Collection of Abstracts
of all Lectures
given at the
Workshop on Schrödinger Operators,
Vienna, December 8–12, 1993

Editor: Thomas Hoffmann-Ostenhof
A PERTURBATION OF A PERIODIC SCHRODINGER OPERATOR BY A MODULATED DECAYING POTENTIAL

M. Sh. BIRMAN

Dept. of Physics, St. Petersburg University, Ulianov St. 1, Petrodvoretz, St. Petersburg, 198904 Russia

ABSTRACT. We consider a perturbation of a periodic Schrödinger operator $H_0$ by a nonnegative potential $V$ with asymptotics

$$V(x) \sim f\left(\frac{x}{\sqrt{\lambda T}}\right) |x|^{2\sigma} g(x), \quad \sigma > 0, \quad |x| \to \infty$$

where $g$ is also periodic. The total number of eigenvalues of the operator $H(\alpha) = H_0 + \alpha V$, $\alpha > 0$, in a gap of $H_0$ is studied. We discuss a difference of its asymptotics as $\alpha \to \infty$ compared to the case $g(x) = 1$. 

Typeset by AMSTeX
A METHOD TO COMPARE OPERATORS. APPLICATION TO SCHRÖDINGER AND DIRAC OPERATORS

M. Bordoni

Dipartimento Di Matematica "G. Catelnuovo", Università ' Degli Studi "La Sapienza", P.le Aldo Moro 2, I-00185 Roma

Abstract. Let \((M, \mu) (M', \mu')\) be two measure spaces and let \(\pi : M' \to M\) be any mapping verifying Fubini's property. From an Hilbertian theorem (which generalizes a theorem of S. Gallot and D. Meyer), we deduce a general criterion to compare the spectra of two semibounded self-adjoint operators \(T\) and \(T'\) acting on the Hilbert spaces \(L^2(M)\) and \(L^2(M')\) Res \(P\), under the assumption that they verify Kato's inequality with respect to the mapping \(\tilde{\pi} : L^2(M') \to L^2(M)\) defined, for \(x \in M\), by

\[
(\tilde{\pi}(f))(x) = \|f_{\pi^{-1}(x)}\|_{L^2(\pi^{-1}(x))}.
\]

We apply this principle in the case of a Riemannian manifold vector bundle \(E \to M\) (\(M\) is a Riemannian manifold), endowed with a compatible connection \(D\), to compare the spectra of the operator \(D^*D + \pi\), where \(\pi\) is any field of symmetric endomorphisms of the fibers and of the Schrödinger operator \(\Delta_M + \pi\), whose potential function is \(R_{\text{min}}(x) = \text{minimal eigenvalue of } \pi_x : E_x \to E_x\). Our results contain a particular cases, previous estimates given by other authors (S. Gallot and D. Meyer for the Hodge-de-Rham Laplacian acting on differential forms, T. Friedrich for Dirac operator acting on spinors).

Proofs, remarks, other applications and references may be found in:

SEMI-CLASSICAL TUNNELING FOR SCHRODINGER OPERATOR AND GRAPH'S EMBEDDING

YVES COLIN DE VERDIERE
Institut Fourier, BP74, F-38402 St. Martin d'Heres Cedex

ABSTRACT. We describe a new proof of a planarity criterium for graph using the Helffer-Sjöstrand analysis of semi-classical tunnel effect. We introduce a spectral invariant $\mu(F)$ for a finite graph which is monotonic with respect to minors.

POSITIVE COMMUTATORS AND
PETURBATION OF SINGULAR SPECTRUM

J. M. COMBES

Dept. Mathématiques, Faculté des Sciences et Techniques, University de Toulon, F-83957 La Garde

ABSTRACT. A general criteria is given ensuring that for a given one parameter family of self-adjoint operators \( \{ H_\lambda, \lambda \in \Gamma = [\lambda_0, \lambda_1] \} \) with spectral projections \( P_\lambda(\cdot) \) one has \( P_\lambda(J) = 0 \) almost everywhere \( \lambda \) if \( J \) has zero Lebesgue measure. The conditions involve twice differentiability with respect to \( \lambda \), a local positivity assumption on
\[
\frac{dH_\lambda}{d\lambda}
\]
and a simple type of relative bound on
\[
\frac{d^2H_\lambda}{d\lambda^2}
\]
This criteria provides the so called Kotani’s trick when applied to the perturbation theory of singular spectrum à la Simon-Woelfl-Howland. If the \( H_\lambda \)’s are unitarily equivalent it also provides absolute continuity results of the positive commutator type. This is a joint work with P. Hislop and E. Mourre.
LOCALIZATION VERSUS DIFFUSION FOR ‘KICKED’ QUANTUM SYSTEMS

M. Combescure

Université de Paris Sud, Centre d’Orsay, Laboratoire de Physique Théorique et Hautes Energies, Bâtiment 211, F-91405 Orsay Cedex

