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On the relativistic field theory model of the deuteron. II

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Abstract

The relativistic field theory model of the deuteron suggested in [1] is revised and adjusted to the calculation of processes of low–energy interactions like the radiative neutron–proton capture n + p → D + γ, the low–energy proton–proton scattering p + p → D + e^+ + ν_e and so on.

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Introduction

In a recent publication [1] we have suggested a relativistic field theory model of the deuteron based on the Nambu–Jona–Lasinio (NJL) model [2].

The basic idea of the model is in the assumption that the physical deuteron state should be produced due to integration over low–energy proton–neutron fluctuations at energies restricted by the scale $\Lambda_{\rm D}$. Following the NJL model prescription we have suggested the one–nucleon loop approximation for the integration over proton–neutron fluctuations. In this case the scale $\Lambda_{\rm D}$ has the meaning of the cut–off. We have defined $\Lambda_{\rm D}$ in terms of the effective radius of the deuteron $r_{\rm D}$, i.e., $\Lambda_{\rm D} = 1/r_{\rm D}$. For the estimate of $\Lambda_{\rm D}$ we have applied the non–relativistic formula: $r_{\rm D} = (\varepsilon_{\rm D}\, M_{\rm N})^{-1/2} = 4.319\,{\rm fm}$ [3,4], where $\varepsilon_{\rm D} = 2.225\,{\rm MeV}$ is the binding energy of the physical deuteron [4] and $M_{\rm N} = 938\,{\rm MeV}$ is the mass of the nucleon. We used equal masses for the proton and neutron, i.e., $M_{\rm p} = M_{\rm n} = M_{\rm N} = 938\,{\rm MeV}$. This corresponds to the chiral limit when masses of current u– and d–quarks vanish, i.e., $m_{0\rm u} = m_{0\rm d} = 0$. Our estimate of $\Lambda_{\rm D}$ gives: $\Lambda_{\rm D} = 1/r_{\rm D} = 46\,{\rm MeV}$.

The interactions of the deuteron with proton and neutron were described in terms of two coupling constants $g_{\rm V}$ and $g_{\rm T}$, defining the interactions by vector and tensor nucleon currents, respectively. In the one–nucleon loop approximation we calculated the anomalous magnetic moment of the deuteron $\kappa_{\rm D}$ in units of the nucleon magneton $\mu_{\rm N} = e/2\,M_{\rm N}$ where e is the proton charge, and the electric quadrupole moment $Q_{\rm D}$. Imposing the constraint $\kappa_{\rm D} = 0$, being valid in the lowest approximation, we found the correlation between coupling constants $g_{\rm V}$ and $g_{\rm T}$, i.e., $g_{\rm T}/g_{\rm V} = -\sqrt{3/8}$. Then the coupling constant $g_{\rm V}$ has been fixed in terms of the electric quadrupole moment $Q_{\rm D}$. Eventually we calculated in the one–nucleon loop approximation the binding energy of the physical deuteron in terms of $g_{\rm V}$ and $\Lambda_{\rm D}$. The theoretical result was obtained in good agreement with experimental value: $(\varepsilon_{\rm D})_{\rm exp} = 2.225\,{\rm MeV}$ [4].

The physical nature of the coupling constants $g_{\rm V}$ and $g_{\rm T}$ is connected with one–meson exchange. Of course, π–meson exchange should give the main contribution. This assumption has been confirmed by the magnitude of $g_{\rm V}$, i.e., $g_{\rm V} \simeq g_{\pi{\rm NN}}$ where $g_{\pi{\rm NN}}$ is the coupling constant of the πNN–interactions.

The application of the model to the calculation of processes of low–energy interactions of the deuteron encounters the problem of the unambiguous evaluation of one–nucleon loop diagrams that should describe the interactions of the deuteron other particles in the suggested model.

Indeed, since these are fermion loops, most of them, contributing to processes like radiative neutron–proton capture ${\rm n} + {\rm p} \to {\rm D} + \gamma$, low–energy proton–proton scattering ${\rm p} + {\rm p} \to {\rm D} + e^+ + \nu_e$ and so on, depend strongly on the shift of virtual momenta of nucleons in the loop. This introduces substantial ambiguities interfering with the direct application of the model.

In this paper we revise our model [1], especially to adjust it to the calculation of low–energy processes like ${\rm n} + {\rm p} \to {\rm D} + \gamma$, ${\rm p} + {\rm p} \to {\rm D} + e^+ + \nu_e$ and so on. We impose the constraints on the evaluation of one–nucleon loops, caused by the requirement of electromagnetic gauge invariance of contributions for individual loops. This distinguishes our effective model from a field theory as QED, where the electromagnetic gauge invariance should be required for the complete set of diagrams in fixed order of perturbation theory.

The requirement of electromagnetic gauge invariance applied to the individual nucleon loops allows one to fix ambiguities and use one–nucleon loops as well–defined quantum field theory objects. Of course, this approach should be restricted to the one–nucleon loop approximation only, even for the calculation of the matrix elements of the processes n + p → D + γ, p + p → D + e^+ + ν_e and so on.

The present paper is organized as follows. In Sect. 1 we adduce the starting assumptions that we have put in the foundation of the model, and phenomenological parameters in terms the parameters characterizing the physical deuteron. In Sect. 2 and Sect. 3 we give the detailed derivation of the effective Corben–Schwinger and Aronson Lagrangians, respectively, describing electromagnetic interactions of the deuteron in terms the magnetic and electric quadrupole moments. In Sect. 4 and Sect. 5 we apply the relativistic field theory model of the deuteron to the evaluation of the amplitudes and cross sections of the radiative neutron–proton capture n + p → D + γ and the low–energy proton–proton scattering p + p → D + e^+ + ν_e, respectively. In the final section we discuss the obtained results.

1. Deuteron structure in the one–nucleon loop approximation

In Ref. [1] we have suggested a relativistic field theory model for the deuteron as a bound state of proton and neutron. The main idea, that has been put into the foundation of the method, has been adopted from the Nambu–Jona–Lasinio (NJL) model [2]. Unfortunately, we cannot start directly from a local four–nucleon interaction, since that would lead to a strongly bound proton–neutron state, whereas a physical deuteron is weakly bound. Therefore, we have borrowed from the NJL model only the NJL–prescription concerning the admission of the applicability of the one–loop approximation. This is used for the computation of observed parameters for physical states in leading order in the long–wavelength expansion [5-8].

In our description of the physical deuteron we use the following scheme: We start from the Lagrangian of an unphysical deuteron field $D^{(0)}_{\mu}(x)$, considered as a bound proton–neutron state at zero binding energy and with a mass equal to the sum of the proton and neutron masses. Then in the one–nucleon loop approximation and leading order in the long–wavelength expansion we obtain an effective Lagrangian of a physical deuteron field describing a physical deuteron with observable binding energy $(\varepsilon_{\rm D})_{\rm exp} = 2.225\,{\rm MeV}$ [4], anomalous magnetic moment $(\kappa_{\rm D})_{\rm exp} = -0.023$, determined as $\kappa_{\rm D} = \mu_{\rm D} - \mu_{\rm p} - \mu_{\rm n}$, where $\mu_{\rm D} = 0.857$, $\mu_{\rm p} = 1 + \kappa_{\rm p} = 2.793$ and $\mu_{\rm n} = \kappa_{\rm n} = -1.913$ are the magnetic moments of the physical deuteron, proton and neutron, respectively, and the electric quadrupole moment $(Q_{\rm D})_{\rm exp} = 0.286\,{\rm fm}^2$ [4]. The magnetic moment of the deuteron is measured in nuclear magneton $\mu_{\rm N} = e/2\,M_{\rm N}$ where e and $M_{\rm N}$ are the electric charge of the proton and the mass of the proton and neutron, then $\kappa_{\rm p}$ and $\kappa_{\rm n}$ are the anomalous magnetic moments of the proton and neutron. In the case of the neutron the total magnetic moment coincides with the anomalous one. Below, to simplify matter we neglect the proton–neutron mass difference and use equal masses of the proton and neutron, i.e., $M_{\rm p} = M_{\rm n} = M_{\rm N} = 938\,{\rm MeV}$. This should correspond to the chiral limit approximation with zero masses of current u– and d–quarks, i.e., $m_{0u} = m_{0d} = 0$.

The Lagrangian of the unphysical deuteron field $D^{(0)}_{\mu}(x)$, which interacts strongly with the proton $p(x)$ and neutron $n(x)$ fields, reads

$$\begin{aligned}\mathcal{L}_{\rm st}(x) &= -\frac{1}{2}D^{\dagger(0)}_{\mu\nu}(x)D^{(0)\mu\nu}(x) + M^2_0 D^{\dagger(0)}_{\mu}(x)D^{(0)\mu}(x) - \\ &-ig_{\rm V}\,[\bar{p}(x)\gamma^{\mu}n^c(x) - \bar{n}(x)\gamma^{\mu}p^c(x)]\,D^{(0)}_{\mu}(x) - \\ &-ig_{\rm V}\,[\bar{p^c}(x)\gamma^{\mu}n(x) - \bar{n^c}(x)\gamma^{\mu}p(x)]\,D^{\dagger(0)}_{\mu}(x) + \\ &+\frac{g_{\rm T}}{M_0}\,[\bar{p}(x)\sigma^{\mu\nu}n^c(x) - \bar{n}(x)\sigma^{\mu\nu}p^c(x)]\,D^{(0)}_{\mu\nu}(x) + \\ &+\frac{g_{\rm T}}{M_0}\,[\bar{p^c}(x)\sigma^{\mu\nu}n(x) - \bar{n^c}(x)\sigma^{\mu\nu}p(x)]\,D^{\dagger(0)}_{\mu\nu}(x) + \\ &+\bar{p}(x)(i\gamma^{\mu}\partial_{\mu} - M_{\rm N})p(x) + \bar{n}(x)(i\gamma^{\mu}\partial_{\mu} - M_{\rm N})n(x).\end{aligned} \tag{1}$$

Here $D^{(0)}_{\mu\nu}(x) = \partial_{\mu}D^{(0)}_{\nu}(x) - \partial_{\nu}D^{(0)}_{\mu}(x)$, $M_0 = 2M_{\rm N}$ is the mass of the unphysical deuteron, $\psi^c(x) = C\,\bar{\psi}^T(x)$ and $\bar{\psi}^c(x) = \psi^T(x)\,C$, and $\sigma^{\mu\nu} = \frac{1}{2}\,[\,\gamma^{\mu},\,\gamma^{\nu}]$. The operator C denotes charge conjugation, T is a transposition, $g_{\rm V}$ and $g_{\rm T}$ are the phenomenological constants that will be fixed below. We assume that the coupling constants $g_{\rm V}$ and $g_{\rm T}$ are caused by one–meson exchanges such as the π–meson exchange and should give the main contribution.

To obtain the anomalous magnetic and electric quadrupole moments we have to include the interaction with the electromagnetic field. Having performed this inclusion by a minimal manner we obtain the Lagrangian

$$\mathcal{L}_{\rm tot}\,(x) = \mathcal{L}_{\rm st}\,(x) + \mathcal{L}_{\rm el}\,(x), \tag{2}$$

where

$$\begin{aligned}\mathcal{L}_{\rm el}(x) &= -ieD^{\dagger(0)}_{\mu\nu}(x)A^{\mu}(x)D^{(0)\nu}(x) + ieD^{(0)}_{\mu\nu}(x)A^{\mu}(x)D^{\dagger(0)\nu}(x) + \\ &+ie\frac{2g_{\rm T}}{M_0}\,[\bar{p}(x)\sigma^{\mu\nu}n^c(x) - \bar{n}(x)\sigma^{\mu\nu}p^c(x)]\,A_{\mu}(x)D^{(0)}_{\nu}(x) - \\ &-ie\frac{2g_{\rm T}}{M_0}\,[\bar{p^c}(x)\sigma^{\mu\nu}n(x) - \bar{n^c}(x)\sigma^{\mu\nu}p(x)]\,A_{\mu}(x)D^{\dagger(0)}_{\nu}(x) + \\ &-i\,e\,\frac{\lambda}{M^2_0}\,D^{\dagger(0)}_{\mu\nu}(x)\,D^{(0)\,\nu\,\alpha}(x)\,F_{\alpha}{}^{\mu}(x)\; + \\ &-e\,\bar{p}(x)\gamma^{\mu}p(x)A_{\mu}(x)\; - \\ &-ie\,\frac{\kappa_{\rm p}}{2M_{\rm N}}\,\bar{p}(x)\,\sigma^{\mu\nu}p(x)\,F_{\mu\nu}(x)\;-\;ie\,\frac{\kappa_{\rm n}}{2M_{\rm N}}\,\bar{n}(x)\,\sigma^{\mu\nu}n(x)\,F_{\mu\nu}(x)\;+\;O(e^2)\,.\end{aligned} \tag{3}$$

Here $F_{\alpha}{}^{\mu}(x) = \partial_{\alpha}\,A^{\mu}(x) - \partial^{\mu}\,A_{\alpha}(x)$ and $A_{\mu}(x)$ are the electromagnetic field strength tensor and the electromagnetic potential, respectively. Also we have added the Aronson interaction [9] describing the anomalous magnetic and electric quadrupole moments of the unphysical deuteron: $\Delta g_{{\rm D}^{(0)}} = \lambda$ and $Q_{\,{\rm D}^{(0)}} = 2\,\lambda/M^{\,2}_{\,0}$ [7].

We have taken into account the anomalous magnetic moments of the proton and neutron that play an important role for the radiative neutron–proton capture n + p $\to$ D + γ. The inclusion of λ and the anomalous magnetic moments of the proton and neutron differs in the present model from that given in [1].

