ESI

2013 Erwin Schrödinger International Institute for Mathematical Physics

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Preface—Turning Over the Page

by Joachim Schwermer

Restart. In October 2010, when the Erwin Schrödinger International Institute for Mathematical Physics (ESI) had been in existence as an independent research institute since 1993, the scientific directorate and the international community of scholars had to learn with great distress of the intention of the government of Austria to cease funding for the ESI. Due to budgetary measures affecting a large number of independent research institutions in Austria, funding of the ESI would be terminated as of January 1st, 2011. Since its start it was the mission of the ESI to advance research in mathematics, physics and mathematical physics at the highest international level through fruitful interaction between scientists from these disciplines. An abrupt end for the scientific activities of the Institute and the closure of the ESI appeared on the horizon. Weeks of trembling uncertainty followed, mixed with signs of a solution in which the University of Vienna would be involved. In the wake of a protest action by renowned scholars and academic institutions worldwide, an agreement was achieved in January 2011 that the ESI could continue to exist but now as a research centre ("Forschungsplattform") at the University of Vienna. As a partner in this agreement the Ministry of Science and Research (BMWF) guaranteed to fund the "new" ESI through the University yearly with a reduced budget until 2015. At a time when pure research and scholarly activities are undervalued, the opportunities for scholars and young researchers that the Institute provides have never been more necessary. The University of Vienna took the chance and created a home for "one of the world's leading research institutes in mathematics and theoretical physics", as Peter Goddard, the chair of the international review committee for the Institute, commissioned by the BMWF, and its members put it in 2010 in a letter to the Ministry.

Stabilization. With this new institutional framework in place since June 2011, it was the main task of the new governing board of the ESI, called the "Kollegium", to carry on the mission of the ESI and to retain its international reputation. These aims include, in particular, to support research at the University of Vienna, to contribute to its international visibility and appeal, and to stimulate the scientific environment in Austria. Setting aside all the technical issues the transition process of the ESI involved, and which had to be taken care of, the ESI could begin restoring its fundamental scientific activities. This was and still is the prevalent task: striving to be excellent and thereby keeping its position within the international scientific community of scholars as a research institute with a specific unique character. The ESI is a place that is very conducive to research and, at the same time, integrates scientific education and research in mathematics and mathematical physics.

In retrospect, though the planning horizon was very short, the *Thematic Programmes,* scheduled already far ahead for 2011 and 2012, turned out in the end to be successful scientific events. Additionally, various workshops and other activities could be solicited for the year 2012 on short notice, involving, in particular, young researchers who came to the Institute for the first time as organizers. In addition, by January 1, 2012, the Erwin Schrödinger Institute had established the *Research in Teams Programme* as a new component in its spectrum of scientific activities.

This booklet. With the 20th anniversary of the ESI occurring precisely at this critical time, it is necessary to utilize this moment, as it were, not only to celebrate the rich heritage of the Institute but also to position us for the future. Specifically, this booklet serves as an overview of the institutional structure of the Erwin Schrödinger International Institute for Mathematical Physics, turned into a research centre at the University of Vienna in June 2011, and the various programmatic pillars of its scientific activities. Recollections of scientists involved in some of these activities add an important personal perspective to this catalogue of possible scholarly experiences.

In view of the strategic exchanges between mathematics and mathematical physics that have occurred over the course of the last decades and even beyond, one finds personal reflections of scientists on their experiences as scholars whose work is rooted in mathematics and physics. They found a creative way to capture their thoughts regarding the interaction between mathematics and mathematical physics and specific advances in their work. In the same vein, historical case studies dealing with fundamental facets or specific developments in which the interrelations of mathematics and physics have unfolded extend the discussion. Another note follows the interplay between thought and images in the mathematical sciences and how visual thinking may function in the creative process in mathematics, physics and their crossroads.

Finally, a concluding section in this booklet documents to some extent the scientific activities of the ESI, starting from its beginnings. The Institute has always remained true to its commitment: promoting and supporting original scholarship in the mathematical sciences.

Securing the future. The Erwin Schrödinger International Institute for Mathematical Physics fills a unique role in (post-) graduate education and scientific research in mathematics and mathematical physics, and in Austria, in particular. However, one has to give careful consideration to ways in which the ESI arrangements might be made more stable as it goes into a period of various challenges regarding its financial resources. As mentioned, the Ministry has only guaranteed to fund the ESI through the University until 2015. Thus, in the near future a fundamental decision has to be reached as to whether the government of Austria and the University of Vienna are prepared to host and fund an institution like the ESI, engaged as it is in fundamental research in mathematics and physics. Due to the lack of sufficient funding schemes in Austria in such cases, the usual mechanisms of third party funding are not feasible in this case. The ESI is, of course, firmly bound to seeking funds from other sources but it is only in the position to complement adequate basic financial support in this way. With regard to the latter point, as a first step, the Simons Foundation recently approved funding for the Erwin Schrödinger International Institute for Mathematical Physics to support the five-year appointment of a Simons Junior Professor in Mathematics or Mathematical Physics.

It is the Institute's foremost objective to advance scientific knowledge ranging over a broad band of fields and themes in mathematics and theoretical physics. Creating a space where fruitful collaborations and the exchange of ideas between scientists can unfold is decisive. With the ESI now being a research centre within the University of Vienna, the best way of achieving this is to ensure that the ESI continues to interweave leading international scholars and the local scientific community. The research and the interactions that take place at the Institute will have a lasting impact on those who get their scientific education in Vienna.

Joachim Schwermer
Director
Erwin Schrödinger International Institute
for Mathematical Physics

ESI—Structure and Activities

by Joachim Schwermer

1. The Institute and its Mission

The Erwin Schrödinger International Institute for Mathematical Physics (ESI) was founded in Vienna, Austria, in 1992, and became fully operational in April 1993. On June 1, 2011, the ESI assumed its role as a research centre within the University of Vienna. The mission of the Institute is:

- to advance research in mathematics, physics and mathematical physics at the highest international level through fruitful interaction between scientists from these disciplines;
- to support research at the University of Vienna and surrounding universities and to stimulate the scientific environment in Austria.

The transition of the Erwin Schrödinger Institute from an independent research institute to a "Forschungsplattform" at the University of Vienna was a complicated process. There are far-reaching differences in operation as a consequence of the university's involvement in the running of the Institute. This includes issues concerning payments to participants, modifications to the premises and future funding prospects. However, the Institute has continued to function, even flourish, during the radical changes of its status.

The Institute currently pursues its mission in a number of ways:

- Primarily, by running four to six *thematic programmes* each year, selected about two years in advance on the basis of the advice of the International ESI Scientific Advisory Board.
- By organizing workshops and *summer schools* at shorter notice.
- By a programme of *Senior Research Fellows* (SRF), who give lecture courses at the ESI for graduate students and postdocs.
- By a programme of *Research in Teams,* which offers teams of two to four *Erwin Schrödinger Institute Scholars* the opportunity to work at the Institute for periods of one to four months, in order to concentrate on new collaborative research in mathematics and mathematical physics.
- By inviting *individual scientists* who collaborate with members of the local scientific community.

Even through the transition period the ESI had to go through in the years of 2010 and 2011, the ESI has remained a leading international centre for research in the mathematical sciences. This position has been achieved with a minimal deployment of resources, financial and human, especially when compared with similar institutes in other countries.

2. The Institute's Scientific Management and its Resources

The arrangements that provide for the scientific direction and administration of the Institute are perhaps among the noteworthy features of the ESI. Indeed, the Institute is run in a quite minimalist fashion.

The organizational structure of the ESI is as follows: The ESI is governed by a board ("Kollegium") of six scholars, necessarily faculty members of the University of Vienna. These members of the board are appointed by the President (Rektor) of the University after consultations with the Deans of the Faculties of Physics and Mathematics. It currently consists of Goulnara Arzhantseva (Mathematics), Adrian Constantin (Mathematics), Piotr T. Chruściel (Physics), Joachim Schwermer (Mathematics), Frank Verstraete (Physics), and Jakob Yngvason (Physics). All members of the Kollegium still act as Professors at the University.

In addition, the Scientific Advisory Board of the ESI plays a crucial role in keeping this Institute alive scientifically. The members of the Scientific Advisory Board of the ESI, which currently consists of seven international scholars, have a variety of tasks: they assess the programme proposals submitted to the ESI, they point out interesting scientific developments in the area of mathematics and mathematical physics, and suggest topics and possible organizers for future activities of the Institute, and, most importantly, during the yearly

THE STRUCTURE OF THE ESI

meeting, they review—and criticize—the scientific performance of the Institute during the past year and make suggestions for possible improvements. Though the name of the Institute only contains mathematical physics as the subject of concern, mathematics plays an equally important role in its scientific activities.

On June 1, 2011, the Scientific Advisory Board of the ESI was restructured. Only scholars who are not affiliated with a scientific institution in Austria can be appointed as members. Thus, at the same time, its composition changed. For the sake of continuity, John Cardy (Oxford), Horst Knörrer (ETH Zürich), Vincent Rivasseau (Paris) and Herbert Spohn (München) were reappointed. As new members Isabelle Gallagher (Paris), Helge Holden (Trondheim) and Daniel Huybrechts (Bonn) joined the Board, starting January 2012.

The day-to-day functioning of the ESI is overseen by the Director. The Director is appointed by and accountable to the Rektor of the University. Besides the ongoing oversight of the ESI, the Director chairs the Kollegium, represents the ESI at meetings of the European Institutes and has responsibility for the budget of the ESI. The Director makes sure the ESI functions in a way manner with its mission.

The administrative staff of the Institute, currently consisting of three people, two of them working on part time basis, is also extremely lean but very efficient in handling the approximately 450 visitors per year.

Situated at Boltzmanngasse 9 in Vienna, the ESI is housed in the upper floor of a two hundred-year-old Catholic seminary. This building provides a quiet and secluded environment. By its distinctive character, the ESI is a place that is very conducive to research.

The Institute is still funded by the Austrian Federal Ministry for Science and Research, via the University of Vienna, but it works on the basis of much smaller resources financially than in the years before 2011.

3. Thematic Programmes and Workshops

The Institute's scientific activities are centred around four to six larger thematic programmes per year. Planning for these programmes typically begins two years in advance. About three quarters of the scientific budget are used for these activities each year. In addition, smaller programmes, workshops and conferences are organized at shorter notice, as well as visits of individual scholars who collaborate with scientists of the University of Vienna and the local community.

The list (see page 44) of research areas in mathematics, physics and mathematical physics covered by the scientific activities of the Erwin Schrödinger Institute in the years 1993 to 2012 shows a remarkable variety.

The pages of the annual ESI report, available on its web page, provide ample evidence that the high quality of the scientific programmes was sustained and, in particular, undiminished during and shortly after the radical changes the Institute had to face. Longer thematic programmes and the open approach to research they offer and encourage form a fundamental pillar of the work of the ESI. The Institute provides a place for focused collaborative research and tries to create the fertile ground for new ideas.

It is generally noted, as already the Review Panel of the ESI pointed out in its report in 2008, that over the last years the ESI has widened the range of its thematic programmes and other scientific activities from being originally more narrowly focused within mathematical physics. The Scientific Directorate has increased the scope of the activities mounted by the Institute into areas of mathematics more remote at present from theoretical physics. This process will continue in the same fashion, with special emphasis on the fruitful interactions between mathematics and mathematical physics.

The themes of the programmes which are in place in 2013 range from "The Geometry of Topological D-Branes", "Jets and Quantum Fields for LHC and Future Colliders" over "Forcing, Large Cardinals and Descriptive Set Theory" to "Heights in Diophantine Geometry, Group Theory and Additive Combinatorics".

In 2014 the ESI will host four thematic programmes, the first one dealing with "Modern Trends in Topological Quantum Field Theory", followed by one centred around "Combinatorics, Geometry and Physics". The programmes "Topological Phases and Quantum Matter" and "Minimal Energy Point Sets, Lattices and Designs" cover the second half of the year, supplemented by various workshops.

4. Senior Research Fellowship Programme

In order to stimulate the interaction of the Institute's activities with the local community, the Institute initiated a Senior Research Fellowship Programme in 2000. Its main aim is attracting internationally renown scientists to Vienna for longer visits. These scholars would interact with graduate students and postdocs in Vienna, in particular, by offering lecture courses on an advanced graduate level. This programme enables Ph.D. students and young postdoctoral fellows at the surrounding universities to communicate with leading scientists in their field of expertise.

5. ESI Scholars

By January 1, 2012, the Erwin Schrödinger Institute had established the *Research in Teams Programme* as a new component in its spectrum of scientific activities. The programme offers teams of two to four *Erwin Schrödinger Institute Scholars* the opportunity to work at the Institute in Vienna for periods of one to four months, in order to concentrate on new collaborative research in mathematics and mathematical physics. The interaction between the team members is a central component of this programme. The number of proposals, on themes of topical interest, was high. However, due to limited resources, the Kollegium could only accept four of these applications for the year 2012. The first scholars within this programme were at the ESI in June 2012. Other teams are already accepted for the year 2013.

Of course, by invitation only, the ESI continues to have individual scientists as visitors who pursue joint work with local scientists. In some cases these collaborations originate from previous thematic programmes which took place at the ESI.

6. Junior Fellows and Summer Schools

Funding from the Austrian Federal Ministry for Science and Research (BMWF) enabled the Institute to establish a Junior Research Fellowship Programme (JRF). Its purpose was to provide support for advanced Ph.D. students and postdoctoral fellows to allow them to participate in the activities of the ESI. Grants were given for periods between two and six months. This programme was very successful and internationally held in high esteem but unfortunately it came to an end because funding by the BMWF was terminated with the end of 2010. The presence of the Junior Research Fellows at the Institute, together with the Fellows of the European Post-Doc Institute, had a very positive impact on the ESI's scientific atmosphere through their interaction with participants of the thematic programmes, through lively discussions with other postdocs and also through the series of JRF seminars. In conjunction with the JRF Programme, the ESI had regularly offered Summer Schools which combined series of introductory lectures by international scholars with more advanced seminars in specific research areas. However, even though the JRF programme had to be discarded, the Institute continues its long term policy of vertical integration of scientific education and research. Summer Schools are still essential components of the scientific activities of the ESI.

In 2010, a "May Seminar in Number theory" took place to introduce young researchers to exciting recent developments of current research at the crossroads of arithmetic and other fields. During the summer 2011 a school dealt with recent developments in mathematical physics. Jointly with the European Mathematical Society (EMS) and the International Association of Mathematical Physics (IAMP) the ESI organized in 2012 the "Summer School on Quantum Chaos". This Instructional Workshop attracted more than 45 graduate students, postdocs and young researchers from all over the world. A poster session accompanied this event.

7. Conclusion

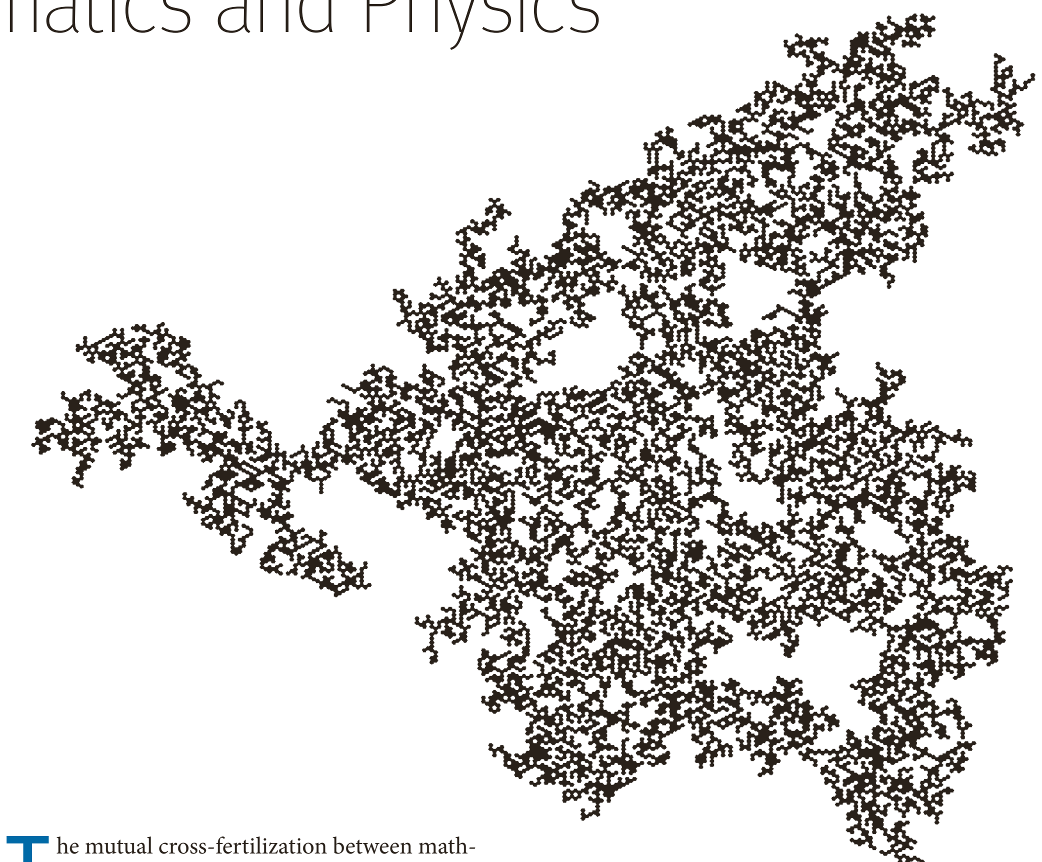
With the ESI now being a research centre within the University of Vienna, the best way of benefiting the local community is to ensure that the ESI continues to function at the highest level internationally and thus attract the world's leading scholars to Vienna where their presence will enhance and stimulate research further. This has been and still is the approach successfully followed by the ESI. ▪

At the Interface of Mathematics and Physics

AT THE INTERFACE OF MATHEMATICS AND PHYSICS

Some Thoughts on the Fruitful and Ongoing Interaction Between Mathematics and Physics

by Wendelin Werner



Sample of a large critical percolation cluster: understanding such random shapes in the fine-mesh limit has been the topic of numerous works in the physics and mathematics communities.

The mutual cross-fertilization between mathematics and physics is a well-documented fact, and has been a constant feature of the history of science. In today's fast-changing world, where spectacular progress is been made in the understanding of Life Science, and where the computing power provided by technology opens new doors, it can be questioned whether the progress driven by this privileged interaction between mathematics and physics belongs to the past or whether it is still, and maybe even more than ever, a major driving force in the evolution of these two disciplines. In these few lines, I would like to explain why, based on my personal mathematical experience, I believe that the latter case holds, and why some institutions such as the Erwin Schrödinger Institute have a pivotal role to play in this direction.