Abstract. We study the quantum long time behaviour for ‘kicked’ Hamiltonian systems, namely for time-dependent Hamiltonians of the form:

$$H(t) = H_0 + \lambda V \sum_{n=-\infty}^{+\infty} \gamma_n \delta(t - nT)$$

where $H_0$ and $V$ are self-adjoint operators in a suitable Hilbert space. $\lambda$ is a real coupling constant, and the time dependence occurs via sums of ‘delta-functions’ located at integer multiples of some period $T$, and is modulated by a deterministic sequence $\gamma_n$ taking values in $\{-1,0,+1\}$. Typically, $\gamma_n$ is chosen to be induced by some substitution rule, like the Fibonacci or Thue-Morse substitution rule. The question is to investigate how the intrinsic disorder of the sequence $\gamma_n$ measured by the nature of its ‘Fourier spectrum’ manifests itself in the quantum long time behaviour, specifically in the suitably defined ‘quantum autocorrelation measure’. Exact results, in the case of the self similar Thue-Morse sequence $\gamma_n$ are presented on two simple quantum systems.

TRACE CLASS CRITERIA IN
STOCHASTIC SPECTRAL ANALYSIS

M. DEMUTH
Max-Planck-Institut, University of Potsdam,
Mathematics Dept., P.O. Box 601553, D-14415 Potsdam

Abstract. Using the framework of stochastic spectral analysis we describe a class of
non-trace-class operators with finite trace. For Feller operators the kernels of the asso-
ciated absorption semigroups can be represented explicitly. The difference between
the free semigroup and the absorption semigroup is determined by the equilibrium
potential. This difference has a finite trace and is a Hilbert-Schmidt operator if the
capacity of the singularity region is finite. But there are natural classes of singularity
regions such that these operators do not belong to the trace ideal.

M. Demuth, P. Stollmann, G. Stolz, J. van Casteren, Trace norm estimates for pro-
ducts of integral operators and diffusion semigroups, Preprint, Max-Planck-
Institute, Bonn (1994).
SCATTERING THEORY FOR TIME-DEPENDENT POTENTIALS

J. DEREZIŃSKI

Dept. Math. Meth. in Phys., Warsaw University, Hoża, P-00-682 Warsaw

ABSTRACT. Scattering theory for time-dependent potentials satisfying

\[ |o^\alpha V(t, x)| \leq c_\alpha < t > ^{-\mu |\alpha|} \quad |\alpha| = 0, 1, 2, \quad \mu > 0 \]

was presented, both in the classical and in the quantum case. In this setting it is easy to illustrate basic ideas of the long-range scattering theory without some of the technical difficulties characteristic of time-independent potentials.
MAGNETIC LIEB-THIRRING INEQUALITIES

László Erdős

Abstract. I study the generalization of the well-known magnetic Lieb-Thirring inequality for the moments of the negative eigenvalues of the Pauli operator describing a nonrelativistic spin-1/2 electron in a nonhomogeneous magnetic field with electric potential. The obtained bound is consistent with the expected semiclassical formula.

The method is probabilistic via the magnetic Feynman-Kac formula. I prove precise pointwise upper bound on the heat kernel of the unperturbed operator which requires nontrivial estimates on a stochastic oscillatory integral. Instead of the usual PDE localization technique in the configurational (or phase) space, I develop a localization method in the path space which relies on the reflection properties of the Brownian bridge. This setup is intrinsically probabilistic and can be used in the study of other diamagnetic phenomena.
RESONANCE COUPLING OF ONE-DIMENSIONAL
SCHRÖDINGER OPERATORS

P. Exner

Nuclear Physics Institute, AS CR, CZ-25068 Rez, near Prague.

ABSTRACT. Several solvable examples of embedded-eigenvalue perturbation theory are constructed in which Schrödinger operators on a pair of halflines are coupled by a contact-type interaction. The resolvents are computed from Krein's formula and the resulting pole condition is solved by means of the implicit function theorem. The results can be used to model low-energy resonance two-channel scattering [1], heavy quarkonia decays [2] or a caricature of the K-capture decay.

SPIN-POLARIZED THOMAS-FERMI THEORY
WITH THE FERMI-AMALDI CORRECTION

Giselle Ruiz Goldstein
Dept. of Math., LSU, Baton Rouge, LA 70803 U.S.A.