To obtain the effective Lagrangian of the physical deuteron field we have to calculate one–nucleon loop contributions according to the NJL model prescription [2,5-8]. The nucleon diagrams describing the contributions to the kinetic term of the deuteron field and leading to the appearance of the non–zero value of the binding energy are depicted in Fig. 1. By calculating these diagrams in leading order in long–wavelength expansion we get

$$\begin{aligned}\delta\mathcal{L}_{\rm eff}(x) &= -\frac{1}{2}\left[\frac{g^2_{\rm V} - 6g_{\rm V}g_{\rm T} + 3g^2_{\rm T}}{3\pi^2}J_2(M_{\rm N})\right]\,D^{\dagger(0)}_{\mu\nu}(x)\,D^{(0)\mu\nu}(x)\,- \\ &- \frac{2}{3}\frac{g^2_{\rm V}}{\pi^2}\left[J_1(M_{\rm N})\,+\,M^2_{\rm N}\,J_2(M_{\rm N})\right]D^{\dagger(0)}_{\mu}(x)\,D^{(0)\mu}(x),\end{aligned} \tag{4}$$

where $J_1(M_{\rm N})$ and $J_2(M_{\rm N})$ are the following divergent integrals

$$\begin{aligned}J_1(M_{\rm N}) &= \int\frac{d^4k}{\pi^2 i}\frac{1}{M^2_{\rm N} - k^2} = 4\int^{\Lambda}_0\frac{d|\vec{k}|\vec{k}^{\,2}}{(M^2_{\rm N}\,+\,\vec{k}^{\,2})^{1/2}}, \\ J_2(M_{\rm N}) &= \int\frac{d^4k}{\pi^2 i}\frac{1}{(M^2_{\rm N} - k^2)^2} = 2\int^{\Lambda}_0\frac{d|\vec{k}|\vec{k}^{\,2}}{(M^2_{\rm N}\,+\,\vec{k}^{\,2})^{3/2}}.\end{aligned} \tag{5}$$

The ultra–violet cut–off Λ restricts the 3–momenta of fluctuations of virtual nucleons taking part in the formation of the physical deuteron field. One should expect that Λ is connected with the region of localization of a wave packet procreated by fluctuations of virtual nucleons, i.e., $\Delta r\,\cdot\,\Lambda \approx 1$. We shall specify the value of Λ below.

Now we discuss the problem of the applicability of the long–wavelength expansion for the calculation of the one–nucleon loop diagrams in Fig. 1. It is well–known that a two–body S–wave bound state being denoted as D with a reduced mass $M_{\rm N}/2$ and a binding energy $\varepsilon_{\rm D}$ is localized in the region restricted by $r_{\rm D} = 1/\sqrt{\varepsilon_{\rm D}M_{\rm N}}$ [3]. This quantity can be considered as the effective radius of the bound state [4]. In the case of the physical deuteron we have $r_{\rm D} = 4.319\,{\rm fm}$ [4]. This value exceeds three times the effective radius of nuclear forces $r_{\rm N} = 1/M_{\pi} = 1.462\,{\rm fm}$, where $M_{\pi} = 134.976\,{\rm MeV}$ is the pion mass [10]. Therefore the physical deuteron looks as a rather "extended" bound state. It should be obvious that such an "extended" bound state can be formed at the expense of the main contributions of long–wavelength fluctuations of the bound proton and neutron. It implies that we can expect a cut–off Λ satisfying the inequality $\Lambda \ll M_{\rm N}$. It should be obvious that Λ has to be identified with $\Lambda_{\rm D} = 1/r_{\rm D} = 45.688$ MeV, i.e., $\Lambda = \Lambda_{\rm D} = 45.688$ MeV.

The infinitesimality of the derivatives $\partial_{\nu}D^{(0)}_{\mu}(x)$ necessary for the validity of the long–wavelength expansion can be justified as follows. We assume that the deuteron field $D^{(0)}_{\mu}(x)$ is a bound state of a proton and neutron at zero binding energy. As has been mentioned above, this means that the field $D^{(0)}_{\mu}(x)$ is localized in the region whose upper boundary $r_{{\rm D}^{(0)}}$ goes to infinity. Obviously it results in a smooth variation of the field $D^{(0)}_{\mu}(x)$ at the scale of the effective radius of the physical deuteron. Thus we have adduced some arguments on behalf of the validity of the long–wavelength expansion, which has been applied for the calculation of the effective Lagrangian (4).

Now we proceed to the computation of the binding energy of the physical deuteron.

First let us introduce the field of the physical deuteron

$$D_\mu(x) = Z_{\rm D}^{1/2} D_\mu^{(0)}(x), \tag{6}$$

where

$$Z_{\rm D} = 1 + \frac{g_{\rm V}^2 - 6 g_{\rm V} g_{\rm T} + 3 g_{\rm T}^2}{3\,\pi^2} J_2(M_{\rm N}) \tag{7}$$

is the wave–function normalization constant. After the renormalization (7) we get the effective Lagrangian of the physical deuteron field $D_\mu(x)$

$${\cal L}_{\rm eff}^{(0)}(x) = -\frac{1}{2} D^{\dagger}_{\mu\nu}(x) D^{\mu\nu}(x) + M_{\rm D}^2 D^{\dagger}_{\mu}(x) D^{\mu}(x), \tag{8}$$

where $M_{\rm D} = M_0 - \varepsilon_{\rm D}$ is the mass of the physical deuteron and $\varepsilon_{\rm D}$ is the binding energy. In the one–nucleon loop approximation and in leading order of the long–wavelength expansion the binding energy $\varepsilon_{\rm D}$ is determined by the expression

$$\varepsilon_{\rm D} = \frac{g_{\rm V}^2}{6\,\pi^2}\,\frac{1}{M_{\rm N}}\,J_1(M_{\rm N}) + \frac{g_{\rm V}^2 - 4 g_{\rm V} g_{\rm T} + 2 g_{\rm T}^2}{2\,\pi^2}\,M_{\rm N}\,J_2(M_{\rm N}). \tag{9}$$

For the derivation of the formula (9) we have used the inequality

$$\frac{g_{\rm V}^2 - 6 g_{\rm V} g_{\rm T} + 3 g_{\rm T}^2}{3\,\pi^2} J_2(M_{\rm N}) \ll 1 \tag{10}$$

being consistent with the inequality $\Lambda \ll M_{\rm N}$ discussed above. For $\Lambda_{\rm D} \ll M_{\rm N}$ the formula (9) reads

$$\varepsilon_{\rm D} \;=\; \Lambda_{\rm D}^3\,\frac{5\,g_{\rm V}^2}{9\,\pi^2}\,\frac{1}{M_{\rm N}^2}\Bigg[1 \;-\; \frac{12}{5}\left(\frac{g_{\rm T}}{g_{\rm V}}\right) \;+\; \frac{6}{5}\left(\frac{g_{\rm T}}{g_{\rm V}}\right)^2\Bigg]. \tag{11}$$

It is seen that the binding energy of the physical deuteron is expressed in terms of three phenomenological parameters of our model: $\Lambda_{\rm D}$, $g_{\rm V}$ and $g_{\rm T}$. There we have to fix the coupling constants $g_{\rm V}$ and $g_{\rm T}$ to obtain the theoretical value of the binding energy.

For this aim we should proceed to the calculation of the anomalous magnetic $\kappa_{\rm D}$ and electric quadrupole $Q_{\rm D}$ moments. We admit that these quantities appear in the one–nucleon loop approximation and only at the expense of interactions taken into account in the Lagrangian (2). The complete set of one–nucleon loop diagrams is depicted in Fig. 2. The non–trivial contributions come from the diagrams in Figs. 2c and 2d. The detailed evaluation of the diagrams depicted in Figs. 2c and 2d is given in Sect. 2 and Sect. 3, respectively. Here we only adduce the results.

The effective Lagrangian, defined by the one–nucleon loop diagrams in in Fig. 2 and describing the electromagnetic interactions of the physical deuteron field through the anomalous magnetic and electric quadrupole moments, is given by

$$\begin{aligned}{\cal L}_{\rm eff}^{\rm el}(x) &= ie\frac{g_{\rm V}^2}{2\pi^2}\,D^{\dagger}_{\mu}(x) D_{\nu}(x) F^{\mu\nu}(x) + \\ &+ ie\Bigg\{\frac{2\,g_{\rm T}^2}{3\pi^2}\Bigg[1 + \frac{9}{4}(\kappa_{\rm p} - \kappa_{\rm n})\Bigg(1 - \frac{8}{9}\frac{g_{\rm V}}{g_{\rm T}}\Bigg)\Bigg] - \lambda\Bigg\}\frac{1}{M_{\rm D}^2}\,D^{\dagger}_{\mu\nu}(x) D^{\nu\alpha}(x) F_{\alpha}{}^{\mu}(x).\end{aligned} \tag{12}$$

The Lagrangian (12) contains only physical deuteron fields. The first term in $\mathcal{L}^{\rm el}_{\rm eff}(x)$ is the Corben–Schwinger interaction [11], while the second term represents the interaction that has first been introduced by Aronson [9]. These interactions describe the anomalous and quadrupole moments of the charged vector field. It should be emphasized that we have neglected the divergent contributions that are small in comparison with the convergent ones due to the restriction (10).

The anomalous magnetic moment $\kappa_{\rm D}$ measured in the nuclear magneton $\mu_{\rm N} = e/2\,M_{\rm N}$ and the electric quadrupole moment $Q_{\rm D}$ are given by [9,11]

$$\kappa_{\rm D} \;=\; -\,\frac{g^2_{\rm V}}{4\,\pi^{\,2}} \;-\; \frac{1}{2}\left\{\frac{2\,g^2_{\rm T}}{3\pi^2}\left[1 + \frac{9}{4}(\kappa_{\rm p} - \kappa_{\rm n})\left(1 - \frac{8}{9}\frac{g_{\rm V}}{g_{\rm T}}\right)\right] \;-\; \lambda\right\}, \tag{13}$$

$$Q_{\rm D} \;=\; \left\{\frac{g^2_{\rm V}}{\pi^{\,2}} \;-\; 2\left\{\frac{2\,g^2_{\rm T}}{3\pi^2}\left[1 + \frac{9}{4}(\kappa_{\rm p} - \kappa_{\rm n})\left(1 - \frac{8}{9}\frac{g_{\rm V}}{g_{\rm T}}\right)\right] \;-\; \lambda\right\}\right\}\frac{1}{M^2_{\rm D}}\,. \tag{14}$$

The experimental value of the anomalous magnetic moment of the deuteron $(\kappa_{\rm D})_{\rm exp} = -0.023$ is small compared with the magnetic moment of the deuteron $\mu_{\rm D} \;=\; 0.857$. Therefore, to express the electric quadrupole moment $Q_{\rm D}$ in terms of the coupling constant $g_{\rm V}$ we can set

$$\frac{g^2_{\rm V}}{2\,\pi^{\,2}} \;=\; \lambda \;-\; \frac{2\,g^2_{\rm T}}{3\pi^2}\left[1 + \frac{9}{4}(\kappa_{\rm p} - \kappa_{\rm n})\left(1 - \frac{8}{9}\frac{g_{\rm V}}{g_{\rm T}}\right)\right]. \tag{15}$$

As a result the electric quadrupole moment $Q_{\rm D}$ takes the form

$$Q_{\rm D} = \frac{2g^2_{\rm V}}{\pi^2}\frac{1}{M^2_{\rm D}}\,. \tag{16}$$

By using the experimental value $(Q_{\rm D})_{\rm exp} = 0.286\,{\rm fm}^2$ and $M_{\rm D} \simeq 1876\,{\rm MeV}$ we can estimate the value of $g_{\rm V}$. This gives one: $g_{\rm V} = \pm 11.3$. Without loss of generality we can use the positive sign, i.e.,

$$g_{\rm V} = 11.3. \tag{17}$$

The coupling constant $g_{\rm V}$ satisfies the relation $g_{\rm V} \;\simeq\; g_{\pi{\rm NN}}$ where $g_{\pi{\rm NN}} \;=\; 13.4 \,\pm\, 0.1$ [4] is the coupling constant of the πNN–interactions. Thus the magnitude of $g_{\rm V}$ corroborates our assumption that the phenomenological interactions of the deuteron and the proton and neutron, given in the Lagrangian (1), are caused by the one–meson exchange, and π–meson exchange gives the main contribution. This admission can be justified by comparing the effective radii of one–meson exchanges. The radii of pion, $\sigma\,(660)$–meson [4,8] and $\rho\,(770)$–meson exchanges are defined in terms of their masses, i.e., $r_{\pi} \;=\; 1/M_{\pi} \;=\; 1.462\,{\rm fm}$ at $M_{\pi} \;=\; 134.976\,{\rm MeV}$ [10], $r_{\sigma} \;=\; 1/M_{\sigma} \;=\; 0.30\,{\rm fm}$ at $M_{\sigma} \;=\; 660$ MeV [4,8] and $r_{\rho} \;=\; 1/M_{\rho} \;=\; 0.27$ fm at $M_{\rho} \;=\; 770\,{\rm MeV}$, respectively. The radii of the $\sigma\,(660)$ and $\rho\,(770)$–meson exchanges are much smaller than the radius of the π–meson exchange. Therefore, these interactions of the proton and neutron with $\sigma\,(660)$ and $\rho\,(770)$ mesons should contribute perturbatively to the deuteron problem.

We have put $\kappa_{\rm D} \;=\; 0$ as the experimental value $(\kappa_{\rm D})_{\rm exp} \;=\; -\,0.023$ is small enough in comparison with the magnetic moment of the deuteron $\mu_{\rm D} \;=\; 0.857$. We assume that the non–zero value of $\kappa_{\rm D}$ can be obtained perturbatively by taking into account non–zero values of current quark masses, for example, within Chiral perturbation theory at the quark level $(\rm CHPT)_q$ [8], based on the extended Nambu–Jona–Lasinio model with linear realization of chiral $U(3)\times U(3)$ symmetry, and interactions of the proton and neutron with $\sigma\,(660)$ and $\rho\,(770)$–mesons within the one–meson exchange approximation.