A first preliminary question is to wonder what the reasons are that lead some minds to choose to devote their life to study abstract mathematical structures, that seem to have little direct applications or even little concrete meaning in the real outside world. Of course, each mathematician has his/her personal history and motivations, but most of the time, they are to a large extent emotionally driven (the recent exhibition by the Fondation Cartier "Mathématiques, un dépaysement soudain" illustrates this very well). Mathematics is, for many of us, a way to express or capture some aspects of our interaction with the outside world, and the questions we study are often in some sense a continuation of those questions that we faced as children or teenagers, when learning about how our physical world is structured and functions. After all, the experimental exploration of the outside world as a baby, when one learns how to move arms in space, where one starts to walk and sense gravity, when one starts to look and feel shapes, textures, colors, occurs simultaneously with some sort of intuitive conceptualization of these experiments in our brain, and one could therefore even argue that the interaction between physics and mathematics is a natural continuation of this very natural dual discovery of the physical world and its interpretation/understanding.

This relation can be very direct (like for instance trying to understand mathematical aspects of relativity, which is still an ongoing hot topic in mathematics) or indirect, more based on analogies or on abstract generalizations. It is very rare to find a recent important mathematical result that is not related in some way or the other to physics. A look at the list of prize-winners (for instance for the Abel Prize, Shaw Prize, Wolff Prize etc.) backs this assertion very convincingly.

My own personal scientific life has in fact not only been shaped by the motivation that comes from the relation to physics of the type that I have just described, but also more directly from the contact with physics as an academic field. Indeed, a number of the questions that I have been studying have been initially either studied or raised by physicists, with motivations that range from the experimental study of phase transitions (how spontaneous magnetization of iron depends on the temperature, say) to theoretical questions about the nature of interactions (related to field theory). In fact, in the particular topic that I have been working on, we were very fortunate that theoretical physicists such as Bertrand Duplantier or John Cardy actually "came to us" with precise mathematical challenges, i.e. precise statements ("conjectures") that were calling not only for a rigorous proof, but also for an intuitive understanding that the physical theories were not able to fully provide. In fact, together with Greg Lawler and Oded Schramm, we also worked on a prediction of Benoît Mandelbrot, who can also be viewed as a mediator between natural sciences and mathematics (see his book "The fractal geometry of nature").
In this area, key mathematical objects that have led to new insight have been the Schramm-Loewner Evolutions (invented by Oded Schramm), that provide random ways to draw curves in the plane, making use of conformal transformations of the plane (these are the angle-preserving distortions of portions of the plane), as they turn out to be the only natural candidates to describe curves that appear in various parts of physics, for instance as boundaries between different randomly created domains, or as level lines of randomly created mountains.

This interaction between mathematical and physical academic communities is not such an easy one. To start with, the vocabulary used, the background material that is assumed to be known, the rules of what information a paper should contain, are very different, so that it is extremely difficult for a mathematician to extract the useful needed information for his research out of a physics paper, and vice-versa. We all spent quite frustrating and long hours looking at a physics paper without understanding its logic. The direct face-to-face interaction can however be much more productive, but it requires time, and repetition. Just one conversation is often not enough… In that respect, mathematicians in our area have been particularly fortunate to be able to benefit from such timely programs that did take place at places such as the ESI or the Isaac Newton Institute for Mathematical Sciences in Cambridge, where one could meet our friends from theoretical physics, and interact with them, in the lecture room as well as in office discussions or around a glass of beer. This has certainly helped a lot in the exchange of ideas.

When one tries to think about scientific strategy, organization and funding of the academic life, the main motivation is to see how to help and stimulate novel creative scientific ideas. It is important to pay a salary to academics and more generally to provide them with good working conditions, but it is also essential to put them in the right "environment", where they can interact with those other scientists that would bring them some fresh, new and maybe unsettling approaches to the questions that they are looking at, to feed their curiosity. This is why such places like the ESI have played, play and will continue to play an essential part in the future developments of our disciplines. ▪

Photo: Antoine Taveneaux

Wendelin Werner is a German-born French mathematician working in the area of self-avoiding random walks, Schramm-Loewner evolution, and related subjects in probability and mathematical physics. In 2006, at the 25th International Congress of Mathematicians in Madrid, he received the Fields Medal. Werner became a member of the French Academy of Sciences in 2008. He is professor at the University Paris-Sud and part-time at the École Normale Supérieure in Paris.

Algebras, Groups and Strings

by Peter Goddard

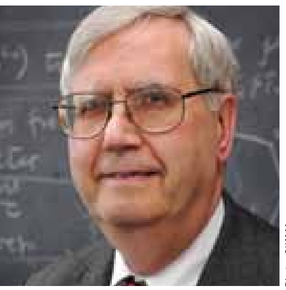


Photo: Cliff Moore

Peter Goddard is a mathematical physicist who has made contributions to quantum field theory, string theory and conformal field theory. His achievements have been recognized by the award, jointly with David Olive, of the Dirac Medal of the International Centre for Theoretical Physics in 1997, and appointment as a Commander of the Order of the British Empire in 2002. He is a Fellow of the Royal Society of London and an Honorary Fellow of Trinity College, Cambridge. From 2004 to 2012, he served as the eighth Director of the Institute for Advanced Study in Princeton. Before that he was Master of St John's College and Professor of Theoretical Physics in the University of Cambridge, where he played a leading role in establishing the Newton Institute and the Centre for Mathematical Sciences.

Emerging and interweaving connections between finite group theory, infinite-dimensional symmetries, and string theory are related from a personal perspective, illustrating the symbiotic relationship between mathematics and theoretical physics.

The birth of string theory, or more accurately its conception, was announced in the Hofburg in Vienna forty-five years ago, in the late summer of 1968. Gabriele Veneziano proposed to the 14th International Conference on High Energy Physics an explicit formula for a 'scattering amplitude' describing the interaction of strongly interacting subnuclear particles, such as pions.[1] His formula was simply expressed in terms of the Beta function introduced by Euler in the 18th century, whose properties are set out in the standard text books on the methods of mathematical physics.

Veneziano's proposal resolved a controversy about how the description of a scattering amplitude in terms of resonances, as a sum of poles in the energy variable, *s,* could be reconciled with the high-energy description of the amplitude, as an asymptotic series of terms which are powers of *s,* with exponents dependent on the angle of scattering, called Regge pole contributions. Some physicists had argued that the resonance terms should be added to the Regge pole contributions, so that there could be interference between these features of the amplitude, but Veneziano's amplitude showed that the sum over resonances and the asymptotic series of Regge pole contributions could be alternative, equivalent descriptions of the amplitude. The resonance and Regge pole descriptions of the amplitude were said to be 'dual'; the extensions and generalizations of the Veneziano amplitude that were made by many theoretical physicists over the next few years were called *dual models.* Dual models proved to be the embryo (or perhaps, it would be more apposite to say, the larval form) of what emerged within a few years to become string theory.

In the summer of 1968, I was a research student in Cambridge, working on other aspects of strong interaction scattering amplitudes and hearing gossip from my friends amongst the pure mathematicians of exciting developments in a very different area, namely, finite group theory. John Conway had become intrigued with the Leech lattice, a lattice of points in 24-dimensional Euclidean space describing an extremely close packing of (hyper-)spheres, and its symmetries. He had calculated precisely how many there were of these symmetries, more than 8.3×10^{18}. With any lattice there is always the obvious symmetry of reflection through the origin and identifying symmetries that differ by such a reflection gives a group of half the size, now known as the Conway group, Co_1.

Conway quickly showed that his group, Co_1, is *simple,* and so as near as one can get to a basic building block of finite group theory. It is now known that the finite simple groups come in 18 infinite series and 26 exceptional *sporadic* cases. Conway's group, Co_1, turned out to be the fifth largest of the sporadic simple groups, the largest one that had been discovered at that time. Furthermore, other new simple groups were found inside it and it provided a crucial step towards the overall classification of finite simple groups, which took over 150 years to complete. As you may know, the number 26 also plays a particular role in string theory, as a special dimension of space-time. Our story is full of coincidences that turn out to be clues of much deeper and unexpected connections, but the occurrence of 26 in these two places really is a coincidence (I suppose!); the 24-dimensionality of the Leech lattice is quite another matter.

Having moved to CERN, Geneva, in 1970, as a postdoctoral fellow, I became engaged in the development of the theory of dual models. Veneziano's original formula, proposed for processes involving the scattering of two particles, had been generalized to processes involving any number of particles. Initially, it might have been regarded as an interesting example, resolving the controversy over whether to add resonance pole contributions to Regge contributions. However, it became apparent that, more ambitiously, it might be considered as the starting point (or 'Born term') for a perturbative series expansion for the scattering amplitude in a fundamental theory of the strong interactions, rather than just a provisional phenomenological description. For this purpose, in addition to generalizations to the scattering of arbitrary number of particles, we also need higher order, or *loop,* contributions to ensure consistency with unitarity, *i.e.* conservation of probability.

Some clear obstacles were present to developing dual models into a consistent fundamental theory. In particular, quantum theories consistent with special relativity are likely to generate negative answers for some of the quantum mechanical probabilities specifying the results of scattering processes, that thus would make no sense. These negative probabilities are associated with certain potential states of the system, known as *ghost* states. To have a satisfactory interpretation, such a theory needs to have a consistent way of excluding ghosts states; that is, we should be able to define a subspace of the potential states of the theory, which would be regarded as the *physical* states, free of ghost states producing negative probabilities, and generating only other physical states, and no ghosts, on scattering.

Typically, the space of physical states is characterized by the vanishing of a set of (linear) conditions, and the consistency of these conditions with scattering amounts to a symmetry of the theory. In quantum electrodynamics (QED), it is gauge symmetry that plays this role in eliminating the ghosts. In the theory of dual models, there are, in a sense, infinitely more ghost states than in QED, and, in response, an infinite-dimensional symmetry is needed to eliminate them. Early on, it was realized that the fundamental constituents scattering in dual models were more analogous in some sense to a sort of vibrating medium, a 'rubber band' or a 'string', rather than to conventional particles, and there are ghost modes associated with each of the infinitely many vibrational modes of the string.

Towards the end of 1969, at the cost of an assumption that necessitated massless spin 1 particles (like the photon), Miguel Virasoro identified an infinite-dimensional symmetry, which had the potential to eliminate the ghost states. The algebra generating this symmetry, with the central term essential in the string theory context, is now named the *Virasoro algebra.*[3] The proposition that Virasoro's conditions did indeed eliminate the ghost states was then a mathematically precise conjecture that could be formulated just in terms of the Virasoro algebra: it was the requirement that a certain space defined by the algebra was positive semi-definite. With the efforts of several theoretical physicists[4], within two and a half years, the result, known as the *No-Ghost Theorem,* was proved.

A second severe challenge in the development of the theory of dual models was the consistent construction of loop contributions. Typically, there are divergences in loops that have to be removed by renormalization (unless, in very special theories, they cancel as the result of supersymmetry). In certain dual model loops, a new problem arose: an unwanted singularity, a branch cut in an energy variable, which would violate the axiom of unitarity unless it could be transformed into a pole, in which case it would signal a new particle in the theory. Late in 1970, Claud Lovelace, desperate to make the theory consistent, considered varying the dimension of space-time, away from the familiar four, and even fantasized about varying the number of sets of Virasoro-like conditions. Relaxing reality in this way, he found that the dual model would be saved if the dimension, *D,* of space-time were 26 (25 space and one time) and the Virasoro conditions were doubled in number.

At the time, to many physicists, Lovelace's speculations seemed at best irrelevant, even a joke, because considering extra dimensions to space-time was viewed as just science fiction. But his Delphic observation was rehabilitated, if not totally explained, by the proof of the No-Ghost Theorem, because it showed that ghosts were absent if and only if D is no greater than 26, and that, when $D = 26$, the Virasoro conditions indeed do effectively behave as though they were two sets of conditions rather than one (mimicking what happens in QED). Thus, Lovelace's consistency requirements are precisely met, as if by miraculous coincidence. But these results encourage another, geometrical interpretation. A string moving in four-dimensional space-time can vibrate in two dimensions transversely and one longitudinally; a string moving in D-dimensional space-time can vibrate in $D - 2$ transverse dimensions and one longitudinal dimension. When $D = 26$, the extra effectiveness of the Virasoro conditions removes the longitudinal

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oscillations and the only significant vibrations are those in the $D - 2 = 24$ transverse dimensions. And, over the next two decades, it would emerge that the occurrences of the special dimension 24 in finite group theory and as the number of transverse dimensions in what was becoming *string theory* are essentially related.

In retrospect, that the longitudinal oscillations of the string should be absent when the passage to quantum theory leaves the symmetries of theory intact, when the theory is free from anomalies, is clear from the description of the mechanics of the string originally proposed by Nambu and Goto.[6] In their action principle, longitudinal motions of the string have no dynamical significance; they are only reparametrizations. When $D \neq 26$, the passage to quantum theory is corrupted by anomalies, and longitudinal modes, which seem impossible to treat consistently, are introduced by quantization. This motivated an understanding[7] of how the quantum mechanics of a relativistic string would lead to the spectrum of states found in the dual model, provided that $D = 26$, and to a precise relationship between the geometrical string picture and the calculational framework of dual models. Gradually it came to be accepted that string theory required more than the familiar four dimensions of space-time, a total of 26 for the original string theory or 10 for the more elaborate and realistic *superstring theory,* and these extra dimensions should be curled up on a scale of 10^{-35}m, with the theory being interpreted as a unified theory of all the fundamental interactions, rather than one describing strong interaction physics on the scale of the nucleus, its original context.

While the attention of theoretical physicists shifted away from string theory, from the mid 1970s to the mid 1980s, there were a number of significant developments in mathematics which were or would become intimately related to string theory. In 1981, Robert Griess announced the construction of the largest sporadic simple group, usually known as the Monster group, *M.* It has about 8.1×10^{53} elements, more than 36 orders of magnitude bigger than Conway's group. As well as being the biggest, it contains all but 6 of the other sporadic simple groups within it in some sense. Just as Co_1 is the group of symmetries of the 24-dimensional Leech lattice, up to reflections, so it is natural to seek to an object, albeit an abstract one, of which *M* is the group of symmetries. This is challenging not least because the dimension of the smallest space, in which *M* can be thought of acting as rotations, is not 24 but 196,883.

In 1974, even before the Monster had been constructed, while the evidence for its existence, although extremely strong, remained circumstantial, John McKay spotted that this dimension differed by one from the coefficient 196,884 in the expansion of the elliptic modular function $j(\tau)$, a function of central importance in complex variable theory, an apparently very different branch of mathematics. He sent his observation to John Thompson, then visiting Princeton from Cambridge, who generalized it to observe that the next four coefficients of $j(\tau)$ could also be expressed as simple sums of a few of the dimensions of spaces in which the putative Monster group might be represented as acting as rotations. Since both the coefficients of $j(\tau)$ and the dimension of these representation spaces get big rather quickly, this would be an extreme coincidence, unless there was a deeper reason. He conjectured that this would be true for all the coefficients of $j(\tau)$, and that this phenomenon reflected the existence of an infinite-dimensional space, sliced or graded into finite-dimensional layers, in which *M* could be represented naturally acting on each layer. Conway together with Simon Norton, another Cambridge group theorist, generalized Thompson's work to conjecture a modular function, and a series connected with the hypothetical natural infinite-dimensional representation space, for each of the elements 8.1×10^{53} elements of the Monster, conjectures which they called *Monstrous Moonshine.*[8]

This posed the challenge of finding the natural infinite-dimensional representation space, later denoted $V^\natural$, and a structure within it of which *M* was the symmetries, so 'explaining' the existence of *M* in the same way that the Leech lattice 'explains' the existence of Co_1, with the important historical difference that, whereas the Leech lattice led to the discovery of Conway's group, it would be the Monster group that was leading to the discovery of $V^\natural$ and whatever lived inside. The construction of $V^\natural$ and the structure inside it which could be regarded as providing the *raison d'être* for the Monster, as its symmetry group, brings together concepts from string theory and another new development from the late 1960s: the infinite-dimensional algebras introduced independently by Victor Kac and Robert Moody[9]. These Kac-Moody algebras generalized the compact Lie algebras, such as those associated with the group of rotations in a Euclidean space, and some of the beautiful and important results from the theory of such algebras generalize to Kac-Moody algebras.

It was realized about 1980 that the basic ways of constructing or representing Kac-Moody algebras involve the same mathematical objects that describe the interaction of strings, called *vertex operators.* This established a two-way flow of information, enabling physicists, in particular, to understand that Kac-Moody algebras played a role in physical theories and could be used to provide new ways of realizing gauge symmetries.[10] In string theory, vertex operators are part of what is called a conformal field theory which describes the structure of the string. Igor Frenkel, James Lepowsky and Arne Meurman[11] found the natural setting for the Monster group by constructing a special conformal field theory, called a vertex operator algebra in this context, acting in $V^\natural$. Their conformal field theory was special amongst conformal field theories in much the same way that the Leech lattice is special amongst 24-dimensional lattice. They showed that the group of symmetries of this vertex operator algebra was indeed *M,* and in this way proved the conjectures of Thompson.

To go further and prove the Moonshine conjectures of Conway and Norton, Richard Borcherds, originally a student of Conway, defined the notion of a Generalized Kac-Moody algebra. He enlarged $V^\natural$, so that it became something like the transverse vibration states of a string and defined a Generalized Kac-Moody algebra associated with the space defined by Virasoro-like physical state conditions imposed in this larger space. In a denouement which brings together the strands of our plot, by applying a generalized form of the Weyl character formula, a central result from the theory of compact Lie algebras which extends to Generalized Kac-Moody algebras, and using the No-Ghost theorem of string theory, Borcherds was able to prove the Moonshine conjectures.[12] For this *tour de force,* he was awarded a Fields medal in 1998.[13]

In this story, extending over thirty years, I have tried to outline briefly how three seemingly disparate theories or themes, which began about 1968—string theory, Kac-Moody algebras, and the developments in finite group theory following from the discovery of Conway's group, Co_1—became intertwined and cross-fertilized, revealing profound connections between previously unrelated areas of research. Vertex operator constructions from string theory were rediscovered or imported into the theory of Kac-Moody algebras, and then used to provide a natural setting for the Monster group, and a framework within which the Moonshine conjectures could be proved using the No-Ghost theorem from string theory; and ideas from Kac-Moody algebras were imported back into string theory to provide new methods of realizing gauge symmetry there.

The story provides a fine illustration of the symbiotic relationship between mathematics and theoretical physics. Its plot could not have been guessed in advance; it is more wonderful than anyone could have imagined. Few of the chapters could have been written as predetermined deliverables on multi-year research programs. Lovelace's suggestion of a 26-dimensional space-time and Thompson's conjecture of a detailed relationship between the elliptic modular functions and representations of the Monster group led others to doubt their reason. And, of course, the story is not yet over. Physicists Tohru Eguchi, Hirosi Ooguri, and Yuji Tachikawa have found a new form of 'Moonshine' relating the representations of the Mathieu sporadic simple group, M_{24}, to a weak Jacobi form, the elliptic genus associated to a K3 surface.[14] To what new questions, connections and concepts this will lead, we have yet to imagine… ▪

I am grateful to Richard Borcherds, Matthias Gaberdiel and Siobhan Roberts for helpful comments on a draft of this text.