ABSTRACT. This work is joint with Jerome A. Goldstein and Wenyas Jia. We investigate the problem of minimizing the energy of the functional

\[ E(\rho_1, \rho_2) = \sum_{i=1}^{2} c_i \int_{\mathbb{R}^3} \rho_i^2(x) dx + \int_{\mathbb{R}^3} V(x) \rho(x) dx 
+ \frac{1}{2} \int_{\mathbb{R}^3} \int_{\mathbb{R}^3} \frac{\rho(x) \rho(y)}{|x - y|} dx dy 
+ \int_{\mathbb{R}^3} B(x)(\rho_1 - \rho_2)(x) dx 
+ \frac{1}{2} \sum_{i=1}^{2} C_{FA} \int_{\mathbb{R}^3} \int_{\mathbb{R}^3} \frac{\rho_i(x) \rho_j(y)}{|x - y|} dx dy \]

where \( \rho_i \in L^1(\mathbb{R}^3) \cap L^p(\mathbb{R}) \), \( \int_{\mathbb{R}^3} \rho_i(x) dx = Ni_i, \rho_i \geq 0, |V|, |B| \) are integrable on \( \mathbb{R}^3 \) and

\[ \rho_i(x) \rho_j(y) \]

is integrable on \( \mathbb{R}^3 \) for \( i, j \in \{1, 2\} \). Here \( \rho_1 \), respectively \( \rho_2 \) represents the density of spin up, respectively spin down, electrons, \( \rho = \rho_1 + \rho_2 \) is the total electron density \( c_1 > 0, p > 1 \) and \( C_{FA} \) is either \( 0 \) (the Hartree approximation) or \( 1 \) (the Fermi-Amaldi correction). Our results are obtained for a general class of potentials \( V \) which includes the molecular Coulomb potential. We also consider a magnetic field \( B(x) \) interacting with our system.

In the case \( C_{FA} = 0 \) is solved by Goldstein and Rieder [1] for more general kinetic energy terms (\( \rho_i^p \) is replaced by any convex function of the density) by converting the convex minimizing problem for (1) to an equivalent nonlinear elliptic system. The more interesting case \( C_{FA} = 1 \) is more difficult since the energy functional (1) is NOT convex. Goldstein, Goldstein and Jia prove that (1) has a minimizer by approximating the problem on \( \mathbb{R}^3 \) by the problem on \( B_R \) and letting \( R \to \infty \); however, uniqueness of the minimizing density \( (\bar{\rho}_1, \bar{\rho}_2) \) is open.

OBSTACLE SCATTERING FOR ELASTIC WAVES

JEROME A. GOLDSTEIN
Dept. of Math., LSU, Baton Rouge, LA 70803 U.S.A.

ABSTRACT. In this joint work with Genbao Shi we present a new framework for the scattering theory for equations of order $n$ in time which factor into first order parts. Let $S_k^{(k)} = (S_1^{(k)}, \ldots, S_n^{(k)})$ be a family of $n$ commuting selfadjoint operators on a Hilbert space $\mathcal{H}_k$, $k = 0, 1$. The equation

$$(*) \quad \Pi_{k=1}^n (\frac{d}{dx} \times iS_k^{(k)}) \mu + u = 0 \quad (t \in \mathbb{R})$$

can be viewed as an abstract Schrödinger equation $i\mu U/\partial t = H_k U$ ($H_k = H_k^*$) on $\mathcal{H}_k$. If $S_j^{(n)} - S_i^{(n)}$ is absolutely continuous for $j \neq i$ and the wave operator $W_{\pm, (S_j^{(1)}, S_j^{(0)}, J_j)}$ exist and are complete for $j = 1, \ldots, n$ then the wave operators governing $(*)$ exist and are complete.

Each component $u_i$ of the elastic wave equation

$$\frac{\partial u_i}{\partial x_i} = \mu \Delta u_i + (\lambda + \mu) \frac{\partial}{\partial x_i} (\text{div } u)$$

satisfies an equation of the form $(\ast)$ with $n = 4$. Here $u = (u_1, \ldots, u_n)$ and $\lambda, \mu$ are the (positive) Lamé parameters. The basic Hilbert spaces are $\mathcal{H}_0 = L^2(\mathbb{R}^N)$ and $\mathcal{H}_1 = L^2(\mathbb{R}^N \setminus 0)$ where $0$ is the (bounded) obstacle. Either Dirichlet or Neumann conditions may be imposed on $\partial \Omega$. The existence and completeness of the wave operators for this obstacle scattering by elastic waves problem follows from our abstract theory combined with the known results for scattering for the usual wave equation $\frac{\partial^2 u}{\partial t^2} = \Delta u$ in $\mathbb{R}^n$ versus the same equation in $\mathbb{R}^n \setminus 0$. 
SOME BOUNDS ON EIGENVALUES AND SPECTRAL GAPS OF SCHRÖDINGER AND LAPLACE OPERATORS

EVANS M. HARRELL II

School of Mathematics, Georgia Institute of Technology, Atlanta, GA 30342, U.S.A.

Abstract. The literature on eigenvalues of differential operators especially Laplacians, contains several theorems in which the gap between successive eigenvalues is dominated by an expression involving the sum of lower eigenvalues. The prototype of these bounds is that of Payne, Pólya, and Weinberger from the 50’s, which stated that

$$\lambda_{n+1} - \lambda_n < \frac{4}{\text{dimension}} \sum_{k=1}^{n} \lambda_k$$

These, and the related inequalities of Hile and Protter, turn out to be special cases of general bounds involving first and second commutators of the operator of interest with certain other operators.