From the formulae (11) and (16) we can express the binding energy $\varepsilon_{\rm D}$ in terms of the electric quadrupole moment $Q_{\rm D}$ and the ratio $g_{\rm V}/g_{\rm T}$

$$\varepsilon_{\rm D} = \frac{10}{9}\,Q_{\rm D}\,\Lambda^3_{\rm D}\Bigg[1\,-\,\frac{12}{5}\left(\frac{g_{\rm T}}{g_{\rm V}}\right)\,+\,\frac{6}{5}\left(\frac{g_{\rm T}}{g_{\rm V}}\right)^2\Bigg]. \tag{18}$$

This formula agrees with the statement that the main contribution to the binding energy of the deuteron comes from the tensor forces producing the non–zero value of the quadrupole moment $Q_{\rm D}$ [3]. For the computation of the magnitude of the binding energy we have to fix the ratio $g_{\rm T}/g_{\rm V}$ that is still free. Putting $g_{\rm T}/g_{\rm V}\;=\;-\sqrt{3/8}$ [1] we obtain the best fit of the binding energy

$$\varepsilon_{\rm D}\;=\;2.273\,{\rm MeV}\,. \tag{19}$$

The accuracy of the fit makes up 2%. Thus, we have fixed all of phenomenological parameters and described all quantities, characterizing the physical deuteron.

The total effective Lagrangian of the physical deuteron describing strong and electromagnetic interactions of the deuteron reads

$$\begin{aligned}
{\cal L}_{\rm tot}(x) \;\;=\;\; & -\frac{1}{2}D^{\dagger}_{\mu\nu}(x)D^{\mu\nu}(x) + M^2_D D^{\dagger}_{\mu}(x)D^{\mu}(x) - ieD^{\dagger}_{\mu\nu}(x)A^{\mu}(x)D^{\nu}(x) + \\
& + ieD_{\mu\nu}(x)A^{\mu}(x)D^{\dagger\nu}(x) + ie\left[\frac{g^2_{\rm V}}{2\pi^2}\right]D^{\dagger}_{\mu}(x)D_{\nu}(x)F^{\mu\nu}(x) + \\
& - ie\left[\frac{g^2_{\rm V}}{2\pi^2}\right]\frac{1}{M^2_{\rm D}}\,D^{\dagger}_{\mu\nu}(x)D^{\nu\alpha}(x)F_{\alpha}{}^{\mu}(x) - \\
& - ig_{\rm V}\Big[\bar{p}(x)\gamma^{\mu}n^c(x) - \bar{n}(x)\gamma^{\mu}p^c(x)\Big]D_{\mu}(x) - \\
& - ig_{\rm V}\Big[\bar{p^c}(x)\gamma^{\mu}n(x) - \bar{n^c}(x)\gamma^{\mu}p(x)\Big]D^{\dagger}_{\mu}(x) + \\
& + \frac{g_{\rm T}}{M_{\rm D}}\Big[\bar{p}(x)\sigma^{\mu\nu}n^c(x) - \bar{n}(x)\sigma^{\mu\nu}p^c(x)\Big]D_{\mu\nu}(x) + \\
& + \frac{g_{\rm T}}{M_{\rm D}}\Big[\bar{p^c}(x)\sigma^{\mu\nu}n(x) - \bar{n^c}(x)\sigma^{\mu\nu}p(x)\Big]D^{\dagger}_{\mu\nu}(x) + \\
& + ie\frac{2g_{\rm T}}{M_{\rm D}}\Big[\bar{p}(x)\sigma^{\mu\nu}n^c(x) - \bar{n}(x)\sigma^{\mu\nu}p^c(x)\Big]A_{\mu}(x)D_{\nu}(x) - \\
& - ie\frac{2g_{\rm T}}{M_{\rm D}}\Big[\bar{p^c}(x)\sigma^{\mu\nu}n(x) - \bar{n^c}(x)\sigma^{\mu\nu}p(x)\Big]A_{\mu}(x)D^{\dagger}_{\nu}(x) + O(e^2) + \ldots.
\end{aligned} \tag{20}$$

The ellipses stand for interactions of the proton and neutron with other fields such as photon, pions, etc.

We have to underline that the suggested field theory model of the deuteron is applicable only at the low–energy limit. Thereby all interactions of the deuteron with other hadrons should run through the one–nucleon loop exchange. This is due to the one–nucleon loop origin of the deuteron in our model. This assertion is very similar to the approximation accepted within the NJL model, where all interactions of hadrons run by one–constituent quark–loop exchange [5-8]. Also, one has to understand that most likely the deuteron cannot be inserted in an intermediate state of any process of low–energy interactions. This is connected with a very sensitive structure of the deuteron as an "extended" bound state with a small binding energy. The representation of such a state in terms of any local quantum field is rather limited. The latter entails an undetermined character of the description for the deuteron in intermediate states in terms of Green functions of these local fields.

Now we can apply this model to the calculation of the processes of low–energy interactions like radiative neutron–proton capture n + p $\to$ D + γ, low–energy proton–proton scattering p + p $\to$ D + e^+ + ν_e and so on. We discuss these processes in Sect. 4 and Sect. 5, respectively.

2. Effective Corben–Schwinger interaction in the relativistic field theory model of the deuteron

In this section we give the detailed derivation of the effective Corben–Schwinger Lagrangian defined by the one–nucleon loop diagram in Fig. 2c and describing the effective electromagnetic interactions of the deuteron. The effective Lagrangian of the diagram in Fig. 2c is defined

$$\int d^4x\,\mathcal{L}_{\rm Fig.2c}(x) = \int d^4x\int\frac{d^4x_1d^4k_1}{(2\pi)^4}\frac{d^4x_2d^4k_2}{(2\pi)^4}D_{\beta}(x)\,D^{\dagger}_{\alpha}(x_1)\,A_{\mu}(x_2)\times$$
$$\times\,e^{-ik_1\cdot x_1}\,e^{-ik_2\cdot x_2}\,e^{i(k_1+k_2)\cdot x}\,\frac{e\,g^2_{\rm V}}{4\pi^2}\,\mathcal{J}^{\beta\alpha\mu}(k_1,k_2;Q) \qquad (21)$$

where

$$\mathcal{J}^{\beta\alpha\mu}(k_1,k_2;Q) = \qquad (22)$$
$$= \int\frac{d^4k}{\pi^2i}{\rm tr}\Big\{\gamma^{\beta}\frac{1}{M_{\rm N}-\hat{k}-\hat{Q}}\gamma^{\alpha}\frac{1}{M_{\rm N}-\hat{k}-\hat{Q}-\hat{k}_1}\gamma^{\mu}\frac{1}{M_{\rm N}-\hat{k}-\hat{Q}-\hat{k}_1-\hat{k}_2}\Big\}.$$

The 4–vector $Q = a\,k_1 + b\,k_2$, where a and b are arbitrary constants, displays the dependence of the k integral in (21) on the shift of the virtual momentum. This ambiguity of the evaluation of the integral over k, which has been found by Gertsein and Jackiw [12], is used to remove undesirable contributions and make the effective Lagrangian gauge invariant. We use the fields of the physical deuteron. This is because the renormalization (6) introduces divergent terms that are small compared with the convergent ones we are following for the evaluation of the Corben–Schwinger Lagrangian.

In order to display the Gertsein–Jackiw ambiguity we follow the Gertsein–Jackiw method and evaluate the difference

$$\delta\mathcal{J}^{\beta\alpha\mu}(k_1,k_2;Q) = \mathcal{J}^{\beta\alpha\mu}(k_1,k_2;Q) - \mathcal{J}^{\beta\alpha\mu}(k_1,k_2;0). \qquad (23)$$

In accordance with the Gertsein–Jackiw method the difference (23) can be represented by the integral

$$\delta\,\mathcal{J}^{\beta\alpha\mu}(k_1,k_2;Q) = \int_0^1 dx\,\frac{d}{dx}\mathcal{J}^{\beta\alpha\mu}(k_1,k_2;xQ) =$$
$$= -\int_0^1 dx\int\frac{d^4k}{\pi^2 i}\,Q^{\lambda}\frac{\partial}{\partial k^{\lambda}}\,\mathrm{tr}\Big\{\gamma^{\beta}\,\frac{1}{M_{\rm N}-\hat{k}-x\hat{Q}}\,\gamma^{\alpha}\,\frac{1}{M_{\rm N}-\hat{k}-x\hat{Q}-\hat{k}_1}\,\gamma^{\mu}$$
$$\times\,\frac{1}{M_{\rm N}-\hat{k}-x\hat{Q}-\hat{k}_1-\hat{k}_2}\Big\}. \tag{24}$$

This shows that the Gertsein–Jackiw ambiguity is just the surface term. Following Gertsein and Jackiw [12] and evaluating the integral over k symmetrically, we obtain

$$\delta\,\mathcal{J}^{\beta\alpha\mu}(k_1,k_2;Q) = -2\int_0^1 dx\,\lim_{R\to\infty}\Big\langle\frac{Q\cdot R}{R^4}\mathrm{tr}\{\gamma^{\beta}(M_{\rm N}+\hat{R}+x\hat{Q})\gamma^{\alpha}\,\times$$
$$\times(M_{\rm N}+\hat{R}+x\hat{Q}+\hat{k}_1)\gamma^{\mu}(M_{\rm N}+\hat{R}+x\hat{Q}+\hat{k}_1+\hat{k}_2)\}\Big\rangle. \tag{25}$$

The brackets $<\ldots>$ denote the averaging over R directions. Due to the limit $R\to\infty$ we can neglect all momenta with respect to R:

$$\delta\,\mathcal{J}^{\beta\alpha\mu}(k_1,k_2;Q) = -2\lim_{R\to\infty}\Big\langle\frac{Q\cdot R}{R^4}\mathrm{tr}\{\gamma^{\beta}\hat{R}\gamma^{\alpha}\hat{R}\gamma^{\mu}\hat{R}\}\Big\rangle. \tag{26}$$

Averaging over R–directions

$$\lim_{R\to\infty}\frac{R^{\lambda}R^{\varphi}R^{\omega}R^{\rho}}{R^4} = \frac{1}{24}(g^{\lambda\varphi}g^{\omega\rho}+g^{\lambda\omega}g^{\varphi\rho}+g^{\lambda\rho}g^{\varphi\omega}), \tag{27}$$

we obtain

$$\delta\,\mathcal{J}^{\beta\alpha\mu}(k_1,k_2;Q) = -\frac{1}{12}\mathrm{tr}(\gamma_{\lambda}\gamma^{\beta}\gamma^{\lambda}\gamma^{\alpha}\hat{Q}\gamma^{\mu}+\gamma^{\beta}\gamma_{\lambda}\gamma^{\alpha}\gamma^{\lambda}\hat{Q}\gamma^{\mu}+ \tag{28}$$
$$+\gamma_{\beta}\hat{Q}\gamma^{\alpha}\gamma_{\lambda}\gamma^{\mu}\gamma^{\lambda}) = \frac{2}{3}(Q^{\alpha}g^{\beta\mu}+Q^{\beta}g^{\mu\alpha}+Q^{\mu}g^{\alpha\beta}).$$

Thus the surface ambiguity noticed by Gertsein and Jackiw contains only finite contributions.

Now we start evaluating $\mathcal{J}^{\beta\alpha\mu}(k_1,k_2;Q)$. To pick up the ambiguity connected with Q one cannot apply the Feynman method of the evaluation of momentum integrals as in (22). This method involves the mergence of the factors in the denominator with the subsequent shift of virtual momentum. On this way one can lose the Q–dependence that is due to the shift at the intermediate stage. Thereby, we have to evaluate the integral over k without any intermediate shift.

One can carry this out by applying a long–wavelength expansion and keeping to the leading terms:

$$\mathcal{J}^{\beta\alpha\mu}(k_1,k_2;Q) =$$

$$= \int \frac{d^4k}{\pi^2 i}\,\mathrm{tr}\Big\{\gamma^\beta \frac{M_{\rm N} + \hat{k} + \hat{Q}}{M^2_{\rm N} - k^2}\Big[1 + \frac{2k\cdot Q}{M^2_{\rm N} - k^2}\Big]\gamma^\alpha \frac{M_{\rm N} + \hat{k} + \hat{Q} + \hat{k}_1}{M^2_{\rm N} - k^2} \times$$
$$\times\Big[1 + \frac{2k\cdot(Q + k_1)}{M^2_{\rm N} - k^2}\Big]\gamma^\mu \frac{M_{\rm N} + \hat{k} + \hat{Q} + \hat{k}_1 + \hat{k}_2}{M^2_{\rm N} - k^2}\Big[1 + \frac{2k\cdot(Q + k_1 + k_2)}{M^2_{\rm N} - k^2}\Big]\Big\} =$$
$$= \int \frac{d^4k}{\pi^2 i}\frac{1}{(M^2_{\rm N} - k^2)^3}\mathrm{tr}\,\{M^2_{\rm N}\gamma^\beta(\hat{k} + \hat{Q})\gamma^\alpha\gamma^\mu + M^2_{\rm N}\gamma^\beta\gamma^\alpha(\hat{k} + \hat{Q} + \hat{k}_1)\gamma^\mu +$$
$$+ M^2_{\rm N}\gamma^\beta\gamma^\alpha\gamma^\mu(\hat{k} + \hat{Q} + \hat{k}_1 + \hat{k}_2) + \gamma^\beta(\hat{k} + \hat{Q})\gamma^\alpha(\hat{k} + \hat{Q} + \hat{k}_1)\gamma^\mu(\hat{k} + \hat{Q} + \hat{k}_1 + \hat{k}_2)\}$$
$$\Big[1 + \frac{2k\cdot(3Q + 2k_1 + k_2)}{M^2_{\rm N} - k^2}\Big] = \tag{29}$$
$$= \frac{1}{2}\int \frac{d^4k}{\pi^2 i}\Big[\frac{1}{(M^2_{\rm N} - k^2)^2} + \frac{M^2_{\rm N}}{(M^2_{\rm N} - k^2)^3}\Big]\mathrm{tr}\,\{\gamma^\beta\hat{Q}\gamma^\alpha\gamma^\mu + \gamma^\beta\gamma^\alpha(\hat{Q} + \hat{k}_1)\gamma^\mu +$$
$$+ \gamma^\beta\gamma^\alpha\gamma^\mu(\hat{Q} + \hat{k}_1 + \hat{k}_2)\} +$$
$$+ 2\int \frac{d^4k}{\pi^2 i}\frac{k\cdot(3Q + 2k_1 + k_2)}{(M^2_{\rm N} - k^2)^3}\mathrm{tr}\,\{M^2_{\rm N}(\gamma^\beta\hat{k}\gamma^\alpha\gamma^\mu + \gamma^\beta\gamma^\alpha\hat{k}\gamma^\mu + \gamma^\beta\gamma^\alpha\gamma^\mu\hat{k}) +$$
$$+ \gamma^\beta\hat{k}\gamma^\alpha\hat{k}\gamma^\mu\hat{k}\} = \mathcal{J}^{\beta\alpha\mu}_{(1)}(k_1, k_2; Q) + \mathcal{J}^{\beta\alpha\mu}_{(2)}(k_1, k_2; Q).$$

For the evaluation of $\mathcal{J}^{\beta\alpha\mu}_{(1)}(k_1, k_2; Q)$ it is sufficient to calculate the trace of the Dirac matrices and integrate over k

$$\mathcal{J}^{\beta\alpha\mu}_{(1)}(k_1, k_2; Q) = [1 + 2J_2(M_{\rm N})]\,[(Q + 2k_1 + k_2)^\alpha g^{\beta\mu} +$$
$$+ (Q + 2k_1 + k_2)^\beta g^{\mu\alpha} + (Q + 2k_1 + k_2)^\mu g^{\alpha\beta} - \tag{30}$$
$$- 2(k_1 + k_2)^\alpha g^{\beta\mu} - 2k^\beta_1 g^{\mu\alpha}],$$

where $J_2(M_{\rm N})$ describes a divergent contribution depending on the cut–off $\Lambda_{\rm D}$. Due to the inequality $M_{\rm N} \gg \Lambda_{\rm D}$ we can neglect $J_2(M_{\rm N})$ with respect to the convergent contribution.