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A Deeper Union? Poincaré on Mathematics and Mathematical Physics

by Jeremy Gray


Photo: Susan Laurence

Jeremy Gray is a British historian of mathematics working on topics in algebra, geometry, analysis, and the philosophy of mathematics in the 19th and 20th centuries. In 2009 he was awarded the Albert Leon Whiteman Memorial Prize of the American Mathematical Society for his work on the history of mathematics. He is presently a Professor of the History of Mathematics at the Open University, and an Honorary Professor at the University of Warwick.

This topic invites the historian to speculate: are mathematics and mathematical physics two subjects, or two aspects of the same subject? Even if we grant that in the modern world both subjects have become so large that no-one can be expected to know all of even one of them, we find surprises such as string theory, with a strong claim to being fundamental in both physics and topology.

If we set aside disciplinary issues, is there a single domain containing both mathematics and mathematical physics, or are there two domains with perhaps some bridges between them? The view that there are two domains was implied as long ago as Greek times. Archimedes in his *Method* regarded some arguments drawn from considerations that would be convincing in physics as having only a heuristic character when used in a strictly mathematical setting. In other words, there might be arguments in mathematical physics that draw their certainty from an understanding of physics that has not been captured in precise mathematical terms. On this view, the world displays a limited range of possibilities, which it presents for reasons that may not be understood. 19th century arguments in potential theory had a similar character. Mathematical physicists such as Helmholtz and Maxwell could not see the point of the cumbersome and restricted proofs offered by Dirichlet, Riemann, and Schwarz.

Henri Poincaré reflected at length on these issues. He argued that more-or-less intuitive proofs of theorems in potential theory that are based on an appeal to Dirichlet's principle are without value for the mathematician, although they are of the right sort to satisfy a physicist because they leave the mechanism of the phenomena apparent. On the other hand, the rigorous proofs he knew, including his own, he regarded as wrong in kind, because they do not mimic the physical process. Moreover, they depended on convergence arguments that were usually too slow, and the approximations involved too complicated for such approaches to yield effective numerical procedures (see Poincaré 1890).

Poincaré did not regard the lack of rigour in mathematical physics as acceptable. He could see no clear place to draw the line between mathematics and mathematical physics, no way to be sure that a less than rigorous proof was not in fact misleading, and argued that in any case a rigorous proof teaches something. As we have just seen, the more important distinction for him was between 'right' and 'wrong' proofs—proofs that capture the essence of the problem and those that do not.

For Poincaré, the problem was always: How to proceed? Isolated facts had no appeal for him, he said, but a class of facts held together by analogy was valuable because it brings us into the presence of a law. He openly echoed Ernst Mach in his 1908 address to the International Congress of Mathematicians in Rome when remarking that "The importance of a fact is measured by the return it gives—that is, by the amount of thought it enables us to economise". The elegance of a good proof reflects an underlying harmony that in turn introduces order and unity and "enables us to obtain a clear comprehension of the whole as well as its parts. But that is also precisely what causes it to give a large return". The aesthetic response to mathematics was chiefly appreciated by Poincaré as a sign of its efficacy—he set less store by unconscious feelings of certainty which could, he admitted, prove on rational analysis to be worthless.

In downplaying the importance of isolated facts, Poincaré was seeking to diminish the inexpert view of physics, or any science, that it serves up strange things we would never have known otherwise. Indeed, there are many strange—and many familiar—facts about the physical world that to this day lack good mathematical explanations. His point

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was that, as a leading expert in mathematical physics, with a particular interest in the theories of magnetism, electricity, and optics, the only way to work was theoretically. And for Poincaré, a theoretical understanding was a mathematical one.

In one crucial respect, he argued, a fact had travelled the opposite way, from mathematical physics to mathematics. This, he said, was the gift of the continuum, the structure being defined in his lifetime as the real numbers, but upon which the calculus had been based for at least a century. Poincaré knew that there were other mathematical continua, but the one we believe to be appropriate for mathematical physics was responsible, in his view, for all of mathematics except some algebra (group theory and combinatorics).

What had flowed from mathematics to mathematical physics was language. As he put it, "Mathematics is the only language the physicist speaks." Poincaré was entirely familiar with the passage from theoretical to experimental physics and back, but his touch was less sure. What he knew about in physics, what he sought to increase our understanding of, was mathematical physics, in which the objects under discussion should have good definitions and be treated according to agreed mathematical rules. The best definitions, the deepest insights, would be those that made the theories work most smoothly and produce new discoveries most effectively.

The rules were not exclusively mathematical. He attached particular significance to Newton's laws of motion, the conservation of energy, and the principle of least action. In his view, these rules had passed beyond the stage where they were to be disputed. He gave the example of a satellite in orbit round a planet and which displayed behaviour inconsistent with Newtonian gravity. We would not, he said, any longer try out alternatives to the inverse square law. We would instead look for other forces acting on the satellite that were distorting its orbit. Certain results, formerly a matter of empirical testing and discussion, had been elevated to the status of axioms in the relevant theory. They could not be contested within the theory.

Nonetheless, they were what he called conventions. They were not laws of nature or absolute truths (in his opinion we have no access to such things), but had been raised to the status of incontestable truths within a theory because they had achieved a high degree of empirical confirmation and because they played a central role in an effective theory.

Theories could change. Poincaré did wonder if the mathematical physics he knew, which he called the physics of principles, was not indeed coming to an end, and would have to be replaced by a more probabilistic form of physics if and when thermodynamics was better understood. He knew very well that the branch of physics he knew best and upon which he worked most extensively, was in trouble in the early years of the 20th century. Hertz's theory of magnetism, electricity, and optics could not account for Fizeau's experiments on the speed of light in water; its only rival, Lorentz's, could only do so by abandoning Newton's third law (to every action there is an equal and opposite reaction).

Lorentz was not much bothered by this problem. He was exploring the novel idea that all matter might be electro-magnetic in nature. Were that to be true, might it not be that a law about the behaviour of matter would have to be rewritten when matter was no longer a fundamental concept? But Poincaré found this approach unsatisfactory. To him it was ad hoc, a hypothesis invented to get people out of a difficulty, but one that did not meet the criteria of leading to new discoveries.

He felt the same way about the hypothesis of the supposed Lorentz contraction as Lorentz had proposed it. To Poincaré, this was as if nature was giving one a nudge in the ribs (a "coup de pouce")—something it never otherwise did. He preferred to reformulate Lorentz's idea of local time and make it part of a theory of space and time in which measurements in different moving frames of reference were handled by the transformations of what we now call the Lorentz group, and which Poincaré described shortly before Einstein presented his theory of special relativity.

This is not the place to analyse why, having done so much, Poincaré in the end chose to keep to a separation of space and time and not move over to space-time, or, if you prefer, to keep to the Galilean group and not switch to the Lorentz group. Famously, he and Einstein did not understand what each other had done, and Poincaré was technically right: one can do everything Einstein described in the special theory with the Galilean group and different behaviours of rods and clocks.(1)

That there was a choice to make, that it is made on grounds of efficacy or convenience, as Poincaré sometimes put it, that we do not so much discover the laws of nature as propose theories—those are the lessons to take from Poincaré's philosophising. Two generations later, the physicist Eugene Wigner speculated on the unreasonable effectiveness of mathematics in physics. Poincaré could not have found it unexpected at all. If mathematical physics is conducted in the language of mathematics, matters could not be otherwise. We might raise the possibility that nature might be too unruly to admit detectable regularities at all, but that invites the reply that then there would be no sentient beings. Could other beings understand the universe with radically different mathematics to ours? Poincaré never addressed that question directly. He did speculate on whether other minds could perceive the universe as non-Euclidean, and his answer was a straight-forward yes. His message was that mathematics and mathematical physics are two aspects of the same enterprise. Each advances the other. And whether string theory succeeds or fails, there can be little doubt that the best physics of the 20th century was written in the language of 20th century mathematics, and one must suppose the new physics of the 21st century will follow the same pattern. ▪

Footnotes

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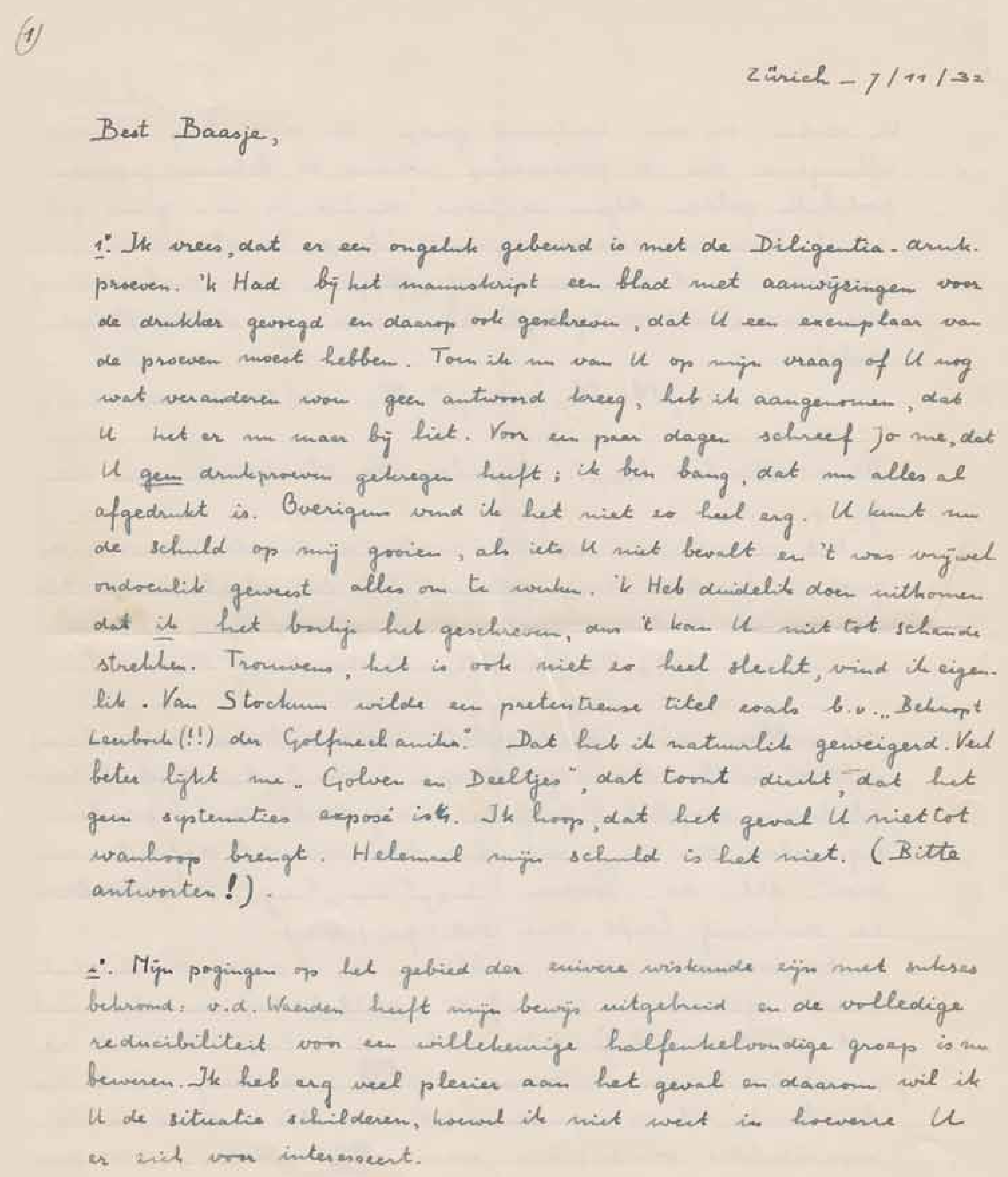


(1)

Zürich – 7/11/32

Best Baasje,

1°. Ik vrees, dat er een ongeluk gebeurd is met de Diligentia-drukproeven. 'k Had bij het manuskript een blad met aanwijzingen voor de drukker gevoegd en daarop ook geschreven, dat U een exemplaar van de proeven moest hebben. Toen ik nu van U op mijn vraag of U nog wat veranderen wou geen antwoord kreeg, heb ik aangenomen, dat U het er nu maar bij liet. Voor een paar dagen schreef Jo me, dat U geen drukproeven gekregen heeft; ik ben bang, dat nu alles al afgedrukt is. Overigens vind ik het niet zo heel erg. U kunt nu de schuld op mij gooien, als iets U niet bevalt en 't was vrijwel ondoenlik geweest alles om te werken. 'k Heb duidelik doen uitkomen dat ik het boekje heb geschreven, dus 't kan U niet tot schande strekken. Trouwens, het is ook niet zo heel slecht, vind ik eigenlik. Van Stockum wilde een pretentieuse titel zoals b.v. „Beknopt Leerboek (!!) der Golfmechanika". Dat heb ik natuurlik geweigerd. Veel beter lijkt me „Golven en Deeltjes", dat toont duidelik, dat het geen systematies exposé is. Ik hoop, dat het geval U niet tot wanhoop brengt. Helemaal mijn schuld is het niet. (Bitte antworten!).

2°. Mijn pogingen op het gebied der zuivere wiskunde zijn met sukses bekroond: v.d. Waerden heeft mijn bewijs uitgebreid en de volledige reducibiliteit van een willekeurige halfenkelvoudige groep is nu bewezen. Ik heb erg veel plezier aan het geval en daarom wil ik U de situatie schilderen, hoewel ik niet weet in hoeverre U er zich voor interesseert.

A Physicist Involved in Purely Algebraic Research: The Case of H. B. G. Casimir

by Martina R. Schneider

The legacy of Paul Ehrenfest (1881–1933), an Austrian physicist, is kept at the Dutch Museum Boerhaave in Leiden and is indeed a treasury of knowledge. Here one can gain insight into Ehrenfest's scientific motivations and problems, learn about his personal opinions on more general (scientific) issues, about the research communities and the research processes he was involved in during the first third of the 20th century. This treasure trove also contains letters to Ehrenfest written by the Dutch theoretical physicist Hendrik Brugt Gerhard Casimir (1909–2000). Casimir was one of Ehrenfest's students who kept Ehrenfest informed about his subsequent scientific activities. In these letters one can gain insight into the development of the first purely algebraic proof of the complete reducibility of finite-dimensional representations of semi-simple Lie-groups (see[1], especially chapter 16). This proof was published in a joint paper by Casimir and the mathematician Bartel Leendert van der Waerden (1903–1996) in 1935[2].

In 1926 Casimir started studying physics, mathematics and astronomy at Leiden university, where Ehrenfest had been teaching since 1912. The bright young student was just in his third semester when he was allowed to participate in Ehrenfest's research colloquium. After passing the "Doctoraalexamen" in June 1928, Casimir turned to theoretical physics: relativity theory and quantum mechanics.

Casimir and van der Waerden probably met in Leiden in 1928/29 during a series of guest lectures organized by Ehrenfest around group theoretical methods in quantum mechanics. At that time a number of articles on the topic by diverse authors as well as the first monograph by the mathematician Hermann Weyl had appeared[3]. Like most physicists, Ehrenfest was unfamiliar with the mathematical theory, i.e. representation theory of groups. The term "group plague" (Gruppenpest) was coined and some physicists, e.g. John Clarke Slater[4], tried to find ways to avoid it. But Ehrenfest was keen to learn about it. He invited leading experts (Wolfgang Pauli, Eugene Paul Wigner, Walter Heitler) as well as the mathematician van der Waerden to Leiden to clarify matters. Casimir who was present during these guest lectures considered van der Waerden's lecture to be brilliant.

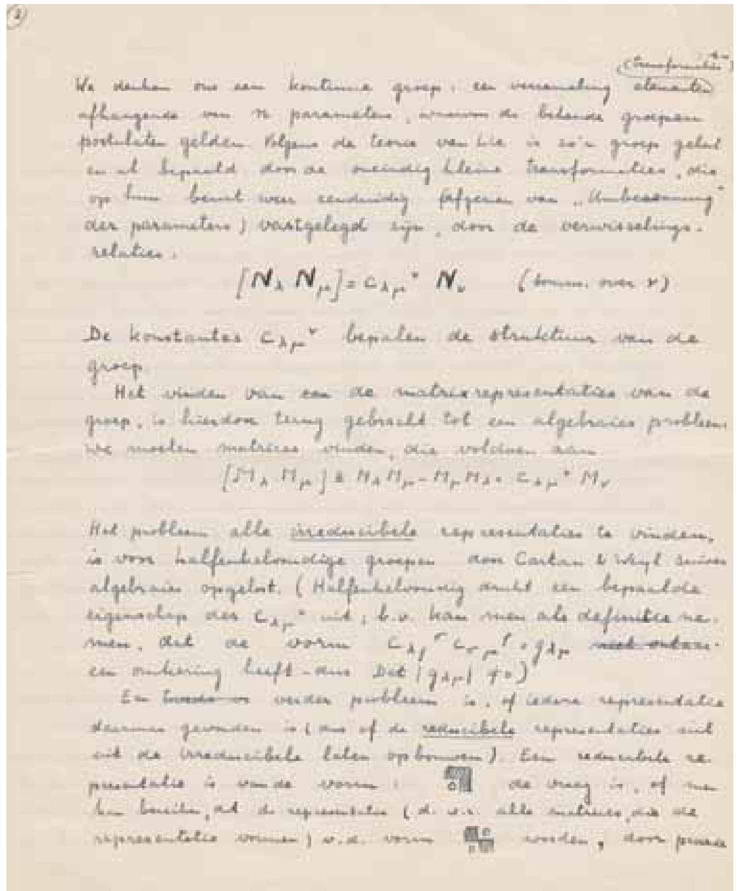
Van der Waerden knew Ehrenfest from his student days. In fact, van der Waerden's first publication ever was a popular science account of a lecture on relativity theory given by Ehrenfest at an institute for workers' education[5]. When van der Waerden became professor at Groningen in October 1928, Ehrenfest frequently wrote him, asking him questions on group theory, especially on certain sections of Weyl's monograph dealing with representations of the rotation group. It was Ehrenfest who urged van der Waerden to develop a calculus which should be analogous to the tensor calculus in special relativity[6]. With the help of this calculus, named spinor-calculus upon Ehrenfest's instigation, the relativistic wave equation of the electron could be handled more easily and conveniently.