By taking this abstract point of view, many of these bounds have been improved, and some new ones, in particular for Schrödinger operators, have been discovered. These bounds are fairly sharp in special cases.

B. HELFFER
DMI, ENS, 45 Rue d’Ulm, 75230 Paris Cedex 05, France

Abstract. In this talk we shall present the known results or open problems concerning the spectral properties of the Schrödinger operator for these models coming mutually from statistical mechanics. In particular we shall study the limit

$$\frac{\lambda_1(m)}{m}$$

of the first eigenvalue and the splitting

$$\lambda_2(m) - \lambda_1(m)$$

as $m \to \infty$, where $m \to \infty$. The recent results I will present are due to J.-Sjöstrand, B. Helffer or V. Sandoni.

STRONG MAGNETIC FIELDS, DIRICHLET BOUNDARIES, AND SPECTRAL GAPS

Rainer Hempel
Dept. of Mathematics, University of Alabama, Birmingham, AL 35294, U.S.A.

Abstract. In a combined work with I. Herbst (Charlottesville, U.S.A.), we consider the magnetic Schrödinger operators

$$H(\lambda \vec{a}) = -i \nabla - \lambda \vec{a}(x)^2$$

in $L^2(\mathbb{R}^n)$, where $\vec{a} \in C^1(\mathbb{R}^n); \mathbb{R}^n$ and $\lambda \in \mathbb{R}$. Letting $M = \{x; B(x) = 0\}$, where $B$ is the magnetic field associated with $\vec{a}$, and $M_2 = \{x; \vec{a}(x) = 0\}$, we prove that $H(\lambda \vec{a})$ converges to the (Dirichlet) Laplacian on the closed set $M$ in the strong resolvent sense, as $\lambda \to \infty$, provided the set $M \setminus M_2$ has measure 0.

Corresponding results on norm resolvent convergence are then used to show that there exist periodic vector potentials $\vec{a}$ with the property that the magnetic Hamiltonian $H(\lambda \vec{a})$ has spectral gaps, for $\lambda$ large enough. We finally address the question of absolute continuity of periodic $H(\vec{a})$. 

14
SELF-ADJOINTNESS OF SCHRÖDINGER OPERATORS

ANDREAS M. HINZ

University of Münich, Theresienstr. 39 D-80333 Munich 2

Abstract. The problem of establishing self-adjointness of the closure of the Schrödinger operator \( T = -\Delta + q \) on \( C_c^\infty \) with a locally square integrable, real-valued potential \( q \) has a long history, leading to the famous result of Simon, Kato and Jensen, which requires the global Kato condition for the negative part \( q_- \) of \( q \). We follow the more natural approach to assume soundness from below of \( T \) and we need a local Kato condition on \( q_- \) only. The essential ingredient of the proof is to establish local boundedness of weak solutions of \( -\Delta u + qu = 0 \). This is achieved with the aid of a mean-value inequality, which may also be employed for proving higher regularity properties of solutions such as (Hölder) continuity. The method can be extended to operators with a vector potential \( b \), i.e. when \( \Delta \) is replaced by \((\lambda - ib)^2\). Kato's inequality being a crucial tool, however, \( b \) has to be continuously differentiable. For more general \( b \) (and if \( q_- \) fulfills the global Kato condition), we had to take the deviation via form techniques. So it would be highly desirable to establish local boundedness of solutions in that case too.

Details and references can be found in:

LOCALISATION FOR THE SCALAR WAVE AND MAXWELL EQUATIONS IN RANDOM MEDIA

P. D. Hislop
Mathematics Dept., University of Kentucky, Lexington, KY 405060027, U.S.A.

ABSTRACT. The propagation of scalar and electromagnetic waves in random media can be described by Families of random self-adjoint operators of the form $H_w = C_w^{-1} H_0 C_w$. The coefficients $C_w = 1 + \eta_w > 0$, where $\eta_w$ is some stochastic process. The operator $H_0 = -\Delta$ for the scalar case, and $H_0 = -\Delta \pi_T$ in the electromagnetic case, where $\pi_T$ is a 3x3 matrix operator projecting onto the transverse modes. We consider the case when $\eta_w$ has the forms

$$\eta_w(x) = \sum_{i \in \mathbb{R}^d} \lambda_i(w) u(x - i),$$

or

$$\eta_w(x) = \sum_{i \in \mathbb{R}^d} \lambda_i(w) u(x - X_i(w)).$$

for some random variables $(J, iid)$ and points $(X_i)$ distributed according to a Poisson process. The single site potential $u$ is a non-negative. We prove that $\Gamma(H_w)$ is a dense pure point a.s. in any compact positive energy interval for all disorders sufficiently large. Furthermore, if $\sup u$ is compact, the corresponding eigenfunctions decay exponentially and the integrated density of states is Lipschitz continuous for the first model. This is joint work with J-M. Combes, C. Shubin, and A. Tip.