To evaluate $\mathcal{J}^{\beta\alpha\mu}_{(2)}(k_1, k_2; Q)$ we have to integrate first over k directions. This gives

$$\mathcal{J}^{\beta\alpha\mu}_{(2)}(k_1, k_2; Q) =$$
$$= \int \frac{d^4k}{\pi^2 i}\Big[\frac{1}{2}\frac{M^2_{\rm N}k^2}{(M^2_{\rm N} - k^2)^4} - \frac{1}{6}\frac{k^4}{(M^2_{\rm N} - k^2)^4}\Big](3Q + k_1 + k_2)_\lambda \times$$
$$\times\mathrm{tr}\,(\gamma^\beta\gamma^\lambda\gamma^\alpha\gamma^\mu + \gamma^\beta\gamma^\alpha\gamma^\lambda\gamma^\mu + \gamma^\beta\gamma^\alpha\gamma^\mu\gamma^\lambda) = \tag{31}$$
$$= -\frac{1}{9}[1 + 6J_2(M_{\rm N})]\,[(3Q + 2k_1 + k_2)^\alpha g^{\beta\mu} + (3Q + 2k_1 + k_2)^\beta g^{\mu\alpha} +$$
$$+ (3Q + 2k_1 + k_2)^\mu g^{\alpha\beta}].$$

Here we have used the integrals

$$\int \frac{d^4k}{\pi^2 i}\frac{1}{(M^2_{\rm N} - k^2)^3} = \frac{1}{2M^2_{\rm N}},$$
$$\int \frac{d^4k}{\pi^2 i}\frac{1}{(M^2_{\rm N} - k^2)^4} = \frac{1}{6M^4_{\rm N}}. \tag{32}$$

Summarizing the contributions, we get

$$\begin{aligned}
&\mathcal{J}^{\beta\alpha\mu}(k_1,k_2;Q) = \frac{2}{3}(Q^\alpha g^{\beta\mu} + Q^\beta g^{\mu\alpha} + Q^\mu g^{\alpha\beta}) + \frac{8}{9}[1 + \frac{3}{2}J_2(M_{\rm N})] \times \\
&\times[(2k_1 + k_2)^\alpha g^{\beta\mu} + (2k_1 + k_2)^\beta g^{\mu\alpha} + (2k_1 + k_2)^\mu g^{\alpha\beta}] + \\
&+[1 + 2J_2(M_{\rm N})][-2(k_1 + k_2)^\alpha g^{\beta\mu} - 2k_1^\beta g^{\mu\alpha}]\,.
\end{aligned} \tag{33}$$

It is seen that the Q–dependence coincides with that obtained by the Gertsein–Jackiw method (28). Due to the arbitrariness of Q we can absorb by the Q–term the terms having the same Lorentz structure. This brings up the r.h.s. of (33) to the form

$$\begin{aligned}
\mathcal{J}^{\beta\alpha\mu}(k_1,k_2;Q) &= \frac{2}{3}(Q^\alpha g^{\beta\mu} + Q^\beta g^{\mu\alpha} + Q^\mu g^{\alpha\beta}) + \\
&+[-2(k_1 + k_2)^\alpha g^{\beta\mu} - 2k_1^\beta g^{\mu\alpha}]\,.
\end{aligned} \tag{34}$$

Also we have neglected the divergent contribution. This approximation is valid due to the inequality $M_{\rm N} \gg \Lambda_{\rm D}$.

The effective Lagrangian $\mathcal{L}_{\rm Fig.2c}(x)$ defined by (34) reads

$$\begin{aligned}
&\mathcal{L}_{\rm Fig.2c}(x) = ie\frac{g_{\rm V}^2}{6\pi^2}[(3 - a)\partial^\alpha D^\dagger_\alpha(x)D_\beta(x)A^\beta(x) - \\
&-(3 - a)D^\dagger_\alpha(x)\partial^\beta D_\beta(x)A^\alpha(x) - bD^\dagger_\alpha(x)D_\beta(x)\partial^\alpha A^\beta(x) - \\
&-(b - a)D^\dagger_\alpha(x)D_\beta(x)\partial^\beta A^\alpha(x) - \\
&-(a - b)\partial^\beta D^\dagger_\alpha(x)D^\alpha(x)A_\beta(x) + bD^\dagger_\alpha(x)\partial^\beta D^\alpha(x)A_\beta(x) + \\
&+3D^\dagger_\alpha(x)D_\beta(x)(\partial^\alpha A^\beta(x) - \partial^\beta A^\alpha(x))].
\end{aligned} \tag{35}$$

Now we can consider a and b as free parameters that can be fixed from the requirement of the gauge invariance of the effective Lagrangian described by the one–nucleon loop diagram in Fig. 2c.

Due to the constraints $\partial^\mu D^\dagger_\mu(x) = \partial^\mu D_\mu(x) = 0$ the corresponding terms in the Lagrangian (34) can be dropped

$$\begin{aligned}
&\mathcal{L}_{\rm Fig.2c}(x) = ie\frac{g_{\rm V}^2}{6\pi^2}[-bD^\dagger_\alpha(x)D_\beta(x)\partial^\alpha A^\beta(x) - \\
&-(b - a)D^\dagger_\alpha(x)D_\beta(x)\partial^\beta A^\alpha(x) - \\
&-(a - b)\partial^\beta D^\dagger_\alpha(x)D^\alpha(x)A_\beta(x) + bD^\alpha(x)\partial^\beta D^\alpha(x)A^\beta(x) + \\
&+3D^\dagger_\alpha(x)D_\beta(x)(\partial^\alpha A^\beta(x) - \partial^\beta A^\alpha(x))].
\end{aligned} \tag{36}$$

The subsequent transformations are performed by applying the identity

$$\begin{aligned}
&D^\dagger_\alpha(x)D_\beta(x)(\partial^\alpha A^\beta(x) - \partial^\beta A^\alpha(x)) = \\
&= \partial^\beta D^\dagger_\alpha(x)D_\beta(x)A^\alpha(x) - D^\dagger_\alpha(x)\partial^\alpha D_\beta(x)A^\beta(x)
\end{aligned} \tag{37}$$

where we have dropped the total divergence and the terms proportional to $\partial^\alpha D^\dagger_\alpha(x)$ and $\partial^\beta D_\beta(x)$.

Then it is convenient to rewrite the Lagrangian (36) as follows

$$\begin{aligned}
{\cal L}_{\rm Fig.2c}(x) &= i\,e\,\frac{g^2_{\rm V}}{6\,\pi^2}[-(a-b)\,D^{\dagger}_{\beta\,\alpha}(x)\,A^{\beta}(x)\,D^{\alpha}(x) + b\,D^{\beta\,\alpha}(x)\,A_{\beta}(x)\,D^{\dagger}_{\alpha}(x) + \\
&-(a-b)\,\partial_{\alpha}\,D^{\dagger}_{\beta}(x)\,D^{\alpha}(x)\,A_{\beta}(x) + b\,D^{\dagger}_{\alpha}(x)\,\partial^{\alpha}\,D^{\beta}(x)\,A^{\beta}(x) - \\
&-b\,D^{\dagger}_{\alpha}(x)\,D_{\beta}(x)\,\partial^{\alpha}\,A^{\beta}(x) - (b-a)\,D^{\dagger}_{\alpha}(x)\,D_{\beta}(x)\,\partial^{\beta}\,A^{\alpha}(x) - \\
&+3\,D^{\dagger}_{\alpha}(x)\,D_{\beta}(x)\,(\partial^{\alpha}\,A^{\beta}(x) - \partial^{\beta}\,A^{\alpha}(x))].
\end{aligned} \tag{38}$$

The first two terms give the the contributions to the renormalization constant of the deuteron electric charge. Then putting $a = 2\,b$ we can decompose the effective Lagrangian (38) into two parts defining the renormalization of the electric charge of the deuteron and the gauge invariant interaction coinciding with that given by Corben and Schwinger [11]

$$\begin{aligned}
{\cal L}_{\rm Fig.2c}(x) &= i\,e\,\frac{g^2_{\rm V}}{6\,\pi^2}[-b\,D^{\dagger}_{\beta\,\alpha}(x)\,A^{\beta}(x)\,D^{\alpha}(x) + b\,D^{\beta\,\alpha}(x)\,A_{\beta}(x)\,D^{\dagger}_{\alpha}(x) + \\
&\quad + (2\,b + 3)\,D^{\dagger}_{\alpha}(x)\,D_{\beta}(x)\,F^{\alpha\,\beta}(x)].
\end{aligned} \tag{39}$$

Putting $b = 0$ we remove the finite contributions to the renormalization constant of the deuteron electric charge coming from the one–nucleon loop diagram in Fig. 2c and get a gauge invariant interaction. As a result the effective gauge invariant interaction reads

$$ {\cal L}_{\rm CS}(x) = i\,e\,\frac{g^2_{\rm V}}{2\,\pi^2}\,D^{\dagger}_{\mu}(x)\,D_{\nu}(x)\,F^{\mu\,\nu}(x)\,. \tag{40}$$

Thus the one–nucleon loop diagram in Fig. 2c defines the effective Corben–Schwinger Lagrangian ${\cal L}_{\rm CS}(x)$ describing interaction of the deuteron with electromagnetic field. The effective Lagrangian ${\cal L}_{\rm CS}(x)$ is defined unambiguously only, if one demands the gauge invariance of the effective interaction. The requirement of the gauge invariance leads to the same result if one would impose the constraint concerning the vanishing of finite contributions to the renormalization constant of the electric charge of the deuteron.

One can show that the contribution of the anomalous magnetic moments of the proton and neutron to the effective Corben–Schwinger Lagrangian is negligibly small compared with that given by Eq. (40).

3. Effective Aronson interaction in the relativistic field theory model of the deuteron

In this Section we give the detailed derivation of the effective Aronson Lagrangian defined by the one–nucleon loop diagram in Fig. 2d and describing the effective electromagnetic interactions of the deuteron. The effective Lagrangian described by the diagram in Fig. 2d is given by

$$\begin{aligned}
\int d^4x\,{\cal L}_{\rm Fig.2d}(x) &= \int d^4x \int \frac{d^4x_1\,d^4k_1}{(2\,\pi)^4}\,\frac{d^4x_2\,d^4k_2}{(2\,\pi)^4}\,D_{\alpha\,\beta}(x)\,D^{\dagger}_{\mu\,\nu}(x_1)\,A_{\lambda}(x_2)\times \\
&\times e^{-i\,k_1\cdot x_1}\,e^{-i\,k_2\cdot x_2}\,e^{i\,(k_1+k_2)\cdot x}\,(-\,e)\,\frac{g^2_{\rm T}}{4\,\pi^2}\,\frac{1}{M^2_{\rm D}}\,{\cal J}^{\alpha\,\beta\,\mu\,\nu\,\lambda}(k_1,k_2;Q)
\end{aligned} \tag{41}$$

where

$$\mathcal{J}^{\alpha\beta\mu\nu\lambda}(k_1,k_2;Q) \;=\; \int \frac{d^4k}{\pi^2 i}\;\times \tag{42}$$
$$\times \mathrm{tr}\left\{\sigma^{\alpha\beta}\,\frac{1}{M_{\rm N}-\hat{k}-\hat{Q}}\,\sigma^{\mu\nu}\,\frac{1}{M_{\rm N}-\hat{k}-\hat{Q}-\hat{k}_1}\,\gamma^{\lambda}\,\frac{1}{M_{\rm N}-\hat{k}-\hat{Q}-\hat{k}_1-\hat{k}_2}\right\}.$$

Here we also use the fields of the physical deuteron, for the renormalization (6) introduces divergent terms that are small compared with the convergent ones that we are following for the evaluation of the Aronson Lagrangian. The Gertsein–Jackiw ambiguity is given by

$$\delta\,\mathcal{J}^{\alpha\beta\mu\nu\lambda}(k_1,k_2;Q) \;=\; \frac{1}{6}\,\mathrm{tr}(\sigma^{\alpha\beta}\,\hat{Q}\,\sigma^{\mu\nu}\,\gamma^{\lambda}\,)\,. \tag{43}$$