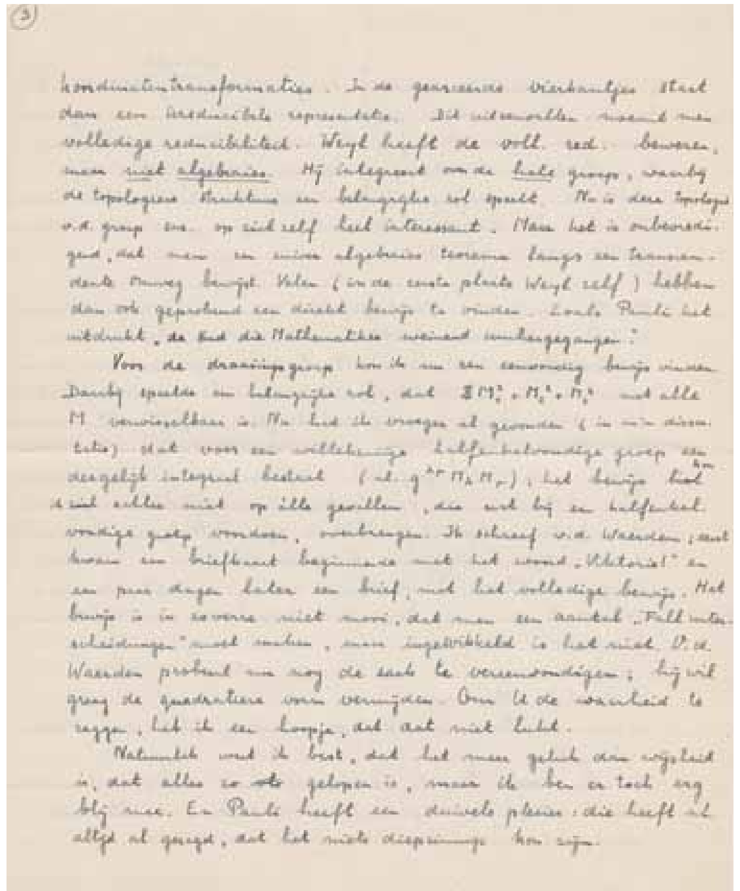
In his Ph.D. thesis, completed in November 1931, Casimir gave an elegant mathematical deduction of the quantum mechanical equations of motion of the spinning top and sketched how the formalism could be applied to describe the (external) rotation of molecules[7]. In this context, he constructed an operator which commuted with any representation of a semi-simple Lie-group[8]—the Casimir-operator was born.

In September 1931 Casimir became assistant to Pauli at the ETH Zurich. Pauli was one of the first to use group theory in quantum mechanics in his attempt to describe the spin of an electron as an intrinsic two-valuedness of the wave function of an electron. During his time as assistant in Hamburg,

Martina R. Schneider is a historian of mathematics at the Institute of Mathematics at Mainz University. She has published several monographs on the interrelationship between mathematics and physics in the 19th and 20th centuries. Her Ph.D. thesis on van der Waerden's group-theoretical contributions to quantum mechanics was awarded with the Nachwuchsförderpreis (young researchers' prize) of the Saxon Academy of Sciences in Leipzig. Recently she has turned to studying the historiography of mathematics by analyzing the developments in the history of ancient Greek mathematics during the 20th century.

Page 20, 22, 23: Letter from Casimir to Ehrenfest; ESC 2, S. 9, 201 (date: November 7, 1932); Archive, Museum Boerhaave, Leiden





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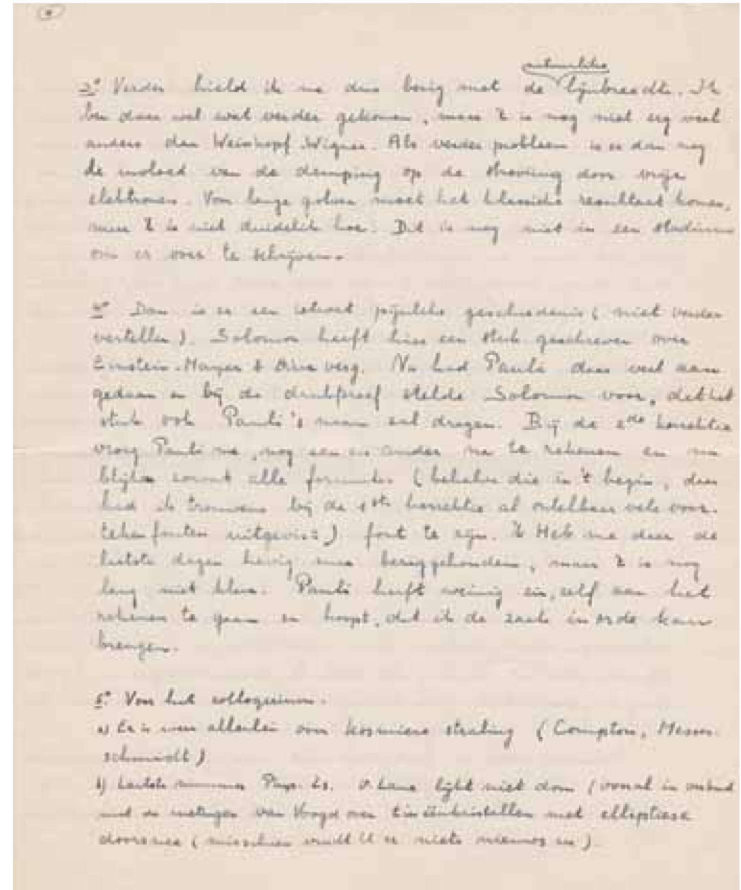
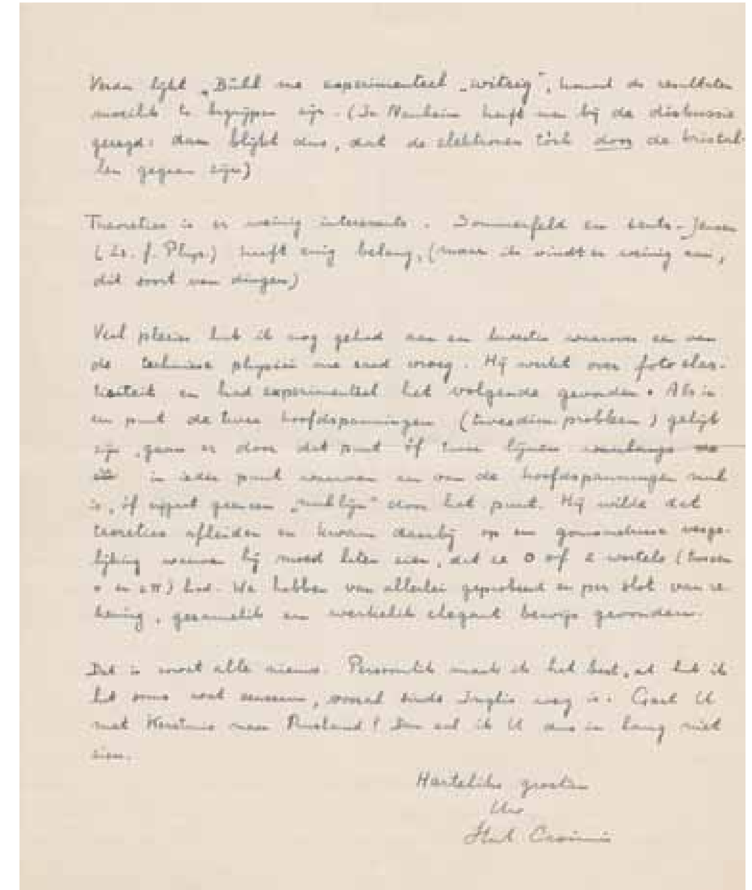
probably in the winter term 1926/27, Pauli attended a series of lectures by Emil Artin on hypercomplex systems (algebras). According to Pauli, Artin started the lecture with a remark on continuous groups, i.e. Lie-groups. Artin could not discuss them because no algebraic proof existed for the complete reducibility of the representations of semi-simple continuous groups. The only proof known was one presented by Weyl in 1925/26. Although Weyl's proof was an important break-through, it unfortunately used integrals and analytic means, instead of algebraic ones. When describing to Ehrenfest how unsatisfied mathematicians, including Weyl himself, were with Weyl's proof, Casimir quoted Pauli as having said: "The mathematicians wandered around in tears."[9]

When Casimir started working in Zurich, Pauli set him the task of proving the full reducibility theorem for semi-simple Lie-groups with algebraic means only. This was a purely mathematical problem. Within one year Casimir had solved the problem for the group of 3-dimensional rotations with the help of the Casimir-operator, but was unable to generalize his proof to arbitrary semi-simple Lie-groups. However, in November 1932, Casimir informed Ehrenfest that a solution had been found. Casimir had written to van der Waerden, who was professor in Leipzig at the time. Van der Waerden had succeeded in proving complete reducibility for arbitrary semi-simple Lie-groups along the lines of Casimir's proof for the rotation group. Van der Waerden was not satisfied because his proof was based on a study of three cases. Only one of the cases could be solved quickly with the help of the Casimir-operator[2]. The delay in the publication of the proof could have been due to van der Waerden's attempt to improve the proof. Casimir was rather proud that van der Waerden could not do without his operator:

"V. d. Waerden is now trying to simplify the thing; he would like to avoid the quadratic form. To tell you the truth, I have hopes that this won't work.

Of course I am aware that it was more luck than intellectual power that everything turned





out that way, but I am very happy about it nevertheless. And Pauli was absolutely delighted: He had always said that it could not be anything profound."[9]

The development of the Casimir-operator is not only an example of a two-way transfer of knowledge and techniques between mathematics and physics, but also represents a methodological shift in the status of group theory: at first physicists had made group theory and quantum mechanics compatible and used group theory mainly as a tool, but then group theory itself became an object of their research in the field of mathematics. In my opinion, what was vital for this transition from being a tool to becoming an object of research was the network of scientists which evolved around group theoretical methods in quantum mechanics. Since group theory was new to the physicists, it needed to be explored by them. Some physicists turned to mathematicians: Pauli went to Artin's lectures, Wigner asked John von Neumann for help, Ehrenfest invited mathematicians to give lectures. Casimir wrote to Weyl and van der Waerden. On the one hand, some of the mathematicians, like von Neumann and van der Waerden, were open to the physical theory and to its special problems. Others like Weyl became even deeper involved in quantum mechanics. On the other hand, physicists, like Pauli, Casimir, Wigner and Guilio Racah, took up mathematical questions and problems. Thus for them group theory also became an object of research—not of physics, but of mathematics, of algebra. Casimir's application of the Casimir-operator to the problem of complete reducibility, however, stands out as an example of research done by a physicist who was not motivated by problems of physics, but of pure mathematics. ▪

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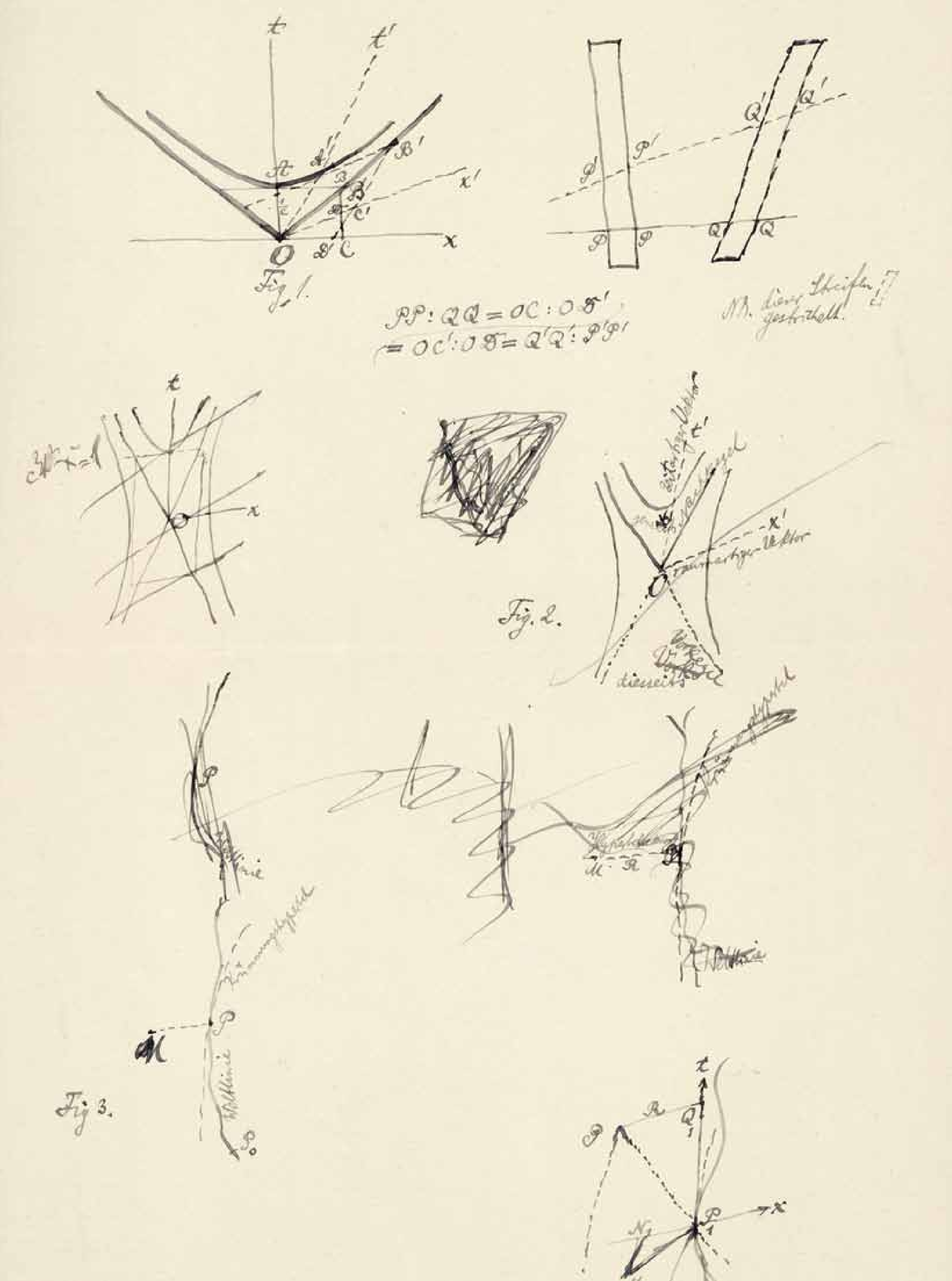


Changing Views—Images and Figures of Thought in the Mathematical Sciences

by Joachim Schwermer

— Part I —

Patterns of scientific discovery. Some sixty years ago thinking about science meant contemplating scientific knowledge. In the case of the mathematical sciences this scholarly work tended to be done from the technical point of view of the history of ideas. This view has changed considerably. To make sense of science, one has to think about both scientific knowledge and the practice with which it engages.

Scientific thought and work is not a single process but a complex pattern of activities aimed towards certain ends. There has been a strong tendency to view scientific discoveries as the outcomes of sudden insights. However, this view of scientific creativity has turned out to be too simple to grasp the many facets and complexity of the scientific enterprise. To some extent this is the inevitable result of the more detailed attention to case studies in the history of sciences. By an analysis of the investigative pathways of a thought process over extended periods of time, one can see the multitude of themes, the gradual growth of certain points of view, repeated encounters with certain structural ideas, their development, their rejection involved in the scientific process.

It has become ever more widely recognized that scientific creativity might be viewed as a process of growth. As a step towards the eventual construction of a "theory of creative thinking" the psychologist Howard E. Gruber called for an "evolving systems approach" in the study of scientific creativity, i.e. a "conceptualization of scientific thought as protracted, purposeful, constructive work".(1) Gruber wrote:

"Issues to be dealt with include: intentionality, the relations between emotions and thought, scientific thinking as a series of structural transformations, metaphoric thought as part of the process of abstraction, differential uptake of complex ideational structures, and the place of insight in an evolving structure of ideas."(2)

Gruber certainly puts forth a perspective that stands in stark contrast to the single "moment" of discovery.

Images and figures of thought as part of the process of abstraction. One of the patterns that play an important role in this intellectual process is the structure of analogies upon which a scientist may have drawn in his or her system of thought. To explore the ongoing mechanisms of innovation and to shed light on the edges of conception it is necessary to reflect on the ways in which "Gedankenbilder", i.e. figures of thought, metaphors and images function in the scientific work. Figures of thought are particularly interesting because they are characterized by both a specific certainty in capturing the invisible hidden behind the appearances and an intuitive uncertainty (or vagueness) to open further explanation. It is the aim of this short note to follow the interplay of thought and image in the mathematical sciences by examining certain cases.

Perception of reality, or, what is an image? This is an apparently simple question, and some people might have an easy answer but I do not. We live in a culture where images are produced at a rate that is nothing short of bewildering, where photographs, films, advertisements, computer pictures are churned out so quickly that our heads start to spin. In addition, we have the treasures of art, the contemporary works of artists on one hand, and we have the scientific images, simple drawings in mathematics or the physical sciences, we have images generated out of large sets of data gained by scanning tunnelling microscope or representing events in high energy physics. We are in the middle of a still unfolding

Photo: Philipp Steinkellner

Joachim Schwermer is Professor of Mathematics at the University of Vienna and Director of the Erwin Schrödinger International Institute for Mathematical Physics, Vienna. His research focuses on questions arising in arithmetic algebraic geometry and the theory of automorphic forms. He takes a keen interest in the mathematical sciences in the 19th and 20th centuries in their historical context.

Page 24: Sketch of the transparency for the lecture "Raum und Zeit", September 21, 1908, Cologne. Cod. Ms. Math. Arch. 60: 2, Bl. 25. SUB Göttingen, Spezialsammlungen und Bestandserhaltung, Abt. Handschriften und Seltene Drucke

AT THE INTERFACE OF MATHEMATICS AND PHYSICS

debate[3] which focuses on the role of images in art and sciences, focuses on the ways in which images help to shape our perception of reality. At the same time, there is the urgent need to have precise distinctions in dealing with images in different contexts—historical, cultural, psychological, philosophical, aesthetical. What is the relation between "picturing science" and "producing art", what are the differences?[4] Moreover, we have to explore an old distrust of images, questioning both the truth of imagination and means of representation.

Visual perception—its ambiguity. In his book "Art and Visual Perception"[5] Rudolf Arnheim discussed some of the virtues of vision. In particular, he described how the foundations of our present knowledge of visual perception are rooted in the studies of the Gestalt psychology. Vision is more than a mechanical recording of elements. It is a creative grasp of significant patterns in reality. The simple act of looking at a given object is characterized by a complex interaction between properties supplied by the object and the intellectual perception of the object. Over the centuries, artists, scientists and each of us have struggled to explore the nature of this relationship of tension and to come up with some satisfying visual concepts to represent objects.

To be more explicit: the elementary task of depicting on a two-dimensional plane a three-dimensional object is a difficult one. Just try to represent a chair on a piece of paper. Different people will get different drawings.

The situation gets even more complicated or interesting if the notion of space is involved. One might discuss how distortions create space, how objects create space, one might trace back how various spatial constructions have developed over the generations and cultures, one might focus on the discovery of central perspective, of the "punta di fuga". Central perspective is a product of visual imagination, a key to solving the problem of spatial organization.

Räumliche Anschauung—spatial intuition. On the one hand, numerous examples make it evident that there is one decisive side of the endless intellectual struggle scientists have to face in their work: images deceive, they lead us to argue on ambiguous premises. On the other hand, we need scientific images because only images as visual elements of thought can help us to develop the intuition needed in the process of conceptualization.[6] This is not an accident. Our brains seem to be organized in such a way that they are extremely concerned with vision. We are highly capable of recognizing, understanding, and making sense of the world that we see upon visual patterns. Therefore, spatial intuition, or spatial perception, (*räumliche Anschauung,* as one says in German) is an enormously powerful tool. Pattern recognition, the capacity to take in a large amount of information by an instantaneous visual action, is a fundamental virtue of the human mind. Images can capture a richness of relations within a given setting in a way that even accurate verbal description can never unfold. We have to acknowledge the dynamic and polysemic signifying power from images.