REGULARITY PROPERTIES OF THE ZERO-SET OF SOLUTIONS OF SCHRÖDINGER EQUATIONS

M. Hoffmann-Ostenhof

Institute for Mathematics, University of Vienna, Vienna, Austria


Let $u \not\equiv 0$ be a real valued distributional solution of the Schrödinger equation

$$(-\Delta + V)u = 0 \text{ in } \Omega,$$

$\Omega$ a domain in $\mathbb{R}^n$ and $V \in L^1_{\text{loc}}(\Omega)$, $V$ real valued. For $V \in K^{n,\delta}(\Omega)$ for some $\delta \in (0,1)$, $u \in C^{0,\delta}(\Omega)$. Let $B_R$ denote the ball centered in the origin with radius $R$ and $B_R \subset \Omega$ and let $N_u^{(1)}$ denote the set of points in $B_R$ where $u$ vanishes of first order. Based on a recent result [M. H.-O. and T. H.-O. 1992] we show that there is a constant $C < \infty$ such that $\forall x_0 \in N_u^{(1)} \cap B_{R/2}$ and $\forall \delta' < \delta$

$$|u(x) - \nabla u(x_0)(x - x_0)| \leq C|x - x_0|^{1+\delta'} \quad \forall x \in B_R,$$

where $C = C(\|V\|_{K^{n,\delta}}, \delta', \delta, n, R, \sup\|u(x)||x \in B_R\|)$. Using this estimate we then prove that $N_u^{(1)}$ is locally $n(1-\delta')$-dimensional hypersurface which is the graph of a $C^{1,\delta'}$-function, and hence "more regular" than $u$ itself.
ACCURATE SPECTRAL ASYMPTOTICS FOR OPERATORS
WITH SINGULARITIES AND SCOTT CORRECTION TERM(S)

V. Ivrii
Dept. of Mathematics, University of Toronto, 100 St. George St., Toronto, ONT. M5S 1A Canada

Abstract. Semi-classical asymptotics for the Riesz' means of the spectral function are derived for a class of operators with singularities at a point. These asymptotics can contain few non-Weylian "Scott correction" terms generated by the singularity solely.
RIESZ SUMMABILITY OF MULTIPLE HERMITE SERIES IN $L^p$ SPACES

G. E. Karadzhov
Svoboda bl. 40, ap. 78, 1231 Sofia, Bulgaria

Abstract. Let $E^\alpha f(y) = \sum (1 - \mu_n / \lambda)^\alpha f(y_n)$, $\mu_n < \lambda$, be the Riesz means of order $\alpha$ of the multiple Hermite series of the function $f$. Here it is proved that $E^\alpha f \to f$ as $\lambda \to \infty$ in the following cases: (i) In the space $L^p(\mathbb{R}^n)$, $n \geq 2$, under the sharp condition on parameter $\alpha$, $\alpha > \alpha(p)$, where $\alpha(p) = \max(0, n + 1/p - 1/2)$ and $1 \leq p \leq 2n/(n + 2)$ or $2n/(n + 2) \leq p \leq \infty$. (ii) In the space $L^2_m$ with a norm $(\int |f(x)|^2 (1 + |x|^m)^m dx)^{1/2}$ if $2\alpha > |m|$; or, equivalently, in the Sobolev space $H^m$ with the usual norm $(\int |f(\xi)|^2 (1 + |\xi|^m)^m d\xi)^{1/2}$.
EFFECTIVE MASSES AND CONFORMAL MAPPINGS

E. Korotyaev

5 Prof. Popov St., Dept. of Maths. 2, Electrotechnical University, St. Petersburg, 197376, Russia

ABSTRACT. Let \( G_n, n \in \mathbb{N} \), denote the set of gaps of the Hill operator. We solve the following problems: (1) Find the effective masses \( M^H_{\pm}, 2 \). (2) Compare the effective mass \( M^H_{\pm}, 2 \) with the length of the gap \( G_n \), and with the height of the corresponding slit on the quasimomentum plane (both with fixed number \( n \) and their sums). (3) Consider the problems (1) and (2) for more general cases (the Dirac operator with periodic coefficients, the Schrödinger operator with a limit periodic potential). To obtain (1) - (3), we use a conformal mapping corresponding to the quasimomentum of the Hill operator or the Dirac operator.
QUANTUM SCATTERING IN GAUGE FIELDS OF ADIABATIC REPRESENTATIONS

Yu. A. Kuperin

c/o Prof. I. Antoniou, Campus Plaine ULB, CP.231, Bol. du Triomphe, B-1050 Brussels