Now we should set about evaluating $\mathcal{J}^{\alpha\beta\mu\nu\lambda}(k_1,k_2;Q)$. By analogy with (22) we get

$$\begin{aligned}
&\mathcal{J}^{\alpha\beta\mu\nu\lambda}(k_1,k_2;Q) \;=\\
&=\int\frac{d^4k}{\pi^2 i}\,\mathrm{tr}\Big\{\sigma^{\alpha\beta}\,\frac{M_{\rm N}+\hat{k}+\hat{Q}}{M^2_{\rm N}-k^2}\left[1\;+\;\frac{2\,k\cdot Q}{M^2_{\rm N}-k^2}\right]\sigma^{\mu\nu}\,\frac{M_{\rm N}+\hat{k}+\hat{Q}+\hat{k}_1}{M^2_{\rm N}-k^2}\;\times\\
&\times\left[1\;+\;\frac{2\,k\cdot(Q+k_1)}{M^2_{\rm N}-k^2}\right]\gamma^{\lambda}\,\frac{M_{\rm N}+\hat{k}+\hat{Q}+\hat{k}_1+\hat{k}_2}{M^2_{\rm N}-k^2}\left[1\;+\;\frac{2\,k\cdot(Q+k_1+k_2)}{M^2_{\rm N}-k^2}\right]\Big\}\;=\\
&=\int\frac{d^4k}{\pi^2 i}\,\frac{1}{(M^2_{\rm N}-k^2)^3}\mathrm{tr}\,\{M^2_{\rm N}[\sigma^{\alpha\beta}(\hat{k}+\hat{Q})\sigma^{\mu\nu}\gamma^{\lambda}+\sigma^{\alpha\beta}\sigma^{\mu\nu}(\hat{k}+\hat{Q}+\hat{k}_1)\gamma^{\lambda}\;+\\
&+\sigma^{\alpha\beta}\sigma^{\mu\nu}\gamma^{\lambda}(\hat{k}+\hat{Q}+\hat{k}_1+\hat{k}_2)]+\sigma^{\alpha\beta}(\hat{k}+\hat{Q})\sigma^{\mu\nu}(\hat{k}+\hat{Q}+\hat{k}_1)\gamma^{\lambda}\;\times\\
&\times(\hat{k}+\hat{Q}+\hat{k}_1+\hat{k}_2)\}\left[1\;+\;\frac{2\,k\cdot(3Q+2k_1+k_2)}{M^2_{\rm N}-k^2}\right]\;=\\
&=\int\frac{d^4k}{\pi^2 i}\frac{1}{(M^2_{\rm N}-k^2)^3}\mathrm{tr}\,\{M^2_{\rm N}[\sigma^{\alpha\beta}\hat{Q}\sigma^{\mu\nu}\gamma^{\lambda}+\sigma^{\alpha\beta}\sigma^{\mu\nu}(\hat{Q}+\hat{k}_1)\gamma^{\lambda}\;+\\
&+\sigma^{\alpha\beta}\sigma^{\mu\nu}\gamma^{\lambda}(\hat{Q}+\hat{k}_1+\hat{k}_2)]-\frac{1}{2}k^2\sigma^{\alpha\beta}\hat{Q}\sigma^{\mu\nu}\gamma^{\lambda}\}\;+\\
&+2\int\frac{d^4k}{\pi^2 i}\frac{1}{(M^2_{\rm N}-k^2)^4}\mathrm{tr}\,\{\frac{1}{2}M^2_{\rm N}k^2[\sigma^{\alpha\beta}(3\,\hat{Q}+2\,\hat{k}_1+\hat{k}_2)\sigma^{\mu\nu}\gamma^{\lambda}\;+\\
&+\sigma^{\alpha\beta}\sigma^{\mu\nu}(3\,\hat{Q}+2\,\hat{k}_1+\hat{k}_2)\gamma^{\lambda}+\sigma^{\alpha\beta}\sigma^{\mu\nu}\gamma^{\lambda}(3\,\hat{Q}+2\,\hat{k}_1+\hat{k}_2)]\;-\\
&-\frac{1}{6}\,k^4\sigma^{\alpha\beta}(3\,\hat{Q}+2\,\hat{k}_1+\hat{k}_2)\sigma^{\mu\nu}\gamma^{\lambda}\}\;=\;\mathcal{J}^{\alpha\beta\mu\nu\lambda}_{(1)}(k_1,k_2;Q)\;+\;\mathcal{J}^{\alpha\beta\mu\nu\lambda}_{(2)}(k_1,k_2;Q)\,.
\end{aligned} \tag{44}$$

Integrating over k we obtain

$$\begin{aligned}
&\mathcal{J}^{\alpha\beta\mu\nu\lambda}_{(1)}(k_1,k_2;Q)\;=\;\frac{1}{4}\,[1+2\,J_2(M_{\rm N})]\,\mathrm{tr}(\sigma^{\alpha\beta}\hat{Q}\sigma^{\mu\nu}\gamma^{\lambda})\;+\\
&+\frac{1}{2}\,\mathrm{tr}\,[\sigma^{\alpha\beta}\sigma^{\mu\nu}(\hat{Q}+\hat{k}_1)\gamma^{\lambda}+\sigma^{\alpha\beta}\sigma^{\mu\nu}\gamma^{\lambda}(\hat{Q}+\hat{k}_1+\hat{k}_2)]\,,
\end{aligned} \tag{45}$$

$$\begin{aligned}
&\mathcal{J}^{\alpha\beta\mu\nu\lambda}_{(2)}(k_1,k_2;Q)\;=\;-\frac{1}{6}\,[-\frac{5}{6}+J_2(M_{\rm N})]\,\mathrm{tr}[\sigma^{\alpha\beta}(3\hat{Q}+2\hat{k}_1+\hat{k}_2)\sigma^{\mu\nu}\gamma^{\lambda}]\;-\\
&-\frac{1}{6}\,\mathrm{tr}\,[\sigma^{\alpha\beta}(3\hat{Q}+2\hat{k}_1+\hat{k}_2)\sigma^{\mu\nu}\gamma^{\lambda}\;+\\
&+\sigma^{\alpha\beta}\sigma^{\mu\nu}(3\hat{Q}+2\hat{k}_1+\hat{k}_2)\gamma^{\lambda}+\sigma^{\alpha\beta}\sigma^{\mu\nu}\gamma^{\lambda}(3\hat{Q}+2\hat{k}_1+\hat{k}_2)]\,.
\end{aligned} \tag{46}$$

Now we should summarize the contributions and collect similar terms

$$\begin{aligned}\mathcal{J}^{\alpha\beta\mu\nu\lambda}(k_1,k_2;Q) &= \frac{1}{6}\,\mathrm{tr}[\sigma^{\alpha\beta}(\hat{Q}-\frac{1}{6}(2\,\hat{k}_1+\hat{k}_2))\sigma^{\mu\nu}\gamma^{\lambda}] + \\ &+ \frac{1}{6}\,\mathrm{tr}\,[\sigma^{\alpha\beta}\sigma^{\mu\nu}(\hat{k}_1-\hat{k}_2)\gamma^{\lambda}] + \frac{1}{6}\,\mathrm{tr}\,[\sigma^{\alpha\beta}\sigma^{\mu\nu}\gamma^{\lambda}(\hat{k}_1+2\,\hat{k}_2)]\,.\end{aligned} \tag{47}$$

It is seen that the Q–dependence coincides with that obtained by the Gertsein–Jackiw method. Due to the arbitrariness of Q the vector $(2\,\hat{k}_1+\hat{k}_2)/6$ can remove by the redefinition of Q. Thereby we get

$$\begin{aligned}\mathcal{J}^{\alpha\beta\mu\nu\lambda}(k_1,k_2;Q) &= \frac{1}{6}\,\mathrm{tr}(\sigma^{\alpha\beta}\,\hat{Q}\,\sigma^{\mu\nu}\,\gamma^{\lambda}) + \\ &+ \frac{1}{6}\,\mathrm{tr}\,[\sigma^{\alpha\beta}\,\sigma^{\mu\nu}(\hat{k}_1-\hat{k}_2)\gamma^{\lambda}] + \frac{1}{6}\,\mathrm{tr}\,[\sigma^{\alpha\beta}\sigma^{\mu\nu}\gamma^{\lambda}(\hat{k}_1+2\,\hat{k}_2)]\,.\end{aligned} \tag{48}$$

After the calculation of the traces of the Dirac matrices we obtain $\mathcal{J}^{\alpha\beta\mu\nu\lambda}(k_1,k_2;Q)$ that leads to the following effective Lagrangian

$$\begin{aligned}&\mathcal{L}_{\rm Fig.2d}(x) = (-\,i\,e)\,\frac{g^2_{\rm T}}{4\,\pi^2}\,\frac{1}{M^2_{\rm D}}\\ &\Big[\frac{8}{3}\,a\,\partial_{\lambda}\,D^{\dagger\,\lambda\nu}(x)\,D_{\nu\mu}(x)\,A^{\mu}(x) + \frac{8}{3}\,a\,D^{\dagger\,\mu\nu}(x)\,\partial_{\lambda}\,D^{\lambda\nu}(x)\,A_{\mu}(x) + \\ &+\frac{8}{3}\,(b+a)\,D^{\dagger}_{\mu\nu}(x)\,D^{\nu\lambda}(x)\,\partial^{\mu}\,A_{\lambda}(x) + \frac{8}{3}\,(b-a)\,D^{\dagger}_{\mu\nu}(x)\,D^{\nu\lambda}(x)\,\partial_{\lambda}\,A^{\mu}(x) - \\ &-\frac{16}{3}\,D^{\dagger}_{\mu\nu}(x)\,D^{\nu\lambda}(x)\,\partial^{\mu}\,A_{\lambda}(x) + 8\,D^{\dagger}_{\mu\nu}(x)\,D^{\nu\lambda}(x)\,(\partial^{\mu}\,A_{\lambda}(x) - \partial_{\lambda}\,A^{\mu}(x))\Big]\,.\end{aligned} \tag{49}$$

For the derivation of the effective Lagrangian (49) we have used the equation of motion

$$\partial_{\lambda}\,D_{\mu\nu}(x) + \partial_{\mu}\,D_{\nu\lambda}(x) + \partial_{\nu}\,D_{\lambda\mu}(x) = 0\,. \tag{50}$$

The analogous equation of motion is valid for the conjugated field. By collecting similar terms in (49) we get

$$\begin{aligned}&\mathcal{L}_{\rm Fig.2d}(x) = (-\,i\,e)\,\frac{g^2_{\rm T}}{4\,\pi^2}\,\frac{1}{M^2_{\rm D}}\,\Big[\frac{8}{3}\,(b\,+\,a\,-\,1)\,D^{\dagger}_{\mu\nu}(x)\,D^{\nu\lambda}(x)\,\partial^{\mu}\,A_{\lambda}(x) + \\ &+\frac{8}{3}\,(b\,-\,a\,-\,3)\,D^{\dagger}_{\mu\nu}(x)\,D^{\nu\lambda}(x)\,\partial_{\lambda}\,A^{\mu}(x) + \\ &+\frac{8}{3}\,a\,\partial_{\lambda}\,D^{\dagger\,\lambda\nu}(x)\,D_{\nu\mu}(x)\,A^{\mu}(x) + \frac{8}{3}\,a\,D^{\dagger\,\mu\nu}(x)\,\partial_{\lambda}\,D^{\lambda\nu}(x)\,A_{\mu}(x)\Big]\,.\end{aligned} \tag{51}$$

The third and the fourth terms can be reduced by applying the equation of motion

$$\partial_{\lambda}\,D^{\lambda\nu}(x) = -\,M^2_{\rm D}\,D^{\nu}(x) \tag{52}$$

and analogous for the conjugated field. Then putting $b\,+\,a\,-\,1\,=\,-\,b\,+\,a\,+\,3$, we obtain $b\,=\,2$. that brings up the effective Lagrangian (51) to the following irreducible form

$$\begin{aligned}&\mathcal{L}_{\rm Fig.2d}(x) = \\ &= i\,e\,\frac{2\,g^2_{\rm T}}{3\,\pi^2}\,a\,\Big[\,D^{\dagger}_{\mu\nu}(x)\,A^{\mu}(x)\,D^{\nu}(x) - D^{\mu\nu}(x)\,A_{\mu}(x)\,D^{\dagger}_{\nu}(x)\Big] + \\ &+ i\,e\,\frac{2\,g^2_{\rm T}}{3\,\pi^2}\,\frac{1}{M^2_{\rm D}}\,(1\,+\,a)\,D^{\dagger}_{\mu\nu}(x)\,D^{\nu\lambda}(x)\,(\partial_{\lambda}\,A^{\mu}(x) - \partial^{\mu}\,A_{\lambda}(x))\,.\end{aligned} \tag{53}$$

Putting $a = 0$ we remove the finite contributions to the renormalization constant of the deuteron electric charge and obtain the gauge invariant interaction

$$\mathcal{L}_{\rm Fig.2d}(x) = i\,e\,\frac{2\,g^2_{\rm T}}{3\,\pi^2}\,\frac{1}{M^2_{\rm D}}\,D^{\dagger}_{\mu\nu}(x)\,D^{\nu\lambda}(x)\,F_{\lambda}{}^{\mu}(x). \qquad (54)$$

The effective Lagrangian (54) coincides fully with that suggested by Aronson [9]. This means that the one–nucleon loop diagram in Fig. 2d defines unambiguously the effective Lagrangian if one imposes the requirement of the gauge invariance. The same result can be gained if one requires the vanishing of finite contributions to the renormalization constant of the electric charge of the deuteron.

By analogy with the evaluation of the Lagrangian (54) one can obtain the contribution of the anomalous magnetic moments of the proton and neutron to the effective Aronson interaction

$$\delta\mathcal{L}_{\rm Ar}(x) = i\,e\,(\kappa_{\rm p} - \kappa_{\rm n})\left(1 - \frac{8}{9}\frac{g_{\rm V}}{g_{\rm T}}\right)\frac{3\,g^2_{\rm T}}{2\,\pi^2}\,\frac{1}{M^2_{\rm D}}\,D^{\dagger}_{\mu\nu}(x)\,D^{\nu\lambda}(x)\,F_{\lambda}{}^{\mu}(x). \qquad (55)$$

As a result the complete expression of the Aronson effective Lagrangian is given by

$$\mathcal{L}_{\rm Ar}(x) = i\,e\,\frac{2\,g^2_{\rm T}}{3\,\pi^2}\left[1 + \frac{9}{4}\,(\kappa_{\rm p} - \kappa_{\rm n})\left(1 - \frac{8}{9}\frac{g_{\rm V}}{g_{\rm T}}\right)\right]\frac{1}{M^2_{\rm D}}D^{\dagger}_{\mu\nu}(x)D^{\nu\lambda}(x)F_{\lambda}{}^{\mu}(x). \qquad (56)$$

Thus we have shown that in the relativistic field theory model of the deuteron and in one–nucleon loop approximation one can unambiguously evaluate effective electromagnetic interactions of the deuteron in terms of anomalous magnetic and electric quadrupole moments. This can be obtained under requirement of gauge invariance to every one–nucleon loop diagram separately.