— Part II —

Visualization. Text books in geometry covering Euclidean geometry, passing through various stages of non-Euclidean geometry to Riemannian geometry provide a menagerie of drawings and images to enrich our understanding of the geometrical objects we are interested in. Sketches or drawings, given in personal discussions or seminar talks, motivate the ideas for rigorous proofs of mathematical results. Even more, in a dynamic process, teaching mathematics at the blackboard intertwines writing and talking, symbols play the role of an object, diagrams are decisive components in the interplay of thinking and writing. In any case, the visualization of geometrical configurations, the figurative marking of analogies between different mathematical structures or fields, the change of the point of view, are used as a device to transmit mathematical knowledge and to inspire an intuitive understanding.

— Part III —

Spatial intuition in Minkowski's work. A case study of one scientist, Hermann Minkowski, may help us to understand the role of visual thinking in the evolving structures of knowledge.

Minkowski presented his views and his theory of an absolute world *(= Postulat der absoluten Welt)* to a wider public on September 21, 1908, when he delivered a lecture to the meeting of the Assembly of Natural Scientists and Physicians in Cologne, later published under the title *"Raum und Zeit"* in 1909.

An analysis of the content of this talk and other sources reveals substantial evidence that Minkowski's visual-geometric insight had a significant influence on the formation of his concept of space and time. Basic geometric notions as symmetry, invariance, familiar to him as a mathematician, play key roles in his thinking on the confusing world of the electron theory at that time. Minkowski's views of the nature of physical reality are directed by his capability of "Räumliche Anschauung", i.e. his extraordinary spatial intuition. Beyond the separation of space and time, which is imposed on us by given experience, the postulate of relativity presents physical reality in its frame as a union of space and time.

Minkowski started his lecture with a quite radical statement:

"Gentlemen, the conceptions of space and time, which I would like to develop before you, arise from the soil of experimental physics. Therein lies their strength. Their direction is radical. From this hour on, space by itself and time by itself are to sink fully into the shadows and only a kind of union of the two should preserve their autonomy.

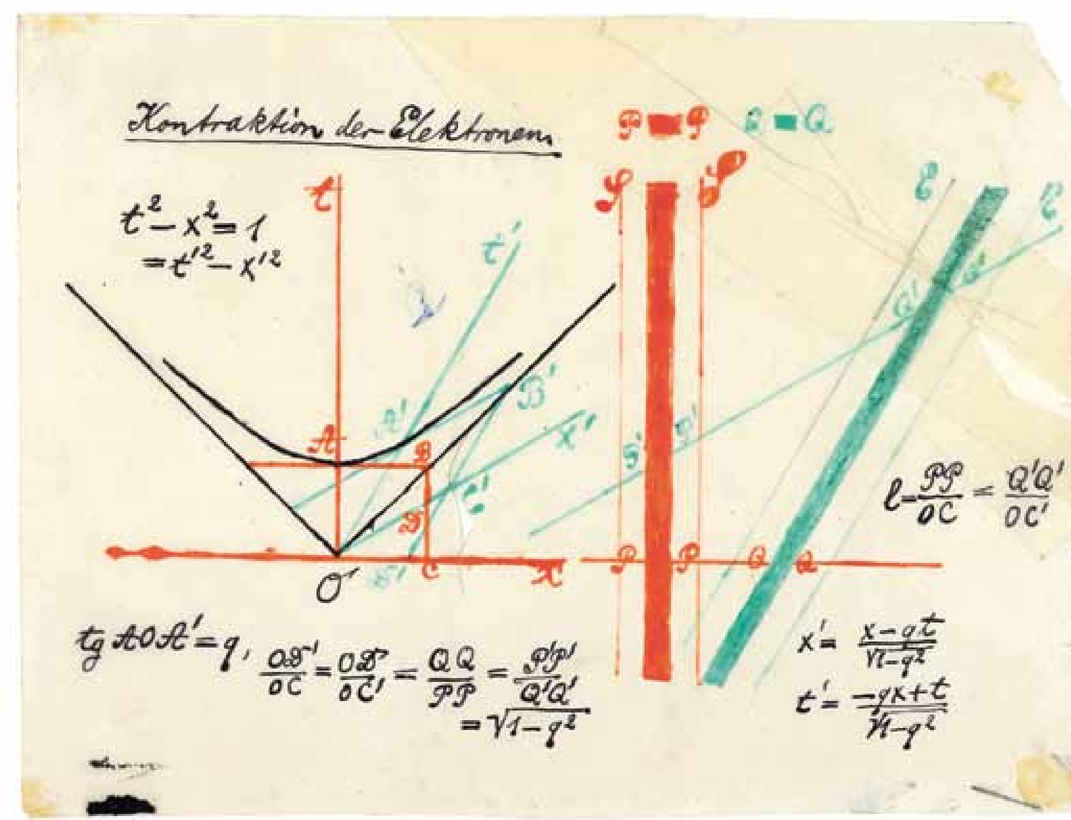
First of all I would like to indicate how, [starting] from mechanics at present, one could arrive through purely mathematical considerations at changed ideas about space and time".[7]

An important point in Minkowki's exposition was the following where he drew attention to the physical space encoded in three orthogonal coordinates and, simultaneously, an arbitrarily-directed temporal axis. He introduced the equation, given in terms of a quadratic form of signature (1, 3),

$$c^2t^2 - x^2 - y^2 - z^2 = 1$$

where c was an unspecified weight assigned to the time coordinate. Suppressing two of the space axes in y and z, he derived his decisive space-time diagram.

This mathematical formulation resembled the approach Minkowski was familiar with in his studies in the arithmetic theory of quadratic forms. In his



Transparency for the lecture "Raum und Zeit", September 21, 1908, Cologne. Cod. Ms. Math. Arch. 60: 2, Bl. 26. SUB Göttingen, Spezialsammlungen und Bestandserhaltung, Abt. Handschriften und Seltene Drucke

"Probevorlesung" for his Habilitation in Bonn, given on March 15, 1887, Minkowski had successfully laid out his geometric approach to questions in the theory of quadratic forms. Following a suggestion of Carl Friedrich Gauss, made in 1840 and pursued by Gustav L. Dirichlet in 1848, Minkowski interpreted quadratic forms as lattices in space. This geometric point of view was already increasingly apparent in Minkowski's early papers when he re-interpreted results obtained in an arithmetical mode as results about lattices in space. But now, as outlined in his Probevorlesung, the approach evolved into a direct treatment of the arithmetically defined objects [that is, the quadratic forms] in the geometric framework. This was quite a decisive step, a turn from arithmetic to geometry. At the same time, Minkowski introduced the concept of volume into his analysis. This simple idea led to a fundamental result, the so-called lattice point theorem. The method of investigation was definitely led by some kind of spatial intuition. This capability of visual-geometric thinking is a unique aspect of Minkowski's approach to questions in number theory. It is steadily directed through geometric concepts and notions. In this way his "Geometry of Numbers" unfolded. It is viewed as one of Minkowski's fundamental contributions to mathematics. However, this visual thinking also forms one pillar in Minkowski's later investigations in the theory of special relativity.

— Part IV —

By examining the case of Minkowski, I indicated in some detail how images and visual thinking may function in the creative process in mathematics or physics. However, images form the core of art. By creating their own reality, images enable the scientist as well as the artist to capture the invisible and help to shape our perception of reality. In light of this connection I would like to conclude with some remarks on similarities, dissimilarities between art and mathematics, remarks towards certain distinctions which have to be made in dealing with images in different contexts.

Sketches matter. Initial sketches to capture the idea of a prospective painting, drawings to bring out the formal organization of a composition, scattered configurations of lines to fix distortions of the pictorial elements so that, e.g. the viewers eye is unconsciously directed towards a certain perception, all these elements constitute essential steps in the artistic creative work. But artists have the tendency to hide their sketches, they view them as their most private sources of inspiration. They view a sketch book as the battlefield where their artistic imagination has to find its formal expression, where compository or aesthetic demands have to match their artistic "Willen", i.e. their conviction and desire. But this is the difficult pathway along which an artwork comes to fruition.

On the other side, mathematics asks for a strict logical-analytical exposition of its results, a clear deduction of arguments in giving a proof. It is difficult to trace the true motivations, the idea behind an argument or a specific point of view, because the publication is convoluted with technical details, condensed to the mathematical subject matter. But the mathematical practice is very different. Mathematicians at work need sketches, they sketch an idea by drawing an image in a discussion with a colleague, they use this device, hopefully, in the classroom, sketches show up in talks to the scientific community. Sketches or images in mathematics capture patterns of thought, they can bring to light some underlying structures by suppressing the unimportant related to a specific question. But the way that sketches are used and how open they are to be perceived depends on the personal and social context.

Rigorous concepts matter. A work of art can convey some of the essential mysteries of life, beyond the frames of ideas. It can unfold the depth of our feelings, it can make us rethink the truth of imagination and the means of representation. At its best its meaning is open ended, it reveals mystery. In contrast, in its best form, a mathematical work represents an idea, and its value lies in the rigorous foundation of the inherent concepts. Revealing the precise nature of an object of our mathematical inquiry within a given framework, making clear the constraints of a given representation or description, capturing beyond the necessary verification the essence of a correspondence between objects in apparently different fields, these tasks mark the heart of mathematical work. The role of mathematics is to understand the mystery and communicate that understanding to others. ▪

Footnotes

(1) cf. Howard E. Gruber, The evolving systems approach to creative scientific work: Charles Darwin's early thought. In: T. Nickles (ed.), Scientific Discovery: Case Studies, 113–130, 1980.

(2) Howard. E. Gruber, ibid. p. 114.

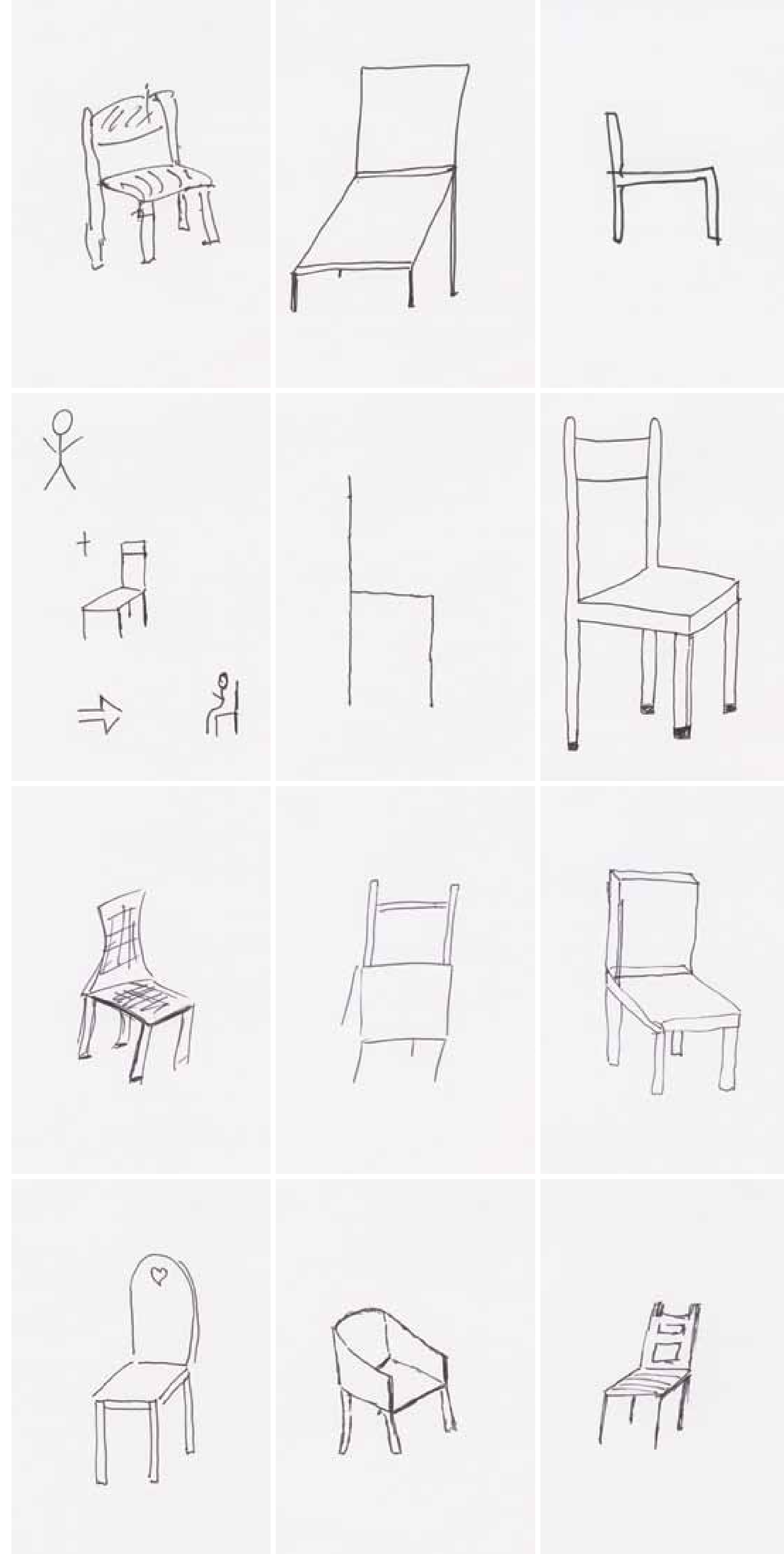
(3) cf. e.g. the exhibition (and the accompanying catalogue) "Iconoclash—beyond the image wars in science, religion and art", ZKM, Karlsruhe, 2002, or, Bettina Heintz, Jörg Huber (ed.), Mit dem Auge denken – Strategien der Sichtbarmachung in wissenschaftlichen und virtuellen Welten, Wien – New York 2001.

(4) This pair of notions is taken from the title of the book: Caroline A. Jones, Peter Galison (ed.), Picturing Science, Producing Art. New York, London 1998.

(5) Rudolf Arnheim, Art and Visual Perception—a Psychology of the Creative Eye, Berkeley-Los Angeles, 1957.

(6) As one testimony one can refer to A. Einstein's "Autobiographical Notes" where he explicitly stated how images become an ordering element of thought and, finally, a concept. Another example is discussed by Peter Galison in "The Surpressed Drawing: Paul Dirac's Hidden Geometry", *Representations* 72 (2000), 145–166.

(7) „M. H.! Die Anschauungen über Raum und Zeit, die ich Ihnen entwickeln möchte, sind auf experimentell-physikalischem Boden erwachsen. Darin liegt ihre Stärke. Ihre Tendenz ist eine radikale. Von Stund' an sollen Raum für sich und Zeit für sich völlig zu Schatten herabsinken und nur noch eine Art Union der beiden soll Selbständigkeit bewahren. Ich möchte zunächst ausführen, wie man von der gegenwärtig angenommenen Mechanik wohl durch eine rein mathematische Überlegung zu veränderten Ideen über Raum und Zeit kommen könnte."
In: H. Minkowski, Gesammelte Abhandlungen [ed. D. Hilbert], Leipzig 1911, vol. II, p. 431–444.



Results of the request to draw a chair.

Scientific Activities

Thematic Programmes

Thematic programmes are the main scientific activities taking place at the ESI. These programmes are extending over a longer period of time, including several workshop periods and research stays of individual scientists. There are usually four to six thematic programmes per year that have to be proposed about two years in advance and are chosen by the Scientific Advisory Board.

Thematic programmes in 2013:

Teichmüller Theory
(organized by L. Funar, Y. Neretin, A. Papadopoulos, R. Penner),
January 28–April 21, 2013

The Geometry of Topological D-Branes, Categories and Applications
(organized by S. Gukov, A. Kapustin, L. Katzarkov and Y. Soibelman),
April 22–July 6, 2013

Jets and Quantum Fields for LHC and Future Colliders
(organized by A. H. Hoang and I.W. Stewart),
July 1–July 31, 2013

Forcing, Large Cardinals and Descriptive Set Theory
(organized by S. Friedman, M. Goldstern, A. Kechris and W. H. Woodin),
September 2–October 25, 2013

Heights in Diophantine Geometry, Group Theory and Additive Combinatorics
(organized by R. Tichy, J. Vaaler, M. Widmer and U. Zannier),
October 21–December 20, 2013

Thematic Programmes in 2014:

Modern Trends in Topological Quantum Field Theory
(organized by J. Fuchs, L. Katzarkov, N. Reshitikin and C. Schweigert),
February 2–March 29, 2014

Combinatorics, Geometry and Physics
(organized by A. Abdesselam, C. Krattenthaler, A. Tanasa, F. Vignes-Tourneret),
June 2–July 31, 2014

Topological Phases and Quantum Matter
(organized by N. Read, J. Yngvason, and M. Zirnbauer),
August 4–September 12, 2014

Minimal Energy Point Sets, Lattices and Designs
(organized by C. Bachoc, P. Grabner, E. Saff and A. Schürmann),
September 29–November 22, 2014

Don Zagier,
Collège de France, Paris, France
and MPI Mathematik, Bonn,
Germany

Research in Teams Programme

The Erwin Schrödinger Institute Research in Teams Programme is in place since the beginning of 2012. It offers teams of two to four ESI scholars the opportunity to work at the Institute in Vienna for periods of one to four months, in order to concentrate on new collaborative research in mathematics and mathematical physics. The interaction between the team members is a central component of this programme.

Teams at the ESI in 2012:

Disordered Oscillator Systems
Bruno Nachtergaele (UC Davis), *Robert Sims* (U Arizona), *Günter Stolz* (U Alabama, Birmingham),
June 18–August 5, 2012

Whittaker Periods of Automorphic Forms
Erez Lapid (Hebrew U), *Zhengyu Mao* (Rutgers U Newark),
July 1–July 31, 2012

Twisted Conjugacy Classes in Discrete Groups
Alexander Fel'shtyn (Szczecin U), *Evgenij Troitsky* (Moscow State U),
July 23–August 23, 2012

Resolution of Surface Singularities in Positive Characteristic
Dale Cutkosky (U Missouri), *Herwig Hauser* (U Vienna), *Hiraku Kawanoue* (Kyoto U), *Stefan Perlega* (U Vienna), *Bernd Schober* (U Regensburg),
November 5–December 21, 2012

Teams at the ESI in 2013:

Degenerate Eisenstein Series for *GL(n)*
Marcela Hanzer (U Zagreb), *Goran Muic* (U Zagreb),
January 7–Feburary 7, 2013

On the First-order Theories of Free Pro-p Groups, Group Extensions and Free Products of Groups
Montserrat Casals-Ruiz (U Oxford), *Ilya Kazachkov* (U Oxford), and *Vladimir N. Remeslennikov* (Russian Academy of Science),
February 18–March 16, 2013

Non-Commutative Geometry and Spectral Invariants
Alan Carey (Australian National U), *Harald Grosse* (U Vienna), *Jens Kaad* (U Bonn),
May 27–June 23, 2013

Nuclear Dimension and Coarse Geometry
Erik Guentner (U Hawaii), *Jan Spakula* (U Münster), *Rufus Willett* (U Hawaii),
May 27–June 23, 2013

Junior Research Fellowship Programme

From 2004 to 2010, the ESI Junior Research Fellowship Programme was in place. Its purpose was providing support for advanced Ph.D. students and postdoctoral fellows to allow them to participate in the activities of the ESI. Grants were given for periods between two and six months.