ABSTRACT. A geometric approach to the method of representations is developed for a class of relativistic Hamiltonians. The theory is used to analyze the associated dynamical equations with effective nonabelian interactions that can be regarded as gauge fields, induced by dimensional reduction of the initial problem in a special representation. It is shown that the approach can be used to study $2 \rightarrow (2, 3)$ quantum scattering process in a three-body system, and a one-to-one relation between the complete and the effective $S$-matrices is derived. Asymptotic expressions are found for the solutions of the effective dynamical equation and for the gauge field in the adiabatic representations. The method is illustrated for systems admitting adiabatic representations with a one-dimensional base; in several cases the field operator is proved to be Hilbert-Schmidt.
FLUXES, LAPLACIANS AND KASTELEYN’S THEOREM

MICHAIL LOSS

Georgia Technical School of Mathematics, Atlanta, Georgia, GA 30332, U.S.A.

Abstract. A joint work with Elliott Lieb, in which the following problem, which stems from the "flux phase" problem in condensed matter physics, is analysed and extended: One is given a planar graph (or lattice) with prescribed vertices, edges and a weight $|txy|$ on each edge $(x, y)$. The flux phase problem (which we partially solve) is to find the real phase function on the edges, $\theta(x, y)$, so that the matrix $T = \left( |txy| e^{i\theta(x, y)} \right)$ minimizes the sum of the negative eigenvalues of $-T$. One extension of this problem which is also partially solved is the analogous question for the Falicov-Kimball model. There one replaces the matrix $-T$ by $-T + V$ where $V$ is a diagonal matrix representing a potential. Another extension of this problem which we solve completely for planar, bipartite graphs, is to maximize $|\det T|$. Our analysis of this determinant problem is closely connected with Kasteleyn's 1961 theorem (for arbitrary planar graphs) and, indeed, yields an alternate, and, we believe, more transparent proof of it.

NON-LINEAR STARK EFFECT
AND MOLECULAR LOCALIZATION

André Martinez
University of Paris, Paris 13, Paris, France

ABSTRACT. In this joint work with V. Grechki, we consider a non-linear Stark effect as a model for localization and symmetry breaking of a molecule in a gas. By a comparison method with respect to the linear Stark effect, we prove the existence of level bifurcation and symmetry breaking at a critical value of the gas pressure exponentially small for large nuclear masses.
WEAKLY COUPLED ELLIPTIC SYSTEMS AND POSITIVITY

ENZO MITIDIERRI

Dipartimento di Scienze Matematiche, Università degli Studi di Trieste, P. Europa 7, Trieste, Italy

ABSTRACT. In this talk we will study under which conditions the positive cone, or part of the positive cone, is preserved when solving a weakly coupled system of elliptic partial differential equations. Such a system will be as follows:

\[
\begin{pmatrix}
-\Delta_1 & 0 \\
0 & \ddots & 0 \\
0 & \cdots & -\Delta_\kappa
\end{pmatrix}
\begin{pmatrix}
u_1 \\
\vdots \\
u_\kappa
\end{pmatrix} = \begin{pmatrix}
c_{11} & \cdots & c_{1\kappa} \\
\vdots & \ddots & \vdots \\
c_{\kappa 1} & \cdots & c_{\kappa\kappa}
\end{pmatrix}
\begin{pmatrix}
u_1 \\
\vdots \\
u_\kappa
\end{pmatrix} + \begin{pmatrix}
f_1 \\
\vdots \\
f_\kappa
\end{pmatrix}
\]

on a bounded domain in \(\mathbb{R}^n\), with zero Dirichlet boundary condition. The operators \(\Delta_i\) will be strictly elliptic such as the Laplacian. The system is said to preserve the positive cone if \(f \geq 0\) implies \(u \geq 0\). We will classify such systems, for noncooperative systems we need and show pointwise estimates for iterates of the Green function.
ON THE SPECTRA OF SELF-ADJOINT EXTENSIONS

H. NEIDHARDT

Technische Universität Berlin, Fachbereich 3, Math., Strasse des 17 Juni 136, D-10623 Berlin

ABSTRACT. Let $H$ be a symmetric operator with gap $(a, b)$, i.e.

$$\| (H - \frac{a + b}{2} ) f \| \geq \frac{b - a}{2} \| f \| \quad f \in \mathcal{D},$$

and infinite deficiency indices. It is shown that there exists a self-adjoint extension with a prescribed point spectrum inside the gap. Moreover, up to a discrete spectrum we can find self-adjoint extensions with a given singularity and absolutely continuous spectrum inside the gap. Furthermore, extensions exist which have a given point, singularly and absolutely continuous spectra. The results can be applied to partial differential operators on a bounded region.
PERTURBATION THEORY FOR TIME DEPENDENT HAMILTONIANS: RIGOROUS REDUCTION THEORY