The procedure having been expounded above should be applied to the evaluation of one–nucleon loop diagrams describing processes of low-energy interaction of the deuteron such as the radiative neutron–proton capture n + p → D + γ, the low–energy proton–proton scattering p + p → D + e^+ + ν_e and so on.

4. Radiative neutron–proton capture for thermal neutrons

At low energies, the process of the radiative neutron–proton capture n + p → D + γ proceeds through electric and magnetic dipole transitions. In the usual notations ${}^{2S+1}{\rm L}_{\rm J}$, the deuteron is a ${}^3{\rm S}_1$ state, and the possible transitions are

$${}^3{\rm S}_1 \to {}^3{\rm S}_1\,(\mu) \quad , \quad {}^3{\rm P}_{0,1,2} \to {}^3{\rm S}_1\,(d) \quad , \quad {}^1{\rm S}_0 \to {}^3{\rm S}_1\,(\mu)$$

where μ denotes magnetic–dipole and d electric–dipole transitions. If the energies are low enough, the nucleons are in an S–state and only magnetic dipole transitions are possible.

In the s–wave the magnetic moment operator acts only on spin variables. This implies that the transition ${}^3{\rm S}_1 \to {}^3{\rm S}_1$ is forbidden [13]. Thereby the only allowed transition is ${}^1{\rm S}_0 \to {}^3{\rm S}_1$. This means that the anomalous magnetic moments of the proton and neutron should give the main contribution [13].

In the relativistic field theory model of the deuteron the interactions of the deuteron with other fields should proceed through one–nucleon loop diagrams. Therefore, for the evaluation of the effective Lagrangian of the radiative neutron–proton capture we have first to evaluate the effective Lagrangian, describing low–energy neutron–proton scattering. Keeping to the one–pion exchange we obtain [1]

$$\begin{aligned}\mathcal{L}^{\rm np}_{\rm eff}(x) = \frac{g^2_{\pi \rm NN}}{4M^2_{\pi}}\{[\bar{p}(x)n^c(x)][\bar{n^c}(x)p(x)] + [\bar{p}(x)\gamma^5 n^c(x)][\bar{n^c}(x)\gamma^5 p(x)] + \\ +[\bar{p}(x)\gamma_{\mu}\gamma^5 n^c(x)][\bar{n^c}(x)\gamma^{\mu}\gamma^5 p(x)] + 3[\bar{p}(x)\gamma_{\mu} n^c(x)][\bar{n^c}(x)\gamma^{\mu} p(x)] + \\ +\frac{3}{2}[\bar{p}(x)\sigma_{\mu\nu} n^c(x)][\bar{n^c}(x)\sigma^{\mu\nu} p(x)]\Big\}.\end{aligned} \tag{57}$$

Only terms $[\bar{p}(x)\gamma^5 n^c(x)][\bar{n^c}(x)\gamma^5 p(x)]$ and $[\bar{p}(x)\gamma_{\mu}\gamma^5 n^c(x)][\bar{n^c}(x)\gamma^{\mu}\gamma^5 p(x)]$ contribute to the S–wave of the neutron–proton scattering in the low–energy limit. Therefore, these terms should dominate the ${}^1{\rm S}_0 \to {}^3{\rm S}_1$ transition in the radiative neutron–proton capture at low energies. The corresponding one–nucleon loop diagrams are depicted in Figs. 3 and 4.

First let us consider the contribution of the diagram in Fig. 3a. The corresponding Lagrangian reads

$$\begin{aligned}&\int d^4x\,\mathcal{L}_{\rm Fig.3a}(x) = \\ &= \int d^4x \int \frac{d^4x_1\,d^4k_1}{(2\pi)^4}\,\frac{d^4x_2\,d^4k_2}{(2\pi)^4}\,[\bar{n^c}(x)\gamma^5 p(x)]\,D^{\dagger}_{\mu}(x_1)\,A_{\nu}(x_2)\,\times \\ &\times e^{-ik_1\cdot x_1}\,e^{-ik_2\cdot x_2}\,e^{i(k_1+k_2)\cdot x}\,i\,e\,\frac{g^2_{\pi \rm NN}}{M^2_{\pi}}\,\frac{g_{\rm V}}{32\pi^2}\,\mathcal{J}^{\mu\nu}_5(k_1,k_2;Q)\end{aligned} \tag{58}$$

where

$$\begin{aligned}&\mathcal{J}^{\mu\nu}_5(k_1,k_2) = \\ &= \int \frac{d^4k}{\pi^2 i}{\rm tr}\Big\{\gamma^5\frac{1}{M_{\rm N}-\hat{k}-\hat{Q}}\gamma^{\mu}\frac{1}{M_{\rm N}-\hat{k}-\hat{Q}-\hat{k}_1}\gamma^{\nu}\frac{1}{M_{\rm N}-\hat{k}-\hat{Q}-\hat{k}_1-\hat{k}_2}\Big\},\end{aligned} \tag{59}$$

and $Q = a\,k_1 + b\,k_2$ is an arbitrary shift of virtual momentum. Fortunately, the integral $\mathcal{J}^{\mu\nu}_5(k_1,k_2;Q)$ does not depend on the shift of the virtual momentum and can be evaluated unambiguously

$$\mathcal{J}^{\mu\nu}_5(k_1,k_2;Q) = \frac{2i}{M_{\rm N}}\,\varepsilon^{\mu\nu\alpha\beta}\,k_{1\alpha}\,k_{2\beta}\quad(\varepsilon^{0123} = 1). \tag{60}$$

This gives the following effective Lagrangian

$$\mathcal{L}_{\rm Fig.3a}(x) = -\frac{e}{2M_{\rm N}}\,\frac{g^2_{\pi \rm NN}}{M^2_{\pi}}\,\frac{g_{\rm V}}{16\pi^2}\,D^{\dagger}_{\mu\nu}(x)^*F^{\mu\nu}(x)\,[\bar{n^c}(x)\gamma^5 p(x)] \tag{61}$$

where ${}^*F^{\mu\nu}(x) = \frac{1}{2}\varepsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}(x)$.

Now let us consider the contribution of the diagram in Fig. 3b. This contribution is caused by the anomalous magnetic moment of the proton. The effective Lagranigan is defined

$$
\begin{aligned}
&\int d^4 x\, \mathcal{L}_{\rm Fig.3b}(x) = \\
&= \int d^4 x \int \frac{d^4 x_1\, d^4 k_1}{(2\pi)^4}\frac{d^4 x_2\, d^4 k_2}{(2\pi)^4}\,[\bar{n^c}(x)\gamma^5 p(x)]\, D^{\dagger}_{\mu}(x_1)\, F_{\alpha\beta}(x_2)\,\times \qquad (62)\\
&\times e^{-i k_1\cdot x_1}\, e^{-i k_2\cdot x_2}\, e^{i(k_1+k_2)\cdot x}\,(-e)\,\frac{\kappa_{\rm p}}{2M_{\rm N}}\frac{g^2_{\pi{\rm NN}}}{M^2_{\pi}}\frac{g_{\rm V}}{32\pi^2}\,\mathcal{J}^{\mu\alpha\beta}_5(k_1,k_2;Q)
\end{aligned}
$$

where

$$
\begin{aligned}
&\mathcal{J}^{\mu\alpha\beta}_5(k_1,k_2;Q) = \qquad (63)\\
&= \int \frac{d^4 k}{\pi^2 i}{\rm tr}\left\{\gamma^5\frac{1}{M_{\rm N}-\hat{k}-\hat{Q}}\gamma^{\mu}\frac{1}{M_{\rm N}-\hat{k}-\hat{Q}-\hat{k}_1}\sigma^{\alpha\beta}\frac{1}{M_{\rm N}-\hat{k}-\hat{Q}-\hat{k}_1-\hat{k}_2}\right\}.
\end{aligned}
$$

Unfortunately, the integral $\mathcal{J}^{\mu\alpha\beta}_5(k_1,k_2;Q)$ depends on the shift of the virtual momentum. Therefore, the leading contribution in the momentum expansion is fully arbitrary and is given

$$
\mathcal{J}^{\mu\alpha\beta}_5(k_1,k_2;Q) = -2i\,\varepsilon^{\mu\alpha\beta\nu}Q_{\nu} = -2i\,\varepsilon^{\mu\alpha\beta\nu}(a\,k_1 + b\,k_2)_{\nu} \qquad (64)
$$

where a and b are arbitrary parameters. The contribution of the momentum k_2 produces in the Lagrangian the operator $\partial_{\mu}{}^{*}F^{\mu\nu}(x)$ that vanishes due to Maxwell′s equation of motion, i.e., $\partial_{\mu}{}^{*}F^{\mu\nu}(x) = 0$. Thus only the contribution of the momentum k_1 matters. The effective Lagrangian produced by $\mathcal{J}^{\mu\alpha\beta}_5(k_1,k_2;Q)$ given by (64) reads

$$
\mathcal{L}_{\rm Fig.3b}(x) = -a\,\kappa_{\rm p}\frac{e}{2M_{\rm N}}\frac{g^2_{\pi{\rm NN}}}{M^2_{\pi}}\frac{g_{\rm V}}{16\pi^2}D^{\dagger}_{\mu\nu}(x)^{*}F^{\mu\nu}(x)\,[\bar{n^c}(x)\gamma^5 p(x)]\,. \qquad (65)
$$

The effective Lagrangian corresponding to the diagrams in Figs. 3a,b is given by

$$
\mathcal{L}_{\rm Fig.3a,b}(x) = -(1 + a\,\kappa_{\rm p})\frac{e}{2M_{\rm N}}\frac{g^2_{\pi{\rm NN}}}{M^2_{\pi}}\frac{g_{\rm V}}{16\pi^2}D^{\dagger}_{\mu\nu}(x)^{*}F^{\mu\nu}(x)\,[\bar{n^c}(x)\gamma^5 p(x)]\,. \qquad (66)
$$

Now let us discuss how we can fix the parameter a. As has been mentioned above the radiative capture n + p → D + γ at low energies is a magnetic dipole transition ${}^1{\rm S}_0 \to {}^3{\rm S}_1\,(\mu)$. This implies that the contribution of the proton and neutron to amplitude of the radiative capture n + p → D + γ should be proportional to their magnetic moments. In the case of the proton the total magnetic moment equals $(1+\kappa_{\rm p})$, i.e., $\mu_{\rm p} = 1+\kappa_{\rm p}$, whereas the total magnetic moment of the neutron is just its anomalous magnetic moment, i.e., $\kappa_{\rm n} = \mu_{\rm n}$. The effective Lagrangian (65) describes the contribution of the proton, therefore the quantity $(1 + a\,\kappa_{\rm p})$ should be nothing more than the total magnetic moment of the proton $\mu_{\rm p}$. This fixes the arbitrariness of the contribution of the diagram in Fig. 3b. That means that we have to put $a = 1$. As a result the complete contribution of the diagrams in Figs. 3a,b reads

$$
\mathcal{L}_{\rm Fig.3a,b}(x) = -\mu_{\rm p}\frac{e}{2M_{\rm N}}\frac{g^2_{\pi{\rm NN}}}{M^2_{\pi}}\frac{g_{\rm V}}{16\pi^2}D^{\dagger}_{\mu\nu}(x)^{*}F^{\mu\nu}(x)\,[\bar{n^c}(x)\gamma^5 p(x)]\,. \qquad (67)
$$

Thus we have fixed the ambiguity introduced by the diagram in Fig. 3b by applying the selection rule ${}^1{\rm S}_0 \to {}^3{\rm S}_1\,(\mu)$.

By extending the procedure of the evaluation of the diagrams in Figs. 3a,b to the other diagrams in Fig. 3 we get the following complete effective Lagrangian

$$\mathcal{L}_{\rm Fig.3}(x) \;=\; -\,(\mu_{\rm p}-\mu_{\rm n})\,\frac{e}{2M_{\rm N}}\,\frac{g^2_{\pi\rm NN}}{M^2_{\pi}}\,\frac{g_{\rm V}}{16\pi^2}\,D^{\dagger}_{\mu\nu}(x)^*F^{\mu\nu}(x)\,[\bar{n^c}(x)\gamma^5 p(x)]\,. \qquad (68)$$

The contributions of the diagrams describing the interaction of the deuteron with the anomalous magnetic moments of the proton and neutron through the tensor nucleon current, i.e., proportional to the constant $g_{\rm T}$, are divergent and due to the inequality $M_{\rm N} \gg \Lambda_{\rm D}$ are small compared with the convergent ones. Thereby the coupling constant $g_{\rm T}$ does not contribute to the amplitude of the radiative neutron–proton capture n + p $\to$ D + γ.

The contribution of the $[\bar{p}(x)\gamma_{\mu}\gamma^5 n^c(x)][\bar{n^c}(x)\gamma^{\mu}\gamma^5 p(x)]$ interaction can be evaluated by analogy to that given above. As a result we obtain

$$\mathcal{L}_{\rm Fig.4}(x) \;=\; i\,e\,(\mu_{\rm p}-\mu_{\rm n})\,\frac{g^2_{\pi\rm NN}}{M^2_{\pi}}\,\frac{g_{\rm V}}{16\pi^2}\,D^{\dagger}_{\mu}(x)^*F^{\mu\nu}(x)\,[\bar{n^c}(x)\gamma_{\nu}\gamma^5 p(x)]\,. \qquad (69)$$

In the low–energy limit the amplitude of the radiative neutron–proton capture, defined by the effective Lagrangians (68) and (69) reads

$$\mathcal{M}({\rm n}+{\rm p}\to{\rm D}+\gamma)_{s's} \;=\; (\mu_{\rm p}-\mu_{\rm n})\,\frac{3eg_{\rm V}}{8\pi^2}\,\frac{g^2_{\pi\rm NN}}{M^2_{\pi}}\,M_{\rm N}\,\vec{e}^{\,*}_{\rm D}(\vec{q}\,)\cdot(\vec{k}\times\vec{e}^{\,*}(\vec{k}\,))\,\chi^{\dagger}_{s'}\,\varphi_s\,, \qquad (70)$$

where $\vec{e}^{\,*}_{\rm D}(\vec{q}\,)$ and $\vec{e}^{\,*}(\vec{k}\,)$ are the polarization vectors of the deuteron and photon, $\vec{k}$ is the photon momentum, and $\chi_{s'}$ and φ_s are the spinorial wave–functions of the neutron and proton.