When the programme had to be terminated due to lack of funding in 2010, there had been more than 150 Junior Research Fellows at the ESI, coming from over 30 different countries.

I am a number theorist a little over halfway through a six month stay in Vienna, working at the ESI as a *Junior Research Fellow,* and what follows are a few thoughts on my time here so far.

Despite being called an Institute for Mathematical Physics, the ESI has a diverse range of mathematical activities, and has certainly kept this particular number theorist stimulated and occupied with organised events—there have been number theoretic seminars and lecture courses running non-stop for over three months. All of these activities have been conducted in a relaxed atmosphere, with all those in attendance happy to talk to a young researcher like myself. The Institute's support for those of us just beginning our research careers is admirable.

The ESI, occupying the top floor of a priests' seminary, is a wonderfully tranquil place to sit and think—it is close to the centre of the city and easily accessible by public transport, yet feels peaceful and calm. It is too easy to spend all of one's time at the Institute! Nothing gets in the way of doing mathematics. The staff at the ESI, both academic and administrative, have been unfailingly friendly. The mathematicians, though busy, have always involved me in the local events and found time for a mathematical chat. I was invited to and took part in several of the seminars taking place at the University of Vienna.

One of the great benefits of a visit to the ESI is the chance it provides to spend some time in the city of Vienna. Of course, I have been to Schönbrunn, seen 'The Kiss' and been to the opera—like all good tourists should—but I have been fortunate to be shown a few things not in the guide books. The friends I have made amongst the staff at the ESI and students of the University have been kind enough to show me some of their own favourite places in Vienna—be they tucked-away restaurants, coffee houses or nightclubs. I am not even sure the six months of my stay will be long enough to sample all of the astonishing array of coffee houses. Apart from sating my worrying new addiction to coffee and cake, I have found them a pleasant setting for doing mathematics (though not as pleasant as the ESI, of course!). Now, if I can only finish this tome of Musil whilst I am here…

Adam Joyce
Spring 2006

Summer Schools

Summer Schools are short programmes of one to two weeks that are aimed at young researchers such as graduate students and postdocs. These schools usually consist of a series of basic introductory lectures in the beginning, complemented by some more advanced talks on ongoing research, and are organized by renown scientists in mathematics and mathematical physics.

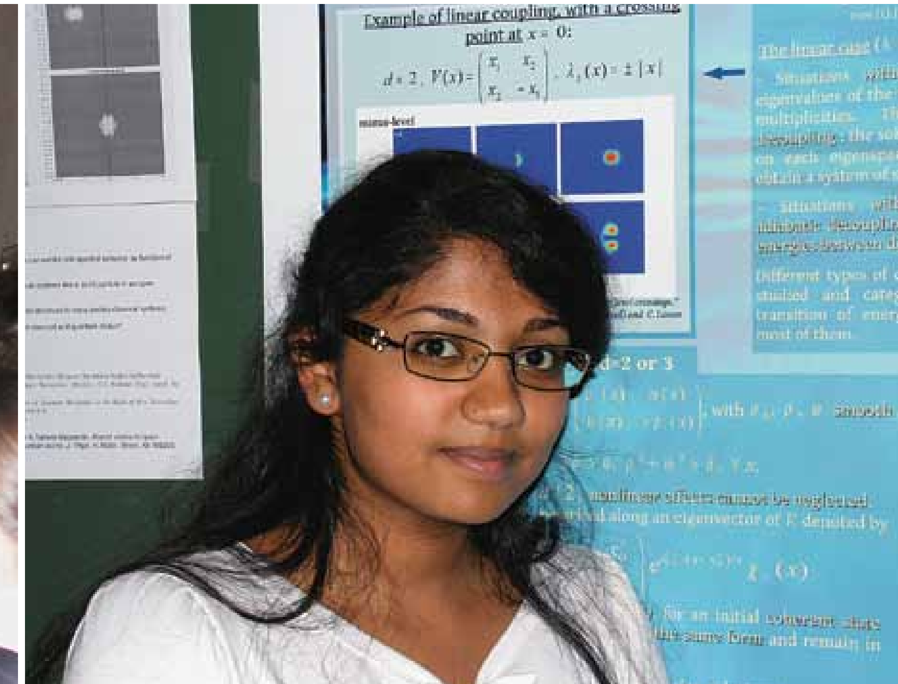
EMS IAMP Summer School on Quantum Chaos. From July 30 to August 3 2012 a Summer School on Quantum Chaos took place at the ESI. Organized by Nalini Anantharaman, Stéphane Nonnenmacher, Zeév Rudnick and Steve Zelditch, it was the first one of a series of schools on various topics of mathematical physics supported by the IAMP and the EMS. Young researchers from more than 10 different countries spent a week in Vienna to learn about the research field of "Quantum Chaos".

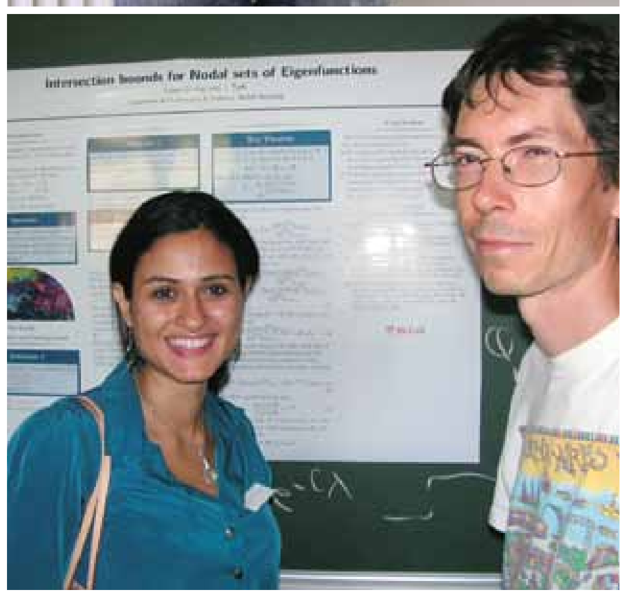
This research field aims at understanding the dynamics of quantum (or wave) systems admitting a chaotic classical counterpart and is situated at the interface of mathematics and physics. As the summer school was mainly adressed to graduate students and postdocs, the week started with several basic courses that introduced the subject and the basic methods and concepts. These introductory lectures were held from Monday to Wednesday and covered topics such as the elementary theory of dynamical systems, semiclassical analysis, and the theory of random matices, amongst others. The second part of the week was devoted to a series of more advanced talks, presenting some recent work and new developments.

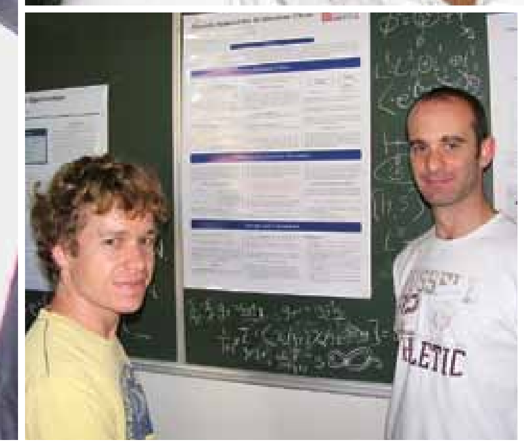
A special event of the summer school was the poster session on Tuesday evening, in which the participants presented their own research. Especially for younger researchers at the beginning of their academic career this served as a good opportunity to present and discuss their research interests and problems with experts of the field in an informal setting. The poster session turned out to be well-attended and very lively and it also showed the many different research interests and backgrounds that were represented at the summer school. ▪

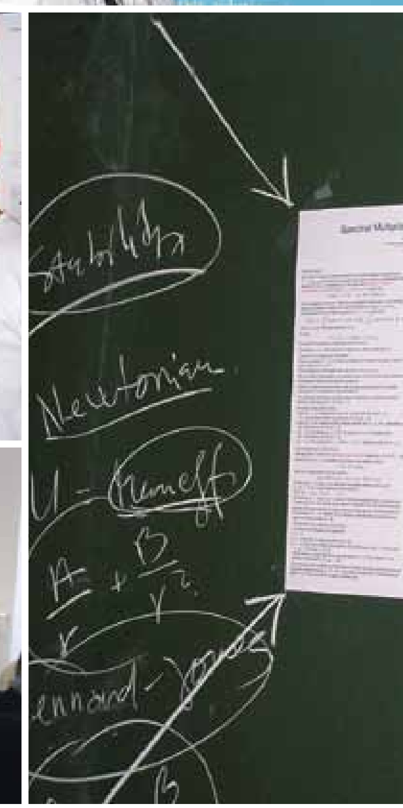
Most of the ESI Junior Research Fellows, May 2006

IMPRESSIONS



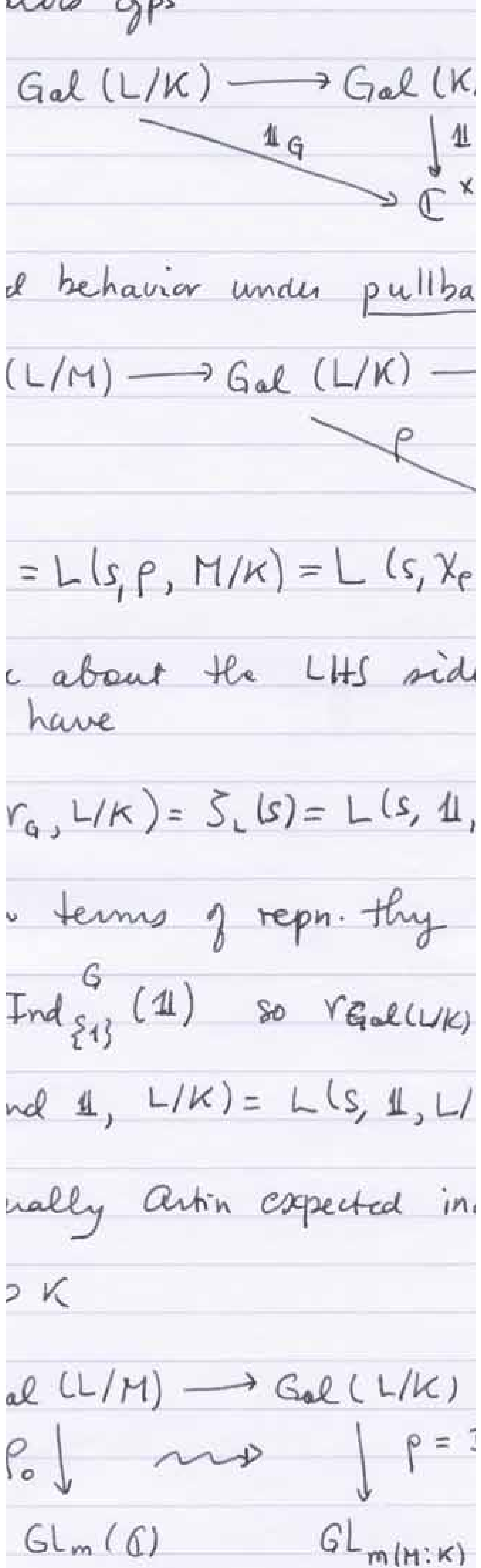






Senior Research Fellowship Programme

The Erwin Schrödinger Institute regularly offers lecture courses and research seminars at an advanced graduate level. These courses and seminars are taught by Senior Research Fellows of the ESI who stay in Vienna for a period of several months. In bringing together young researchers working in Vienna and senior scientists from abroad, the ESI Senior Research Fellows programme contributes in an effective way to the scientific training of graduate students and postdocs of Vienna's universities.



SENIOR RESEARCH FELLOWSHIP PROGRAMME

Gathering Evidence: My Time as a Senior Research Fellow

by James W. Cogdell

The origins of a colony (I). I was a Senior Research Fellow (SRF) at the ESI during the winter term of 2011–2012. As part of my "duties" I gave a course on "*L*-functions and Functoriality". The principle of functoriality is one of the central tenets of the Langlands program; it is a purely automorphic avatar of Langlands vision of a non-abelian class field theory. There are two main approaches to functoriality. The one envisioned by Langlands is through the Arthur-Selberg trace formula, and with the recent work of Ngô, Arthur, and others this is now becoming available. The second method is that of *L*-functions as envisioned by Piatetski-Shapiro and is based on the converse theorem for $GL(n)$. The overall purpose of the series of lectures was to develop and explain the *L*-function approach to functoriality. The course consisted of 12 lectures of 90 minutes each together with the accompanying question periods of 45 minutes each. The first lecture covered the theory of modular forms and their *L*-functions, the classical $GL(2)$ theory, as developed by Hecke in the 1930's. The second lecture began the adelic theory of automorphic representations for $GL(n)$, covering the basic definitions of cuspidal automorphic representations and their decomposition into local representations. Lectures 3 through 9 were spent on the theory of *L*-functions for $GL(n)$ and the twisted *L*-functions for $GL(n)\times GL(m)$. The tenth lecture was devoted to the Converse Theorem for $GL(n)$. The eleventh lecture was devoted to describing an arithmetic family of Euler products that are conjecturally nice, the Artin *L*-functions attached to an *n*-dimensional representation of a Galois group. In this lecture we surveyed the results of class field theory and then the contents of Artin's three papers on his non-abelian *L*-functions from 1923, 1930, and 1931. In the final lecture we applied the "moral theorem" coming from the Converse Theorem to the conjecturally "nice" *L*-functions of Artin to motivate the global and local Langlands correspondences, which roughly state that *n*-dimensional local or global Galois representations should be locally or globally modular, i.e., attached to appropriate representations of $GL(n)$ in such a way that preserves their *L*-functions. This is Langlands' formulation of a "non-abelian class field theory". We discussed how one would formulate such a Langlands correspondence for groups *G* other than $GL(n)$. Putting all this together, we formulated the Langlands Functoriality Conjecture as a process of transferring local and automorphic representations of *G* to $GL(n)$ mediated by the local and global Langlands correspondences, that is, by *L*-functions. Finally we discussed how one would then use the Converse Theorem on $GL(n)$ as a tool for establishing cases of this Functoriality Conjecture.

In attendance I had 5–6 graduate students plus 2 faculty from the University of Vienna. The graduate students ranged from early career to finishing. Besides the lectures, every week I gave out problem sets and a list of historical references for the week's lectures. Even though the problem sessions were designed for student solutions of the exercises—and some were indeed used for that purpose—we also used them as open Q & A sessions where anyone could ask questions. If I asked, they were related to the exercises-detail questions. But when they asked, while there was the occasional detail question about lecture material, they were more often philosophical or historical or questions about relations with other ideas in the area or surrounding areas. For example, while the lectures were centered around integral representations of *L*-functions, the Q & A sessions prompted me to give an impromptu outline of the Langlands-Shahidi method, relating analytic properties of *L*-functions to Fourier coefficients of Eisenstein series. These impromptu expositions are equally challenging and ultimately

Photo: Patty Merry

James W. Cogdell was a student of Piatetski-Shapiro at Yale, finishing in 1981. He has held faculty positions at Rutgers University, Oklahoma State University, and The Ohio State University, as well as several visiting positions through the years. He has spent all of his professional life thinking about *L*-functions, but makes no claims of truly understanding them (and doubts the claims of anyone who says they do).

Page 38 and 40: Clippings from the course notebook that Cogdell used for his lectures.

SENIOR RESEARCH FELLOWSHIP PROGRAMME

satisfying when things go well. It forces an interesting concentration and perspective. Giving the students such opportunity for open ended questions and discussions gives them a sense of control over the direction and makes them an active participant in the course.

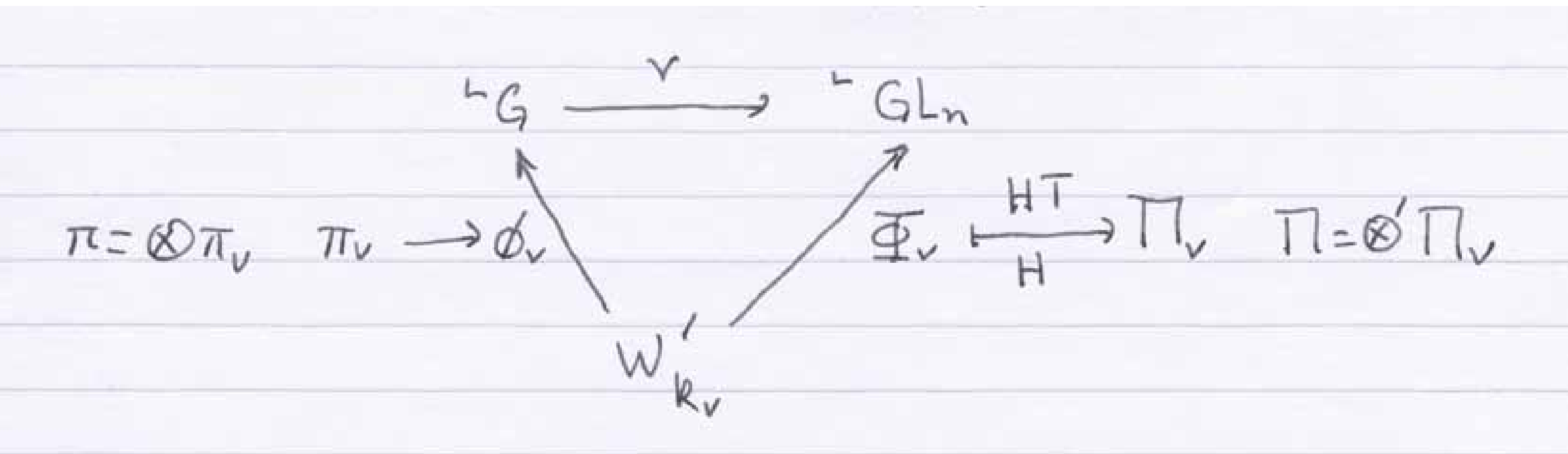
The origins of a colony (II). The SRF program should be very rewarding for both the students and the lecturer. In my case, my SRF tenure coincided with an ESI Programme that I co-organized on "Automorphic Forms: Arithmetic and Geometry" held in January and February of 2012. My course was on *L*-functions for *GL(n)* and was intended as a good preparatory course for the topic of the Programme. Indeed, when the Programme came around, all of my students were regular participants, coming to most of the lectures (when they didn't have other duties) and some of the junior participants in the Programme attended a few of my later course lectures. Two of the more advanced students from the course were invited to speak in the Programme. Their participation in the Programme was very rewarding to me as their instructor in the course. On the other hand, the SRF lectures and their preparation impacted my own work. As part of the course, in order to help motivate the Langlands Program and the connections with arithmetic, I decided to devote one lecture to a survey of Artin *L*-functions. Artin wrote three papers on these *L*-functions in 1923, 1930 and 1931. It was quite a challenge to try to distill these papers down to a single lecture, but in doing this I had several insights into Artin's motivation and sense of structure that he built into these *L*-functions. Not only were these revelations intellectually satisfying, they were necessary for me to be able to give a coherent presentation of Artin's work in my class and I subsequently expanded an article I had written on Artin *L*-functions to include these insights. This article dates from my participation in the meeting "Emil Artin—His Work and His Life" held at the ESI in January of 2006. For the accompanying volume on Artin I had prepared a paper *On Artin L-functions* based on my 2006 lecture. I took this time at ESI to revise this article and in particular I have now included much of the material that I discussed in the lecture on Artin *L*-functions in my course. So for my own intellectual development, the chance to give the SRF course was of significant importance.

