due to this the l.h.s. of eq.(106) describes the vector field with an imaginary mass. For the sign of the quabla is fixed by the kinetic term of the dual–vector field in the Lagrangian (104), so that just dual properties of the $C_\mu$–field matter the imaginary mass obtained within the dual Higgs model.
References