The cross section of the radiative neutron–proton capture is given by

$$\sigma({\rm n}+{\rm p}\to{\rm D}+\gamma) \;=\; \frac{1}{v}\,(\mu_{\rm p}-\mu_{\rm n})^2\,\frac{9\alpha\,Q_{\rm D}}{256\pi^2}\left[\frac{g^2_{\pi\rm NN}}{M^2_{\pi}}\right]^2\varepsilon^3_{\rm D}\,M_{\rm N} \qquad (71)$$

where we have used the relation (16) and v is a laboratory velocity of the neutron and $\alpha = e^2/4\pi = 1/137$ is the fine structure constant.

The cross section $\sigma({\rm n}+{\rm p}\to{\rm D}+\gamma)$ of the radiative neutron–proton capture has been measured for thermal neutrons at the laboratory velocities $v/c = 7.34\cdot 10^{-6}$ (the absolute value is $v = 2.2\cdot 10^5\,{\rm cm/sec}$) [14]

$$\sigma({\rm n}+{\rm p}\to{\rm D}+\gamma)_{\rm exp} \;=\; 334.2\pm 0.5\,{\rm mb}\,. \qquad (72)$$

Putting $v = 7.34\times 10^{-6}$ we get the following theoretical value of $\sigma({\rm n}+{\rm p}\to{\rm D}+\gamma)$:

$$\sigma({\rm n}+{\rm p}\to{\rm D}+\gamma) \;=\; 225.2\,{\rm mb}\,. \qquad (73)$$

It is seen that the theoretical value agrees with the experimental data within 33%.

However, it should be stressed that we do not have taken into account the resonance contribution to the ${}^1{\rm S}_0$ low–energy neutron–proton scattering [15,16]. The account of the resonance contribution gives the amplitude of the radiative neutron–proton capture in the form [17]

$$\begin{aligned}\mathcal{M}(\mathrm{n}+\mathrm{p}\to\mathrm{D}+\gamma)_{s's} &= (\mu_{\mathrm{p}}-\mu_{\mathrm{n}})\,\frac{3eg_{\mathrm{V}}}{8\pi^2}\,\frac{g^2_{\pi\mathrm{NN}}}{M^2_{\pi}}\left(1-\frac{16\pi}{3}\frac{M^2_{\pi}}{g^2_{\pi\mathrm{NN}}}\frac{a_{\mathrm{S}}}{M_{\mathrm{N}}}\right)M_{\mathrm{N}}\,\times\\ &\quad\times\vec{e}^{\,*}_{\mathrm{D}}(\vec{q}\,)\cdot(\vec{k}\times\vec{e}^{\,*}(\vec{k}\,))\,\chi^{\dagger}_{s'}\,\varphi_s\end{aligned} \tag{74}$$

where $a_{\mathrm{S}}=-23.748\,\mathrm{fm}$ [4] is the ${}^1\mathrm{S}_0$ neutron–proton scattering length that is solely due to the resonance contribution [15,16].

As a result the cross section of the radiative neutron–proton capture is given by

$$\begin{aligned}&\sigma(\mathrm{n}+\mathrm{p}\to\mathrm{D}+\gamma)\;=\\ &=\;\frac{1}{v}\,(\mu_{\mathrm{p}}-\mu_{\mathrm{n}})^2\,\frac{9\alpha\,Q_{\mathrm{D}}}{256\pi^2}\left[\frac{g^2_{\pi\mathrm{NN}}}{M^2_{\pi}}\right]^2\left(1-\frac{8\pi}{3}\frac{M^2_{\pi}}{g^2_{\pi\mathrm{NN}}}\frac{a_{\mathrm{S}}}{M_{\mathrm{N}}}\right)^2\varepsilon^3_{\mathrm{D}}\,M_{\mathrm{N}}\;=\;334.2\,\mathrm{mb}\,.\end{aligned} \tag{75}$$

The theoretical value agrees well with the experimental data. The cross section (75) predicted in the relativistic field theory model of the deuteron agrees also well the value given by the potential model [18].

5. Low–energy p + p → D + e^+ + ν_e scattering

The process of the two–proton fusion p + p → D + e^+ + ν_e plays an important role for the nucleosynthesis of deuterons in stars. The deuterons are destroyed agian by the reaction p + D → ^{3}He + γ [20]. In nuclear physics the process p + p → D + e^+ + ν_e is a Gamow–Teller transition governed by the weak axial–vector nucleon current [21].

In the relativistic field theory model of the deuteron the low–energy process p + p → D + e^+ + ν_e is closely connected with the low–energy proton–proton scattering, i.e., p + p → p + p. Low–energy proton–proton scattering differs much from low–energy neutron–proton scattering. This is mostly due to the strong contribution of the Coulomb repulsion [22]. As a result the low–energy proton–proton scattering is a non–resonant reaction. The small relative velocities v in the proton–proton system are suppressed by the factor $\exp(-\pi\alpha/v)$. The factor $\exp(-2\pi\alpha/v)$ is known appropriately as the Gamow penetration factor [22,23]. For the earliest evaluation of the cross section of the p + p → D + e^+ + ν_e scattering we refer to the paper by Bethe and Critchfield [24] and the book by Rosenfeld [25].

For the evaluation of the effective Lagrangian describing the low–energy proton–proton we assume the one–pion–exchange approximation

$$\mathcal{L}^{\mathrm{pp}}_{\mathrm{eff}}(x)=-\frac{g^2_{\pi\mathrm{NN}}}{2M^2_{\pi}}\,[\bar{p}(x)\gamma^5 p(x)]\,[\bar{p}(x)\gamma^5 p(x)]\,. \tag{76}$$

Employing the Fierz transformation we bring up the Lagrangian (76) to the form

$$\begin{aligned}\mathcal{L}^{\mathrm{pp}}_{\mathrm{eff}}(x) &= \frac{g^2_{\pi\mathrm{NN}}}{8M^2_{\pi}}\Big\{[\bar{p}(x)p^c(x)][\bar{p^c}(x)p(x)]+[\bar{p}(x)\gamma^5 p^c(x)][\bar{p^c}(x)\gamma^5 p(x)]+\\ &+[\bar{p}(x)\gamma_{\mu}\gamma^5 p^c(x)][\bar{p^c}(x)\gamma^{\mu}\gamma^5 p(x)]+\frac{1}{2}[\bar{p}(x)\sigma_{\mu\nu}p^c(x)][\bar{p^c}(x)\sigma^{\mu\nu}p(x)]\Big\}\,.\end{aligned} \tag{77}$$

The Coulomb repulsion is taken into account by the factor $\exp(-\pi\alpha/v)$ that is included in the amplitude of the low–energy p + p → D + e^+ + ν_e scattering.

The process p + p → D + e^+ + ν_e should proceed through intermediate W–boson exchange, i.e., p + p → D + W^+ → D + e^+ + ν_e. The Lagrangian describing the electroweak interactions of the W–boson with proton, neutron, positron and neutrino reads [10]

$$\mathcal{L}^{\rm W}_{\rm int}(x) = -\frac{g_{\rm W}}{2\sqrt{2}}[\bar{p}(x)\gamma^{\mu}(1 - g_{\rm A}\gamma^5)n(x) + \bar{\nu}_e(x)\gamma^{\mu}(1-\gamma^5)e(x)]\,W_{\mu}(x) + {\rm h.c.}\,. \tag{78}$$

Here $g_{\rm W}$ is the electroweak coupling constant connected with the Fermi constant $G_{\rm F} = 1.166^{-5}\,{\rm GeV}^{-2}$ and the W–boson mass $M_{\rm W}$ by the relation [10]

$$\frac{g^2_{\rm W}}{8M^2_{\rm W}} = \frac{G_{\rm F}}{\sqrt{2}}, \tag{79}$$

and $g_{\rm A} = 1.260 \pm 0.012$ is the axial–vector coupling constant [4], describing the renormalization of the weak axial–vector hadron current by strong interactions.

Only the interaction $[\bar{p}(x)\gamma_{\mu}\gamma^5 p^c(x)][\bar{p^c}(x)\gamma^{\mu}\gamma^5 p(x)]$ gives in our approach the main contribution to the amplitude of the transition p + p → D + W. The corresponding one–nucleon loop diagrams are depicted in Figs. 5.

The effective Lagrangian defined by the diagrams in Fig. 5 is defined

$$\begin{aligned}&\int d^4x\,\mathcal{L}_{\rm Fig.5}(x) = \\ &= \int d^4x \int \frac{d^4x_1\,d^4k_1}{(2\pi)^4}\,\frac{d^4x_2\,d^4k_2}{(2\pi)^4}\,[\bar{p^c}(x)\gamma^{\alpha}\gamma^5 p(x)]\,W^{\dagger}_{\mu}(x_1)\,D^{\dagger}_{\nu}(x_2)\,\times \\ &\times e^{-ik_1\cdot x_1}\,e^{-ik_2\cdot x_2}\,e^{i(k_1+k_2)\cdot x}\,i\,g_{\rm A}\,\frac{g_{\rm W}}{2\sqrt{2}}\,\frac{g^2_{\pi\rm NN}}{M^2_{\pi}}\,\frac{g_{\rm V}}{32\,\pi^2}\,\mathcal{J}^{\alpha\mu\nu}(k_1,k_2;Q)\,,\end{aligned} \tag{80}$$

where

$$\begin{aligned}&\mathcal{J}^{\alpha\mu\nu}(k_1,k_2;Q) = \\ &= \int \frac{d^4k}{\pi^2 i}{\rm tr}\left\{\gamma^{\alpha}\gamma^5\frac{1}{M_{\rm N} - \hat{k} - \hat{Q}}\gamma^{\mu}\gamma^5\frac{1}{M_{\rm N} - \hat{k} - \hat{Q} - \hat{k}_1}\,\gamma^{\nu}\,\frac{1}{M_{\rm N} - \hat{k} - \hat{Q} - \hat{k}_1 - \hat{k}_2}\right\}.\end{aligned} \tag{81}$$

The evaluation of this integral can be reduced to the evaluation of the integral (23) defining the effective Corben–Schwinger Lagrangian. As a result we get

$$\mathcal{L}_{\rm Fig.5}(x) = g_{\rm A}\,\frac{g_{\rm W}}{2\sqrt{2}}\,\frac{g^2_{\pi\rm NN}}{M^2_{\pi}}\,\frac{g_{\rm V}}{16\pi^2}\,D^{\dagger}_{\mu\nu}(x)W^{\dagger\mu}(x)\,[\bar{p^c}(x)\gamma^{\nu}\gamma^5 p(x)]\,. \tag{82}$$

The effective Lagrangian describing the low–energy process p + p → D + e^+ + ν_e reads

$$\mathcal{L}_{\rm eff}(x) = -\,g_{\rm A}\,\frac{G_{\rm F}}{\sqrt{2}}\,\frac{g^2_{\pi\rm NN}}{M^2_{\pi}}\,\frac{g_{\rm V}}{16\pi^2}\,D^{\dagger}_{\mu\nu}(x)\,j^{\mu}(x)\,[\bar{p^c}(x)\gamma^{\nu}\gamma^5 p(x)]\,, \tag{83}$$

where $j_{\mu}(x) = \bar{\nu}_e(x)\gamma_{\mu}(1-\gamma^5)e(x)$ is the leptonic electroweak current.

In the low–energy limit the amplitude of the p + p → D + e^+ + ν_e scattering is given by

$$\mathcal{M}(\mathrm{p}+\mathrm{p}\to \mathrm{D}+e^{+}+\nu_e)_{s's} = i\,e^{-\pi\alpha/v}\,g_{\mathrm{A}}\,\frac{G_{\mathrm{F}}}{\sqrt{2}}\frac{g_{\mathrm{V}}}{8\pi^2}\,\frac{g^2_{\pi\mathrm{NN}}}{M^2_{\pi}}\,M^2_{\mathrm{N}}\,\vec{e}^{\,*}_{\mathrm{D}}(\vec{q}\,)\cdot\vec{j}(\vec{k}\,)\,\varphi^{\dagger}_{s'}\,\varphi_s\,, \quad (84)$$

where $\vec{e}^{\,*}_{\mathrm{D}}(\vec{q}\,)$ is the polarization vector of the deuteron, $\vec{k}$ is the momentum of the leptonic pair, $\vec{j} = \bar{u}_{\nu_e}(p_{\nu_e})\vec{\gamma}(1-\gamma^5)v(p_e)$ is the leptonic current, $\bar{u}_{\nu_e}(p_{\nu_e})$ and $v(p_e)$ are the Dirac bispinors of the neutrino and positron, $\varphi^{\dagger}_{s'}$ and φ_s are the spinorial wave functions of the protons. The factor $\exp(-\pi\alpha/v)$ stands for the contribution of the Coulomb repulsion.

The cross section of the low–energy p + p → D + e^+ + ν_e scattering reads

$$\begin{aligned}\sigma(\mathrm{p}+\mathrm{p}\to \mathrm{D}+e^{+}+\nu_e) &= \frac{e^{-2\pi\alpha/v}}{v}\,\frac{g^2_{\mathrm{A}}Q_{\mathrm{D}}}{10240\pi^5}\,G^2_{\mathrm{F}}\left[\frac{g^2_{\pi\mathrm{NN}}}{M^2_{\pi}}\right]^2\varepsilon^5_{\mathrm{D}}\,M^3_{\mathrm{N}} = \\ &= 8.61\times 10^{-49}\,\frac{e^{-2\pi\alpha/v}}{v}\,\mathrm{cm}^2. \end{aligned} \quad (85)$$

The cross section is calculated in units of $\hbar = c = 1$.