The Senior Research Fellow programme, particularly when coupled with an ESI Thematic Programme, is one important way in which the ESI impacts graduate education at the University of Vienna. As part of its mission to further the mathematical and physical sciences, an impact on the next generation of scientists is paramount. Contact, both formal and informal, with the Senior Research Fellow can be very important for a students development. I still feel the influence of a summer workshop on Complex Varieties that I attended in Montreal in the summer of 1979, where I had the good fortune to interact with and attend lectures by Douady, Hirzebruch, Mumford,… as well as making connections with the algebraic geometers of my generation. I was very pleased to be able to repay a bit of this mathematical debt by participating in the SRF programme of the ESI.

On the open road. While the preparing and giving of the SRF lectures did take time and effort, there was plenty of time to devote to my own research agenda. During my tenure as a SRF I also worked on a joint project with F. Shahidi of Purdue and T-L. Tsai of the National University of Taiwan on the "Local Langlands correspondence for *GL(n)* and the exterior and symmetric square ε-factors". This project was begun before my residence at ESI, but the final work and the writing were done at ESI, both during my SRF tenure and during the subsequent Programme. (Both Shahidi and Tsai were in residence at the ESI in the month of January 2012 as part of the "Automorphic Forms: Arithmetic and Geometry" programme.) In this work, we show that the local Langlands correspondence for *GL(n)* preserves both the exterior square and symmetric square *L*- and ε-factors. This is a measure of robustness of the local Langlands correspondence and hopefully will help us understand the local Artin ε-factors better. As part of this work, we established the local analytic stability of the exterior square γ-factor for supercuspidal representations of *GL(n)*, a result of independent interest in the local theory of automorphic forms. This paper will eventually find a home in the ESI Preprint archive.

On the other shore. I also took advantage of the central location of Vienna and the ESI to take scientific trips both to the west and to the east. Before the start of my SRF duties I traveled to Oberwolfach to take part in the meeting on "Emigration of Mathematicians and Transmission of Mathematics: Historical Lessons and Consequences of the Third Reich" during late October and early November of 2011. This was related to my interest in Artin as mentioned above. In addition, I took a flight to Zurich to speak on my work with Shahidi and Tsai at ETH and then took the train to Lausanne to speak on the same work at EPFL. During the height of the frigid cold spell of the winter of 2011 I boarded the train at Wien Meidling for Budapest to give two talks at the Rényi Institute. While the trips to Oberwolfach, Zurich and Lausanne were planned in conjunction with my trip to ESI, the trip to Budapest was somewhat serendipitous, only arising when a colleague at Rényi learned through e-mail that I was but a train ride away. His reaction was an immediate "when are you coming to give a talk?", and so I made my first trip to Budapest.

At the Blue Unicorn. Alas, the life of a Senior Research Fellow is not all fun and games… there are serious cultural duties to perform in Vienna. Vienna is a musical city: Mozart, Beethoven, Mahler, … But my tastes run more post-Darmstadt to improvised, and Vienna is a perfect city for that as well. Where the more classically inclined might favor the Musikverein, I frequented the Konzerthaus. I attended a Klangforum Wien concert which featured pieces by Boulez, Cerha, Feldman, Furrer, and Neuwirth. Cerha, Furrer and Neuwirth are all contemporary Viennese composers, and Cerha was in attendance. I attended the premier of an organ and electronics piece written and performed by Wolfgang Mitterer, another contemporary Viennese composer, as part of the Wien Modern festival. On the improvised side I heard the pianist Craig Taiborn and then a duo consisting of Theo Beckmann (voice) and Michael Wollny (piano). I also made my first foray into the Wiener Staatsoper to hear a performance of Janáček's "Aus einem Totenhaus". One of the advantages of an interest in contemporary music is that tickets seemed to be readily available and I could decide with very little notice what to attend. (For the more classical fare at the Musikverein or the Staatsoper I understand one must plan and purchase tickets before one even boards the plane to Vienna.) There was even music at the ESI! As part of the Programme on Automorphic Forms, there was a piano recital by Gülsin Onay, a Turkish State Artist, with a program of Beethoven, Bartok, and Saygun. It was attended by the workshop participants, other members of the ESI community and two of the Turkish ambassadors in Vienna. Vienna also has wonderful museums throughout the city. Probably the museum highlight of this stay was the opening of the Claes Oldenburg exhibit at MUMOK. I also finally made it to see "The Last Judgement" triptych by Hieronymus Bosch at the Akademie für die Bildenden Künste. In a more classical vein, I toured the Lichtenstein Museum, near the ESI, sadly during its last week as a public museum. Vienna has a very eclectic food scene, and I did eat most of my meals out. I will admit I have a proclivity for both traditional and contemporary Viennese fare, and Austrian wines, and there are many fine restaurants and cafes both near and far from the ESI to partake from (as well as some very interesting local "hole in the walls"). But when I think back on it, I realize that I also had fine Chinese, Greek, Italian, Japanese, Persian, and Vegetarian meals as well. But, in spite of all the wondrous things one can do in Vienna, I think I still find that what I enjoy most is just wandering the streets, particularly in the first district. I prefer them at night, when they are more likely to be empty, and covered with snow if possible. (Unfortunately, there was little snow during this visit.) My only regrets are the places I didn't get to visit: the Wittgenstein House, the Anselm Kiefer exhibit, Roithamer's Cone, … but there will be future visits. ▪



Data and Statistics

The Foundational Period 1990–1993

The years 1990–1993, preceding the official foundation of the Erwin Schrödinger International Institute for Mathematical Physics in Vienna, had been a time of intense preparations involving many mathematicians and physicists. This phase is very well documented and described in the Appendix A to the Scientific Report for the years 1993–2002 which can be found on the web page of the ESI under www.esi.ac.at/about/reports.html. Walter Thirring, Peter Michor and Heide Narnhofer, acting on behalf of the scientific community, played a decisive role in the foundational period of the ESI and beyond. Their initiative was well taken up by the Ministry of Science and Research.

Three workshops, organized in the years 1991–1992, underlined the importance of the enterprise to found a research institute at the interface of mathematics and physics in Vienna and resulted in considerable momentum. They form the first scientific activities related to the ESI:

Interfaces Between Mathematics and Physics I (P. W. Michor, H. Narnhofer, W. Thirring), May 22–23, 1991

Interfaces Between Mathematics and Physics II (P. W. Michor, H. Narnhofer, W. Thirring), March 2-6, 1992

75 years of Radon Transform (S. Gindikin, P. W. Michor), August 31–September 4, 1992

ESI Thematic Programmes 1993–2013

— 1993 —

Two-Dimensional Quantum Field Theory (H. Grosse), February 15–June 30, 1993

Schrödinger Operators (T. Hoffmann-Ostenhof), 1993

Differential Geometry (P. W. Michor), 1993

— 1994 —

Ergodicity in Non-Commutative Algebras (H. Narnhofer), spring 1994

Mathematical Relativity (P. C. Aichelburg, R. Beig), July 1–September 15, 1994

Gibbsian Random Fields (R. Dobrushin), August 1–December 31, 1994

Spinors, Twistors and Conformal Invariants (A. Trautman, V. Souek, H. Urbantke), September 1–October 31, 1994

Quanternionic and Hyper-Kähler Manifolds (D. Alekseevsky, S. Salamon), September 1–December 31, 1994

— 1995 —

Complex Analysis (F. Haslinger), January 1–March 31, 1995

Noncommutative Differential Geometry (A. Connes, M. Dubois-Voilette, P. W. Michor), spring 1995

Field Theory and Differential Geometry (G. Marmo, P. W. Michor), May 15–July 31, 1995

Geometry of Nonlinear Partial Differential Equations (A. Vinogradov), spring 1995

Gibbs Random Fields and Phase Transitions (R. Dobrushin, R. Kotecký), fall 1995

Reaction-Diffusion Equations in Biological Context (K. Sigmund, R. Bürger, J. Hofbauer), September 1–November 15, 1995

Condensed Matter Physics—Dynamics, Geometry and Spectral Theory (V. Bach, R. Seiler), August 6, 1995–February 24, 1996

— 1996 —

Condensed Matter Physics—Dynamics, Geometry and Spectral Theory (V. Bach, R. Seiler), August 6, 1995–February 24, 1996

Topological, Conformal and Integrable Field Theory (K. Gawedzki, H. Grosse), February 15–May 14, 1996

Representation Theory With Applications to Mathematical Physics (I. Penkov, J. A. Wolf), April 1–June 30, 1996

Mathematical Problems of Quantum Gravity (A. Ashtekhar, P. C. Aichelburg), July 1–August 31, 1996

Hyperbolic Dynamical Systems With Singularities (D. Szász), September 1–December 31, 1996

— 1997 —

Ergodic Theory and Dynamical Systems (A. Katok, K. Schmidt, G. Margulis), January 1–August 30, 1997

Mathematic Relativity (R. Beig), January 1–June 30, 1997

Spaces of Geodesics and Complex Structures in General Relativity and Differential Geometry (L. Mason, P. Nurowski, H. Urbantke), March 1–July 31, 1997

Local Quantum Physics (D. Buchholz, H. Narnhofer, J. Yngvason), September 1–December 31, 1997

Nonlinear Theory of Generalized Functions (M. Oberguggenberger), September–December, 1997

— 1998 —

Spectral Geometry and Its Applications (L. Friedlander, V. Guillemin), February 1–June 30, 1998

Schrödinger Operators With Magnetic Fields (I. Herbst, T. Hoffmann-Ostenhof, J. Yngvason), March 1–June 30, 1998

Number Theory and Physics I. Convexity (P. M. Gruber), September–December, 1998

Number Theory and Physics II. Quantum Field Theory and the Statistical Distribution of Prime Numbers (I. Todorov), September 1–November 30, 1998

Quantization, Generalized BRS Cohomology and Anomalies (R. A. Bertlmann, M. Kreuzer, W. Kummer, A. Rebhan, M. Schweda), September 28–December 31, 1998

Charged Particle Kinetics (C. Schmeiser, P. Markowich), October 5, 1998–January 31, 1999

— 1999 —

Functional Analysis (J. B. Cooper), January 1–July 31, 1999

Nonequilibrium Statistical Mechanics (G. Gallavotti, H. Posch, H. Spohn), February 1–March 31, 1999

Holonomy Groups in Differential Geometry (D. Alekseevsky, K. Galicki, C. LeBrun), September 6–December 31, 1999

Complex Analysis (F. Haslinger, H. Upmeier), August 1–November 15, 1999

Applications of Integrability (A. Alekseev, L. Faddeev, H. Grosse), August 15–October 31, 1999

— 2000 —

Duality, String Theory and *M*-Theory (H. Grosse, M. Kreuzer, S. Theisen), March 15–July 15, 2000

Representation Theory (V. Kac, A. Kirillov), April 1–July 31, 2000

Confinement (W. Lucha, A. Martin, F. Schöberl), May 1–June 30, 2000

Algebraic Groups, Invariant Theory, and Applications (B. Kostant, P. W. Michor, F. Pauer, V. Popov), August 1–December 29, 2000

Quantum Measurement and Information (A. Zeilinger, A. Eckert, P. Zoller), September 3–December 20, 2000

— 2001 —

Scattering Theory (V. Petkov, A. Vasy, M. Zworski), March 1–July 31, 2001

Random Walks (V. Kaimanovich, K. Schmidt, W. Woess), February 19–July 13, 2001

Mathematical Cosmology (P.C. Aichelburg, G. F. R. Ellis, V. Moncrief, J. Wainwright), June 15–August 15, 2001

Mathematical Aspects of String Theory (M. Blau, J. Figueroa O'Farrill, A. Schwarz), September 3–November 16, 2001

Nonlinear Schrödinger and Quantum Boltzmann Equations (P. Gérard, P. Markowich, N. J. Mauser, G. Papanicolau), fall 2001

— 2002 —

Developed Turbulence (K. Gawedzki, A. Kupiainen, M. Vergassola), May 15–July 14, 2002

Arithmetic, Automata, and Asymptotics (R. Tichy, P. Grabner), March 22–July 5, 2002

Quantum Field Theory on Curved Space Time (K. Fredenhagen, R. Wald, J. Yngvason), July 1–August 31, 2002

Aspects of Foliation Theory in Geometry, Topology and Physics (J. Glazebrook, F. Kamber, K. Richardson), July 15–November 30, 2002

Noncommutative Geometry and Quantum Field Theory, Feynman Diagrams in Mathematics and Physics (H. Grosse, J. Madore, D. Kreimer, J. Mickelsson, I. Todorov), August 26–November 22, 2002

— 2003 —

Mathematical Population Genetics and Statistical Physics (E. Baake, M. Baake and R. Bürger), December 1, 2002–February 28, 2003

Kakeya-Related Problems in Analysis (A. Iosevich, I. Laba, D. Müller), February 15–April 15, 2003

Penrose Inequalities (R. Beig, P. Chruściel, W. Simon), June 2–July 29, 2003

Poisson Geometry and Moment Maps (A. Alekseev, T. Ratiu, S. Haller, P. W. Michor), August 1–October 15, 2003

Gravity in Two Dimensions (W. Kummer, H. Nicolai, D. V. Vassilevich), September 8–October 31, 2003

— 2004 —

Geometric and Analytic Problems Related to Cartan Connections (T. Branson, A. Cap, J. Slovak), January 2–April 20, 2004

String Theory in Curved Backgrounds and Boundary Conformal Field Theory (H. Grosse, A. Recknagel, V. Schomerus), March 1–June 30, 2004

Tensor Categories in Mathematics and Physics (J. Fuchs, Y.-Z. Huang, A. Kirillov, M. Kreuzer, J. Lepowsky, C. Schweigert), May 31–July 9, 2004

Singularity Formation in Non-linear Evolution Equations (P. C. Aichelburg, P. Bizoń), July 7–August 15, 2004

Many-Body Quantum Theory (M. Salmhofer, J. Yngvason), September 1–December 31, 2004

— 2005 —

Open Quantum Systems (J. Derezinski, G. M. Graf, J. Yngvason), January 20–March 31, 2005

Modern Methods of Time-Frequency Analysis (J. J. Benedetto, H. G. Feichtinger, K. Gröchenig), April 4–July 8, 2005

Geometric Methods in Analysis and Probability (J. Cooper, P. W. Jones, V. Milman, P. Müller, A. Pajor, D. Preiss, C. Schütt, C. Stegall), May 25–August 5, 2005

Complex Analysis, Operator Theory and Applications to Mathematical Physics (F. Haslinger, E. Straube, H. Upmeier), September 5–November 11, 2005

Geometry of Pseudo-Riemannian Manifolds With Applications to Physics (D. Alekseevsky, H. Baum, J. Konderak), September 1–December 31, 2005

— 2006 —

Arithmetic Algebraic Geometry (S. S. Kudla, M. Rapoport, J. Schwermer), January 2–February 18, 2006

Diophantine Approximation and Heights (D. Masser, H. P. Schlickewei, W. M. Schmidt), February 27–May 12, 2006

Rigidity and Flexibility (V. Alexandrov, I. Sabitov, H. Stachel), April 23–May 6, 2006

Gerbes, Groupoids, and Quantum Field Theory (P. Aschieri, H. Grosse, B. Jurco, J. Mickelsson, P. Xu), May 8–July 31, 2006

Complex Quantum and Classical Systems and Effective Equations (E. Carlen, L. Erdős, M. Loss), May 15–August 15, 2006

Homological Mirror Symmetry (A. Kapustin, M. Kreuzer, A. Polishchuk, K.-G. Schlesinger), June 12–28, 2006

Global Optimization, Integrating Convexity, Optimization, Logic Programming, and Computational Algebraic Geometry (I. Bomze, I. Emiris, A. Neumaier, L. Wolsey), October 1–December 23, 2006

— 2007 —

Automorphic Forms, Geometry and Arithmetic (S. S. Kudla, M. Rapoport, J. Schwermer), February 11–February 24, 2007

Amenability (A. Erschler, V. Kaimanovich, K. Schmidt), February 26–July 31, 2007

Mathematical and Physical Aspects of Perturbative Approaches to Quantum Field Theory (R. Brunetti, K. Fredenhagen, D. Kreimer, J. Yngvason), March 1–April 30, 2007

Poisson Sigma Models, Lie Algebroids, Deformations, and Higher Analogues (H. Bursztyn, H. Grosse, T. Strobl), August 1–September 20, 2007

Applications of the Renormalization Group (G. Gentile, H. Grosse, G. Huisken, V. Mastropietro), October 15–November 23, 2007

— 2008 —

Combinatorics and Statistical Physics (M. Bousquet-Mélou, M. Drmota, C. Krattenthaler, B. Nienhuis), February 1–June 15, 2008

Metastability and Rare Events in Complex Systems (P. Bolhuis, C. Dellago, E. van den Eijnden), February 1–April 30, 2008

Hyperbolic Dynamical Systems (L.-S. Young, H. Posch, D. Szász), May 25–July 5, 2008

Operator Algebras and Conformal Field Theory (Y. Kawahigashi, R. Longo, K.-H. Rehren, J. Yngvason), August 25–December 14, 2008

— 2009 —

Representation Theory of Reductive Groups—Local and Global Aspects (G. Henniart, G. Muic and J. Schwermer), January 2–February 28, 2009

Number Theory and Physics (A. Carey, H. Grosse, D. Kreimer, S. Paycha, S. Rosenberg and N. Yui), March 2–April 18, 2009

Selected Topics in Spectral Theory (B. Helffer, T. Hoffmann-Ostenhof and A. Laptev), May 5–July 25, 2009

Large Cardinals and Descriptive Set Theory (S. Friedman, M. Goldstern, R. Jensen, A. Kechris and W. H. Woodin), June 14–27, 2009

Entanglement and Correlations in Many-Body Quantum Mechanics (B. Nachtergaele, F. Verstraete and R. Werner), August 10–October 17, 2009

The d-bar-Neumann Problem: Analysis, Geometry and Potential Theory (F. Haslinger, B. Lamel, E. Straube), October 27–December 23, 2009

— 2010 —

Quantitative Studies of Nonlinear Wave Phenomena (P. C. Aichelburg, P. Bizoń, W. Schlag), January 7–February 28, 2010

Quantum Field Theory on Curved Space-Times and Curved Target-Spaces (M. Gaberdiel, S. Hollands, V. Schomerus, J. Yngvason), March 1–April 30, 2010