G. NENCIU
Dept. of Theoretical Physics, University of Bucharest, Nagorele, Bucharest, Romania

ABSTRACT. We generalize the standard time independent perturbation theory by constructing time dependent families of orthogonal projections associated with isolated parts $\sigma_0$ of the spectrum of the unperturbed Hamiltonian. The full evolution between times $t_0$ and $t$ intertwines approximately these projections up to an error of order $\lambda^{N+1}|t-t_0|$, where $\lambda$ is the coupling constant and $N+1$ is the lowest order at which $\sigma_0$ becomes resonant with the rest of the spectrum. This provides a rigorous basis for deriving effective Hamiltonians for time dependent perturbations.
HARMONIC ANALYSIS ON RIEMANN SURFACE AND LAX-PHILLIPS THEORY FOR BAND SPECTRUM

B. Pavlov

Dept. of Mathematical Physics, University of St. Petersburg, Uljanovskaja 1, Petrodvorets 198904, St. Petersburg, Russia

Abstract. Lax-Phillips theory reveals the spectral meaning of resonances, which appear as poles of analytical continuation of the resolvent of self-adjoint operator $A$ in Hilbert space $H$, compressed onto fixed subspace $K$, $K \subset H$:

$$P_K(A - \lambda I)^{-1}K.$$  

It also opens the way to studying of completeness of the resonances states system and corresponding spectral decompositions.

The original version of Lax-Phillips approach is valid only for operators, which have a part, unitarily equivalent to the momentum operator. In the lecture a generalization at the theory was suggested, which is valid for operators with band spectrum. The completeness of resonances states system is studied on the base of index theorems for the pair of orthogonal projectors onto the subspaces of the singular and discrete spectrum of contracting semigroup in the generalized Lax-Phillips scheme, generated by the corresponding scattering problem. Applications to the perturbed periodic Jacobian matrix are described.
DISTRIBUTION OF MATRIX ELEMENTS AND LEVEL SPACINGS FOR CLASSICALLY CHAOTIC SYSTEMS

Didier Robert
University of Nantes, Institute for Mathematics, F-44200 Nantes

Abstract. This is a joint work with M. Combescure (Orsay-France).

For quantum systems obtained by quantization of chaotic classical systems like Schrödinger’s Hamiltonians: $P(h) = -h^2 \Delta + V$ on the configuration $\mathbb{R}^n$ we prove some rigorous results concerning the semi-classical behaviour of matrix elements: $A_{jk}(h) := < A(h) \varphi_j, \varphi_k >$ (scalar product in $L^2(\mathbb{R}^n)$) of some observable $A$ on an orthonormal system of bound states of the Hamiltonian. Let $I_C \subset \mathbb{R}$ be a classical energy interval such that the spectrum of $P(h)$ is purely discrete in $I_C$. We have

$$P(h) \varphi_j = E_j(h) \varphi_j$$

where $\{ \varphi_j \}_j$ is an orthonormal system of bound states of energies $E_j(h) \in I_C$.

Let $E \in I_C$ be such that the classical dynamics is ergodic or mixing on the energy surface: $\sum_{E} := \{(x, \xi) ; \xi^2 + V(x) = E \}$. The diagonal elements obey to the ‘Schnirelmann’s law’ [see references [1,2,3,5]]. We consider here the non diagonal elements $j \neq k$ and prove in particular that (roughly speaking) $A_{jk}(h)$ tends to 0 as $h$ tends to 0 for ‘almost all $(j,k)$’ when $E_j(h), E_k(h)$ go to $E$.

CHARGE DEFICIENCY, CHARGE TRANSPORT
AND THE INDEX OF PROJECTIONS

R. Seiler

Technical University of Berlin, Strasse des 17 Juni 136, D-110623 Berlin


The relative index of two projectors is the generalization of the difference of dimensions of the ranges of two projectors from the finite dimensional to the case of infinite dimensions.

The concept is related to the Fredholm index. We apply the relative index to counting the charge sent to infinity by the insertion of one flux unit to the system; this is charge deficiency.

The relation to the adiabatic charge transport is discussed. It is shown that the two concepts are the same under the condition that a certain homogeneity is satisfied.

For Landau Hamiltonians the relative index is computed explicitly for all Landau levels.
ELECTRONIC DENSITIES OF LARGE
ATOMS NEAR THE NUCLEUS

HEINZ SIEDENTOP
Institute of Mathematics, University of Oslo, PB. 1053, Oslo, Norway

Abstract. Let \( \rho_2 \) be the groundstate density of a neutral atom of nuclear charge \( Z \). Then

\[
\rho_2(0) \leq \frac{Z}{\alpha 4} Z^3
\]

Various extensions of this result are given as well.
ANOMALIES IN SPECTRAL ASYMPTOTICS

M. M. Skriganov
Steklov Mathematical Institute,
Fontanka 27, St. Petersburg,
191011 Russia

Abstract. In the talk we give examples of elliptic pseudodifferential operators on a compact manifold with logarithmically small errors in spectral asymptotics. Such anomalies have pure arithmetical nature and illustrate the relationships which exist between Number and Spectral Theories.