Matter and Radiation (V. Bach, J. Fröhlich, J. Yngvason), May 5–July 30, 2010

Topological String Theory, Modularity and Non-Perturbative Physics (L. Katzarkov, A. Klemm, M. Kreuzer, D. Zagier), June 6–August 15, 2010

Anti-de Sitter Holography and the Quark-Gluon Plasma: Analytical and Numerical Aspects (A. Rebhan, K. Landsteiner, S. Husa), August 2–October 29, 2010

Higher Structures in Mathematics and Physics (A. Alekseev, H. Bursztyn, T. Strobl), September 1–November 11, 2010

— 2011 —

Bialgebras in Free Probability (M. Aguiar, F. Lehner, R. Speicher, D. Voiculescu), February 1–April 22, 2011

Nonlinear Waves (A. Constantin, J. Escher, D. Lannes, W. Strauss), April 4–June 30, 2011

Dynamics of General Relativity: Numerical and Analytical Approaches (L. Andersson, R. Beig, M. Heinzle, S. Husa), July 4–September 22, 2011

Combinatorics, Number Theory, and Dynamical Systems (M. Einsiedler, P. Grabner, C. Krattenthaler, T. Ziegler), October 1–November 30, 2011

— 2012 —

Automorphic Forms: Arithmetic and Geometry (J. W. Cogdell, C. Moeglin, G. Muic, J. Schwermer), January 3–February 28, 2012

K-Theory and Quantum Fields (M. Ando, A. Carey, H. Grosse, J. Mickelsson), May 21–July 27, 2012

The Interaction of Geometry and Representation Theory. Exploring New Frontiers (A. Cap, A. L. Carey, A. R. Gover, C. R. Graham, J. Slovak), September 3–14, 2012

Modern Methods of Time-Frequency Analysis II (H. G. Feichtinger, K. Gröchenig), September 10–December 15, 2012

— 2013 —

Teichmüller Theory (L. Funar, Y. Neretin, A. Papadopoulos, R. Penner), January 28–April 21, 2013

The Geometry of Topological D-Branes, Categories and Applications (S. Gukov, M. Herbst, A. Kapustin, L. Katzarkov, Y. Soibelman), April 22–July 6, 2013

Jets and Quantum Fields for LHC and Future Colliders (A. H. Hoang, I. W. Stewart), July 1–31, 2013

Forcing, Large Cardinals and Descriptive Set Theory (S. D. Friedman, M. Goldstern, A. Kechris, J. Kellner, W. H. Woodin), September 2–October 25, 2013

Heights in Diophantine Geometry, Group Theory and Additive Combinatorics (R. Tichy, J. Vaaler, M. Widmer, U. Zannier), October 21–December 20, 2013

Summer and Winter Schools

Summer School on Nonlinear Wave Equations (Y. Brenier, S. Klainerman, N. Mauser, A. Selberg), July 7–11, 2004

Summer School on Vertex Algebras and Related Topics (E. Frenkel, V. Kac, J. Schwermer), June 12–July 2, 2005

Winter School: Langlands Duality and Physics (E. Frenkel, N. Hitchin, J. Schwermer, K. Vilonen), January 9–20, 2007

Summer School on Combinatorics and Statistical Physics (M. Drmota, C. Krattenthaler, B. Nienhuis, M. Bousquet-Mélou), July 7–18, 2008

Summer School on Current Topics in Mathematical Physics (C. Hainzl, R. Seiringer, J. Yngvason), July 21–31, 2008

Winter School: Mathematics at the Turn of the 20th Century: Explorations and Beyond (D. D. Fenster, J. Schwermer), January 7–12, 2009

ESI May Seminar 2010 in Number Theory (J. Schwermer), May 2–9, 2010

Summer School on Cartan Connections, Geometry of Homogeneous Spaces, and Dynamics (A. Cap, C. Frances, K. Melnick), July 10–23, 2011

Summer School in Mathematical Physics (C. Hainzl, R. Seiringer), August 16–24, 2011

EMS-IAMP Summer School Quantum Chaos (N. Anantharaman, S. Morris Zeltditch, St. Nonnenmacher, Z. Rudnick), July 30–August 3, 2012

DATA AND STATISTICS—JUNIOR RESEARCH FELLOWS 2004–2011

Junior Research Fellows 2004–2011

From 2004 to 2011 the ESI Junior Research Fellowships Program was in place. There was a total of 153 fellowships granted out of 515 applications. The total number of months granted was 510. The Junior research fellows came from 38 different countries and stayed for a period of 3 months and 10 days on average.

Below you find a list of all Junior Research Fellows present at ESI between April 26th 2004 and February 28th 2011.

— 2004 —

1st Call for proposals: February 15, 2004
Number of applications: 40
Number of fellowships granted: 18
Number of months granted: 48 for 2004, 19 for 2005

2nd Call for proposals: May 31, 2004
Number of applications: 38
Number of fellowships granted: 9
Number of months granted: 8 for 2004, 18 for 2005, 2 for 2006

3rd Call for proposals: November 15, 2004
Number of applications: 65
Number of fellowships granted: 7
Number of months granted: 20 for 2005

Junior Research Fellows present at the ESI in 2004 (4 female, 14 male):

Wolfgang Angerer, Austria
Jessica Barrett, Great Britain
Matthias Birkner, Germany
Jeremy Clark, USA
Ionas Erb, Germany
Borislav Gajic, Serbia
Alessandro Giuliani, Italy
Marcela Hanzer, Croatia
Bianca Mladek, Austria
Ari Pakman, Argentina
Milena Radnovic, Serbia
Karl Georg Schlesinger, Germany
Jeff Selden, USA
Alexandre Stefanov, Bulgaria
Jesper Tidblom, Sweden
Christian Tutschka, Austria
Matteo Viale, Italy
Vojtěch Žadník, Czech Republic

— 2005 —

1st Call for proposals: June 15, 2005
Number of applications: 36
Number of fellowships granted: 9
Number of months granted: 5 for 2005, 25 for 2006

2nd Call for proposals: October 15, 2005
Number of applications: 34
Number of fellowships granted: 11
Number of months granted: 36 for 2006

Junior Research Fellows present at the ESI in 2005 (5 female, 17 male):

Sarah Bailey, United States
Christian Böhmer, Germany
Jessica Barrett, Great Britain
Matthias Birkner, Germany
Elena Cordero, Italy
Anton Galaev, Russia
Sebastian Guttenberg, Germany
Marcela Hanzer, Croatia
Anne-Katrin Herbig, Germany
Felipe Leitner, Germany
Thomas Neukirchner, Germany
Kasso Okoudjou, Benin
Alexander Powell, United States
Mikhail Pevzner, Russia
Nenad Teofanov, Serbia
Jan Tichavsky, Czech Republic
Alexandre Stefanov, Bulgaria
Stefan Wenger, Switzerland
Marcin Wiesniak, Poland
Michael Wohlgenannt, Austria
Vojtěch Žadník, Czech Republic
Roland Zweimüller, Austria

— 2006 —

1st Call for proposals: March 31, 2006
Number of applications: 27
Number of fellowships granted: 12
Number of months granted: 22 for 2006, 9 for 2007

2nd Call for proposals: October 31, 2006
Number of applications: 42
Number of fellowships granted: 15
Number of months granted: 41 for 2007

Junior Research Fellows present at the ESI in 2006 (7 female, 21 male):

Katie Bloor, Great Britain
Francesco D'Andrea, Italy
Spyridon Dendrinos, Greece
Martyn De Vries, Netherlands
Karla Diaz-Ordaz, Mexico
Pierluigi Falco, Italy
Anton Galaev, Russia
Victor Junwei Guo, China
Eman Hamza, Egypt
Nataliya Ivanova, Ukraine
Adam Joyce, Great Britain
Wolfgang Lechner, Austria
Richard Miles, Great Britain
Thierry Monteil, France
Ian Morris, Great Britain
Milan Mosonyi, Hungary
Tomasz Paterek, Poland
Evangelia Petrou, Greece
Michail Pevzner, Russia
Peter Pickl, Germany
Pietro Polesello, Italy
Catherine Richard, France
Hisham Sati, Lebanon
Jean Savinien, France
Emanuel Scheidegger, Switzerland
Evelina Shamarova, Russia
Mathieu Stienon, Belgium
Alexandr Usnich, Belarus

— 2007 —

1st Call for proposals: April 30, 2007
Number of applications: 37
Number of fellowships granted: 9
Number of months granted: 19 for 2007, 16 for 2008

2nd Call for proposals: November 10, 2007
Number of applications: 41
Number of fellowships granted: 15
Number of months granted: 57 for 2008

Junior Research Fellows present at the ESI in 2007 (3 female, 17 male):

Stuart Armstrong, Canada
Christoph Bergbauer, Germany
Olivier Bernardi, France
Henning Bostelmann, Germany
Pierluigi Falco, Italy
Josh Garretson, USA
Gerald Gotsbacher, Austria
Peggy Kao, Taiwan
Aleksey Kostenko, Ukraine
Gandalf Lechner, Germany
Christian Lübbe, Germany
Sean Murray, Ireland
Tomasz Paterek, Poland
Pietro Polesello, Italy
Julia Reffy, Hungary
Florian Schätz, Austria
Evelina Shamarova, Russia
Wonmin Son, Korea
Mihaly Weiner, Hungary
Michael Wohlgenannt, Austria

— 2008 —

1st Call for proposals: April 11, 2008
Number of applications: 34
Number of fellowships granted: 10
Number of months granted: 16 for 2008, 14 for 2009

2nd Call for proposals: November 14, 2008
Number of applications: 36
Number of fellowships granted: 9
Number of months granted: 34 for 2009

Junior Research Fellows present at the ESI in 2008 (7 female, 16 male):

Hendrik Adorf, Germany
Vasiliki Anagnostopoulou, Greece
Caterina Cusulin, Italy
Philipp Geiger, Austria
Neven Grbac, Croatia
Harald Grobner, Austria
Minh Ha Quang, Vietnam
Eman Hamza, Egypt
Matthieu Josuat-Verges, France
Peggy Kao, Australia
Aleksey Kostenko, Ukraine
Christian Lübbe, Germany
Mate Matolcsi, Hungary
Philippe Nadeau, France
Maryna Nesterenko, Ukraine
Radu Saghin, Romania
Maria Schimpf, Austria
Josef Silhan, Czech Republic
Rafal Suszek, Poland
Balint Vetö, Hungary
Le Anh Vinh, Vietnam
Mihaly Weiner, Hungary
Lenka Zalabova, Czech Republic

— 2009 —

1st Call for proposals: April 17, 2009
Number of applications: 44
Number of fellowships granted: 13
Number of months granted: 32 for 2009, 14 for 2010

2nd Call for proposals: October 11, 2009
Number of applications: 20
Number of fellowships granted: 7
Number of months granted: 24 for 2010

Junior Research Fellows present at the ESI in 2009 (9 female, 18 male):

Lior Alexandra Aermark, Israel
Jose Aliste, Chile
Emanuela Bianchi, Italy
Francis Brown, Great Britain
Claudio Dappiaggi, Italy
Slawomir Dinew, Poland
Zywomir Dinew, Poland
Anastasia Jivulescu, Romania
Lukasz Kosinski, Poland
Rongmin Lu, Singapore
Anca Matioc, Romania
Kostyantyn Medynets, Ukraine
Karin Melnick, USA
Wolfgang Moens, Belgium
Mathieu Molitor, France
Milan Mosonyi, Hungary
Carolina Neira, Colombia
Nicolas Raymond, France
Jean Ruppenthal, Germany
Josef Silhan, Czech Republic
Jean-Charles Sunye, France
Kirsten Vogeler, Germany
Zhituo Wang, China
Jiangyang You, China
Lenka Zalabova, Czech Republic
Lei Zhang, China
Magdalena Zych, Poland

— 2010 —

Call for proposals: February 12, 2010

Number of applications: 21
Number of fellowships granted: 9
Number of months granted: 27 for 2010, 8 for 2011

Junior Research Fellows present at the ESI in 2010 (6 female, 20 male):

Camilo Arias Abad, Colombia
Matteo Cardella, Italy
Claudio Dappiaggi, Italy
Zywomir Dinew, Poland/Bulgaria
Slawomir Dinew, Poland/Bulgaria
Alexander Fish, Israel
Richard Green, Australia
Rika Hagihara, Japan
Myrto Kallipoliti, Greece
Angelika Kroner, Austria
Helge Krüger, Germany
Wojciech Krynski, Poland
Christiane Losert, Austria
Rongmin Lu, Singapore
Kostyantyn Medynets, Ukraine
Wolfgang Moens, Belgium
Vladimir N. Salnikov, Russia
Christian Ortiz, Chile
Piotr Przytyczki, Poland
Chris Rogers, USA
Florian Schätz, Austria
Susanne Schimpf, Germany
Nora Seeliger, Germany
Marcel Vonk, The Netherlands
Matthias Westrich, Germany
Mark Williamson, Great Britain

— 2011 —

Junior Research Fellows present at the ESI in 2011 (2 female, 2 male):

Angelika Kroner, Austria
Wojciech Krynski, Poland
Nora Seeliger, Germany
Mark Williamson, Great Britain

Senior Research Fellows 2002–2012

From 2002 to 2012, there have been 48 lectures held by Senior Research Fellows that were at the ESI for a longer research period.

— 2002/03 —

fall term 2002:

Arkadi Onishchik (Yaroslavl State U, Russia): Real Representation Theory of Lie Algebras and Lie Groups

Anatoly Vershik (Steklov Institute, St. Petersburg, Russia): Measure Theoretic Constructions and Their Applications in Ergodic Theory, Asymptotics, Combinatorics, and Geometry

spring term 2003:

Michael Lacey (Georgia Institute of Technology, Atlanta, USA): Recent Trends in Fourier Analysis

Peter van Nieuwenhuizen (C. N. Yang Institute for Theoretical Physics, Stony Brook U, New York, USA): *N=1* and *N=2* Supersymmetry and Supergravity

— 2003/04 —

fall term 2003:

Vladimir Mazya (Linköping U, Sweden): Sobolev Spaces With Applications to PDE

Jürgen Rohlfs (U Eichstätt, Germany): Algebraic Groups Over Number Fields and Related Geometric Questions

Peter van Nieuwenhuizen (C. N. Yang Institute for Theoretical Physics, Stony Brook U, New York, USA): *N=1* and *N=2* Supersymmetry and Supergravity, continuation of the spring term lecture course

spring term 2004:

Werner Ballmann (U Bonn, Germany): Über die Geometrie der Gebäude

Jürgen Fuchs (Karlstad U, Sweden): Conformal Field Theory

— 2004/05 —

fall term 2004:

Manfred Salmhofer (U Leipzig, Germany): Renormalization Theory—Analysis and Applications

Anton Wakolbinger (U Frankfurt, Germany): Stochastische Prozesse aus der Populationsgenetik

Vlatko Vedral (Imperial College, London, UK): Foundations of Quantum Information

spring term 2005:

Werner Ballmann (U Bonn, Germany): Kählergeometrie

Jan Derezinski (Warsaw U, Poland): Operator Algebras and Their Applications in Physics

Anatoly Vershik (Steklov Institute, St. Petersburg, Russia): Representation Theory of Symmetric Groups, Graphs, Universality

Emil Straube (Texas A&M U, College Station, USA): The L^2-Sobolev Theory of the d-bar-Neumann Problem

— 2005/06 —

fall term 2005:

Emil Straube (Texas A&M U, College Station, USA): The L^2-Sobolev Theory of the d-bar-Neumann Problem, continuation of the spring term lecture course

Bernard Helffer (U Paris Sud-Orsay, France): Introduction to the Spectral Theory for Schrödinger Operators With Magnetic Fields and Application

Boban Velickovic (U Paris Diderot, France): Introduction to Descriptive Set Theory

spring term 2006:

David Masser (U Basel, Switzerland): Heights in Diophantine Geometry

Mathai Varghese (U Adelaide, Australia): *K*-Theory Applied to Physics

— 2006/07 —

fall term 2006:

Ioan Badulescu (U Poitiers, France): Representation Theory of the General Linear Group Over a Division Algebra

Thomas Mohaupt (U Liverpool, UK): Black Holes, Supersymmetry and Strings

Miroslav Englis (Academy of Sciences, Prague, Czech Republic): Analysis on Complex Symmetric Spaces

spring term 2007:

Miroslav Englis (Academy of Sciences, Prague, Czech Republic): Analysis on Complex Symmetric Spaces, continuation of the fall term 2006 lecture course

Vadim Kaimanovich (International U Bremen, Germany): Boundaries of Groups: Geometric and Probabilistic Aspects

Thomas Mohaupt (U Liverpool, UK): Black Holes, Supersymmetry and Strings, continuation of the fall term 2006 lecture course

— 2007/08 —

fall term 2007:

Christos N. Likos (U Düsseldorf, Germany): Introduction to Theoretical Soft Matter Physics

Radoslav Rashkov (Sofia U, Bulgaria): Dualities Between Gauge Theories and Strings

John Barrett (U Nottingham, UK) and *Richard Szabo* (U of Edinburgh, UK): Two short courses on Theoretical Physics

— 2008/09 —

fall term 2008:

Nigel Higson (Penn State U, USA): Index Theory, Groupoids and Noncommutative Geometry

Goran Muic (U Zagreb, Croatia): Selected Topics in the Theory of Automorphic Forms for Reductive Groups

Feng Xu (UC Riverside, USA): Operator Algebras and Conformal Field Theory

spring term 2009:

Michael Loss (Georgia Institute of Technology, Atlanta, USA): Spectral Inequalities and Their Applications to Variational Problems and Evolution Equations

Raimar Wulkenhaar (U Münster, Germany): Spektrale Tripel in nichtkommutativer Geometrie und Quantenfeldtheorie

— 2009/10 —

fall term 2009:

Peter West (King's College, London, UK): Supergravity Theories

Jeff McNeal (Ohio State U, Columbus, USA): L^2-Methods in Complex Analysis

spring term 2010:

Neven Grbac (U Rijeka, Croatia): Eisenstein Series

Stefan Hollands (Cardiff U, UK): Quantum Field Theory on Curved Spacetimes

Peter West (King's College, London, UK): *E*-Theory

— 2010/11 —

fall term 2010:

Tykal Venkataramana (Tata Institute, Mumbai, India): Representations Contributing to Cohomology of Arithmetic Groups

spring term 2011:

Bruno Nachtergaele (UC Davis, USA): Quantum Spin Systems. An Introduction to the General Theory and Discussion of Recent Developments

Michael Baake (U Bielefeld, Germany): Spektraltheorie dynamischer Systeme und aperiodische Ordnung

Kenneth Dykema (Texas A&M U, College Station, USA) and *Roland Speicher* (U Saarland, Germany): Free Probability Theory

Peter West (King's College, London, UK): Symmetries of Strings and Branes, continuation of the spring term lecture course

— 2011/12 —

fall term 2011:

James W. Cogdell (Ohio State U, Columbus, USA): *L*-functions and Functoriality

spring term 2012:

Detlev Buchholz (U Göttingen, Germany): Fundamentals and Highlights of Algebraic Quantum Fields

Eduard Feireisl (Academy of Sciences, Prague, Czech Republic): Mathematics and Complete Fluid Systems

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