THE PRECISE ASYMPTOTICS OF THE DISCRETE SPECTRUM
FOR THE SCHRODINGER OPERATOR WITH COULOMB
SINGULARITIES IN A HOMOGENEOUS MAGNETIC FIELD

A. V. SOBOLEV
University of Nantes, F-44200 Nantes

ABSTRACT. We study the sum of negative eigenvalues $M(h, \mu)$ of the Schrödinger operator $H_V(h, \mu)$ with a potential $V$ decaying at infinity and a homogeneous magnetic field of intensity $\mu$. The parameter $h$ denotes the Planck constant. Assuming that $V$ has a finite number of Coulomb singularities and is smooth outside them, we establish the two-term asymptotics of $M(h, \mu)$ as $h \to 0, \mu \to \infty$. The second term is determined only by the singularities of $V$ and it turns out to be the same as in the case $\mu = 0$.

A. V. Sobolev, The asymptotics of the Riesz means in a moderate homogeneous magnetic field, submitted to Ann. IHP.
A. V. Sobolev, The sum of eigenvalues for the Schrödinger operator with Coulomb singularities in a homogeneous magnetic field, submitted to Bulletin of AMS.
A SUM RULE FOR THE SCHRÖDINGER EQUATION AND APPLICATIONS

JOACHIM STUBBE

Theoretical Physics Division, C.E.R.N., Geneva, Switzerland

Abstract. Consider the Schrödinger operator $H = -\Delta + V(x)$ on $\mathbb{R}^d$. Let $\{\varphi_1, \varphi_2\}$ be any pair of eigenfunctions of $H$ with corresponding eigenvalues $\{E_1, E_2\}$. The following identity holds:

$$\frac{1}{2}(E_2 - E_1)^2 \text{Re} \int_{\mathbb{R}^d} x\varphi_1^* \varphi_2^* \, dx = \text{Re} \int_{\mathbb{R}^d} \nabla V(x) \varphi_1^* \varphi_2^* \, dx.$$

By means of this sum rule/gap formula the following results can be proved:

1. Convexity properties and sharp inequalities of purely angular excitations for spherically symmetric potentials [1].
2. Order of the lowest Landau level of a Hamiltonian for a constant magnetic field and a cylindrically symmetric potential $W(\rho, \tau)$ if $\frac{\partial W}{\partial \tau}$ has a definite sign. In particular, it provides an elementary proof of the fact that the ground state of atomic hydrogen in a constant magnetic field has zero angular momentum. [2]

For further applications: [3] [4]

LIMITS OF STABILITY OF POSITIVE MOLECULAR IONS IN A HOMOGENEOUS MAGNETIC FIELD

SIMEON VUGALTER

Steklov Mathematical Institute,
Fontanka 27, St. Petersburg,
191011, Russia

ABSTRACT. The problem of stability of positive diatomic molecular ions with nuclear charges \( Z_1 \) and \( Z_2, \ Z_2 \geq Z_1 \) and \( N \) electrons in a magnetic field of strength \( B \) is studied for large \( Z_1, \ Z_2, \ N \) and \( B \). We show that in a magnetic field this physical problem can be reduced to that of the existence of discrete spectrum for the restriction of the Hamiltonian (after separation of center of mass motion only in the direction of the magnetic field) onto the subspaces of functions with fixed rotational \( S_0(2) \) and permutational symmetries. Some sufficient conditions for instability are obtained as relations between \( Z_1, \ Z_2, \ N \) and \( B \), including the case of extremely strong magnetic field \((B >> Z_2^2)\)
NEW CHANNELS IN 3-BODY LONG-RANGE SCATTERING

D. R. Yafaev
IRMAR, Dept. of Mathematics,
University of Rennes 1,
F-35042 Rennes

ABSTRACT. A system of three one-dimensional particles with pair potentials vanishing as $|x|^{-p}$, $0 < p < 1/2$, at infinity is considered. It is shown that such systems can possess channels of scattering not included in the usual list of channels called the asymptotic completeness.
ON THE LOCALIZATION OF THE ESSENTIAL SPECTRUM OF $N$-PARTICLE HAMILTONIANS WITH MAGNETIC FIELD

GREGORY M. ZHISLIN

Radio-Physical Research Institute, Bolshaya 25/14,
Nij Novgorod, 603600 Russia

ABSTRACT. We formulate theorems on the localization of the essential spectrum for many-particle Hamiltonians with a magnetic field. For the homogeneous magnetic field, we use the fixation of the type $m$ of $SO(2)$ symmetry, for increasing fields, the result is obtained without the fixation $m$. The theorems are similar to the usual HVZ-theorems.