To compare the obtained cross section with the experimental data we have to average over relative velocities of the proton–proton system with a Maxwell–Boltzmann distribution [24]

$$<\sigma(\mathrm{p}+\mathrm{p}\to \mathrm{D}+e^{+}+\nu_e)\,v> = 2.92\times 10^{-38}\int^{\infty}_{0} du u^2\,e^{-u^2-\frac{\pi\alpha}{u}\sqrt{\frac{M_{\mathrm{N}}}{k\mathrm{T}}}}\,\mathrm{cm}^3\,\mathrm{s}^{-1}\,. \quad (86)$$

where $k = 8.62\times 10^{-11}\,\mathrm{MeV\,K^{-1}}$ is the Boltzmann constant, and T is the temperature. Putting $\mathrm{T} = 1.5\times 10^7\,\mathrm{K}$ [21] we get

$$<\sigma(\mathrm{p}+\mathrm{p}\to \mathrm{D}+e^{+}+\nu_e)\,v> = 1.68\times 10^{-43}\,\mathrm{cm}^3\,\mathrm{s}^{-1}\,. \quad (87)$$

The magnitude (86) agrees qualitatively with that given by the potential model $<\sigma(\mathrm{p}+\mathrm{p}\to \mathrm{D}+e^{+}+\nu_e)\,v> = 1.19\times 10^{-43}\,\mathrm{cm}^3\,\mathrm{s}^{-1}$ [21].

We can diminish this discrepancy by using instead of the quantity $g^2_{\pi\mathrm{NN}}/M^2_{\pi}$ the observed magnitude of the s-wave scattering length of proton–proton scattering. This can be performed by the change $g^2_{\pi\mathrm{NN}}/M^2_{\pi} \to -64\pi a_{\mathrm{S}}/M_{\mathrm{N}}$ where $a_{\mathrm{S}} = -7.828\pm 0.008\,\mathrm{fm}$ [4]. This gives

$$<\sigma(\mathrm{p}+\mathrm{p}\to \mathrm{D}+e^{+}+\nu_e)\,v> = 1.25\times 10^{-43}\,\mathrm{cm}^3\,\mathrm{s}^{-1}\,. \quad (88)$$

It is seen that our prediction for the cross section of the two–proton fusion p + p → D + e^+ + ν_e agrees well with the potential approach.

Conclusion

In the present paper we have developed the relativistic field theory model of the deuteron that has been suggested in Ref. [1]. We have given the elaborate evaluation of the Corben–Schwinger and Aronson effective Lagrangians describing the interactions of the deuteron with electromagnetic fields and defining the anomalous magnetic and electric quadrupole moments of the deuteron. We have adjusted the model to the calculation of the low–energy processes such as the radiative neutron–proton capture n + p $\to$ D + γ for thermal neutrons and the reaction of the fusion of two protons p + p $\to$ D + e^+ + ν_e. This reaction is very important for the synthesis of deuterons in stars where they are destroyed by the reaction p + D $\to$ ^{3}He + γ [20].

We have shown that the model is able to give unambiguous predictions agreeing well with experimental data and predictions of the potential model for the radiative neutron–proton capture as well as for the cross section for p + p $\to$ D + e^+ + ν_e.

We have found that the coupling constant $g_{\rm T}$ does not contribute to the amplitudes of the processes under consideration. One can assume that the interaction of the deuteron with the antisymmetric tensor nucleon current is less important for the physics of low–energy interactions of the deuteron than the interaction with a vector nucleon current. If this is true, this leads to the question for the need of keeping non–zero value of the coupling constant $g_{\rm T}$. What would happen if we would put $g_{\rm T} = 0$?

Putting $g_{\rm T} = 0$ we face the only problem of the fit of the binding energy of the deuteron by the cut–off $\Lambda_{\rm D} = 45.688\,{\rm MeV}$. Using (18) at $g_{\rm T} = 0$ we obtain the best fit of the binding energy at $\Lambda_{\rm D} = 64.843\,{\rm MeV}$. Recall that we have identified $1/\Lambda_{\rm D}$ with the effective radius of the deuteron [3]. By using the new value of the cut–off we get: $r_{\rm D} = 1/\Lambda_{\rm D} = 3.043\,{\rm fm}$. This value agrees well the average value of the deuteron radius, i.e., $< r >= 3.140\,{\rm fm}$ [19].

Thus putting $g_{\rm T} = 0$ we simplify our model reducing the number of free parameters and obtain a cut–off $\Lambda_{\rm D} = 64.843\,{\rm MeV}$ that should define the average value of the deuteron radius $r_{\rm D} = 1/\Lambda_{\rm D} = 3.043\,{\rm fm}$.

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Figure Captions

- Fig. 1 One–nucleon loop diagrams contributing to the binding energy of the physical deuteron, where $n^c = C\bar{n}^T$ is the field of anti–neutron.

- Fig. 2 One–nucleon loop diagrams describing the effective Corben–Schwinger and Aronson interactions that are responsible on the anomalous magnetic and electric quadrupole moments of the physical deuteron, where $n^c = C\bar{n}^T$ is the field of anti–neutron.

- Fig. 3 The contribution of the $[\bar{p}(x)\gamma^5 n^c(x)][\bar{n^c}(x)\gamma^5 p(x)]$ to the amplitude of the radiative neutron–proton capture.

- Fig. 4 The contribution of the $[\bar{p}(x)\gamma_{\mu}\gamma^5 n^c(x)][\bar{n^c}(x)\gamma^{\mu}\gamma^5 p(x)]$ to the amplitude of the radiative neutron–proton capture.

- Fig. 5 The contribution of the $[\bar{p}(x)\gamma_{\mu}\gamma^5 p^c(x)][\bar{p^c}(x)\gamma^{\mu}\gamma^5 p(x)]$ to the amplitude of the p + p → D + e^+ + ν_e scattering.

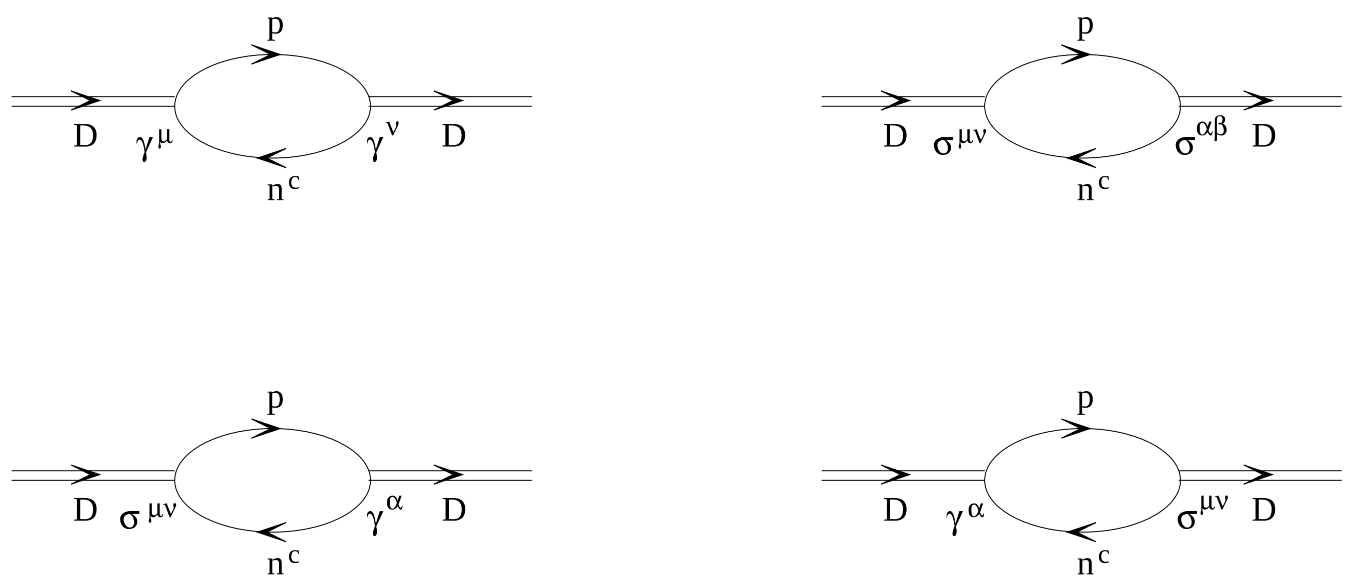


Fig. 1

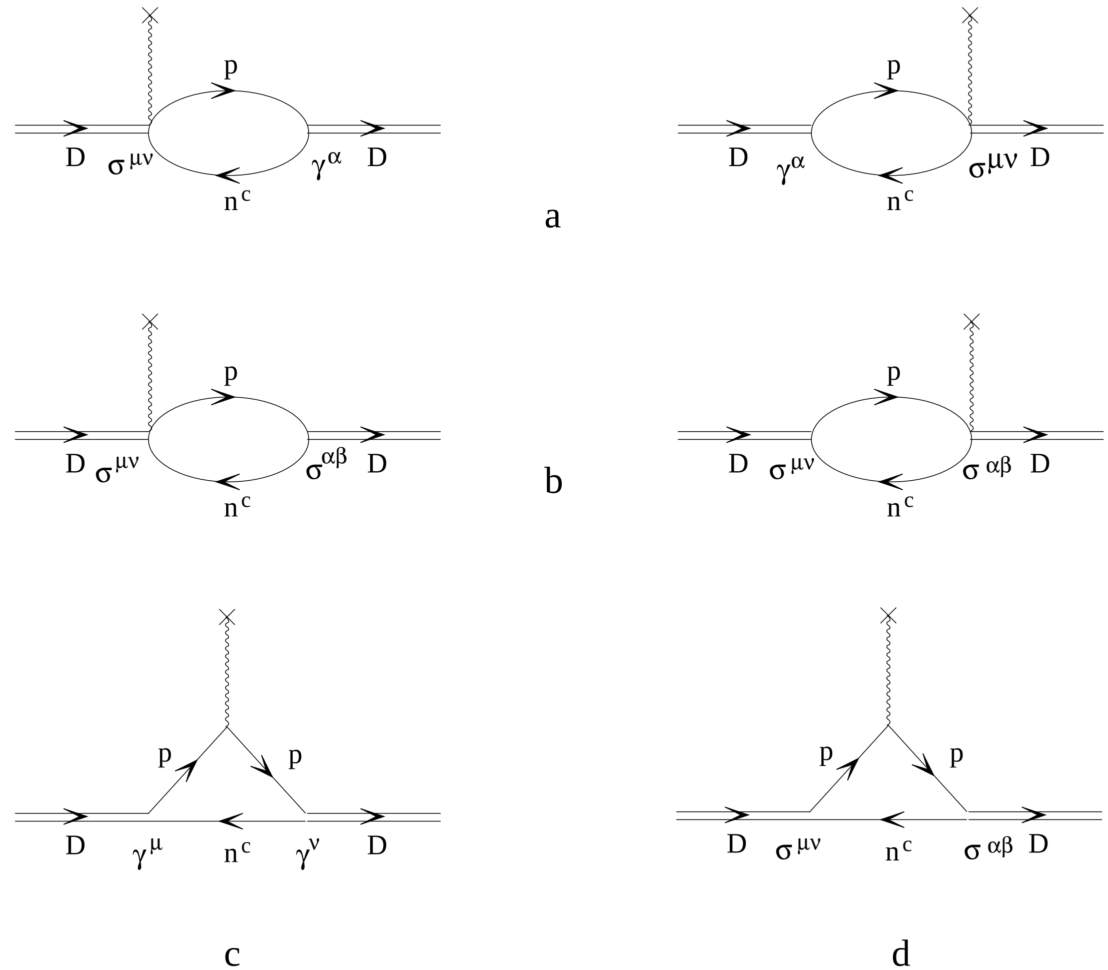
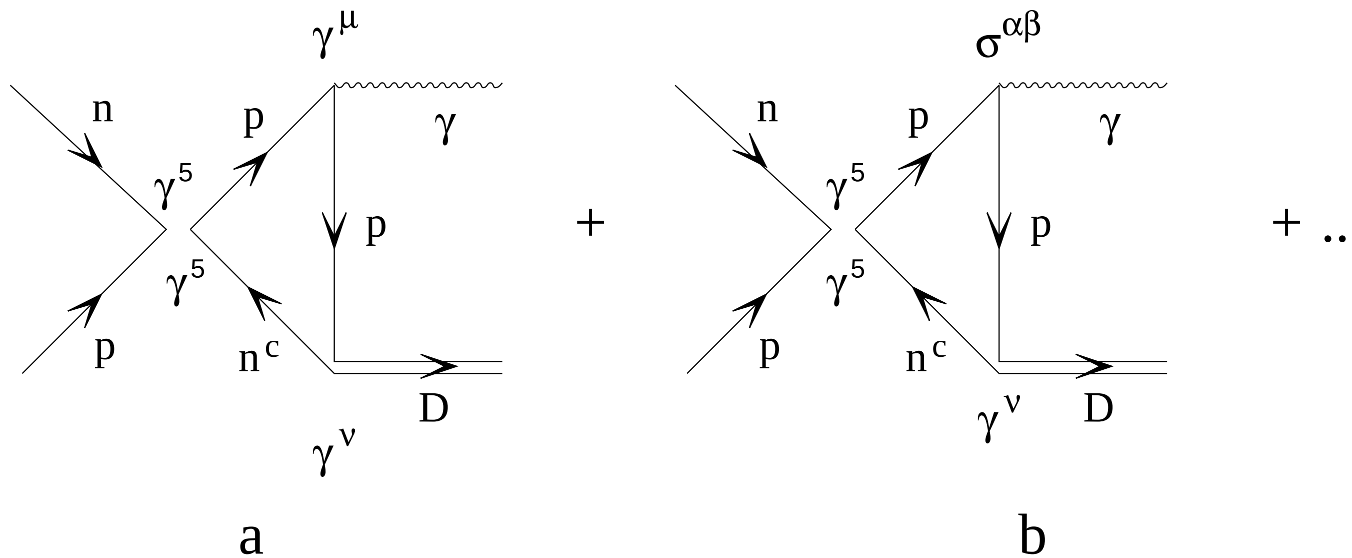


Fig. 2



γ^{μ}
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b
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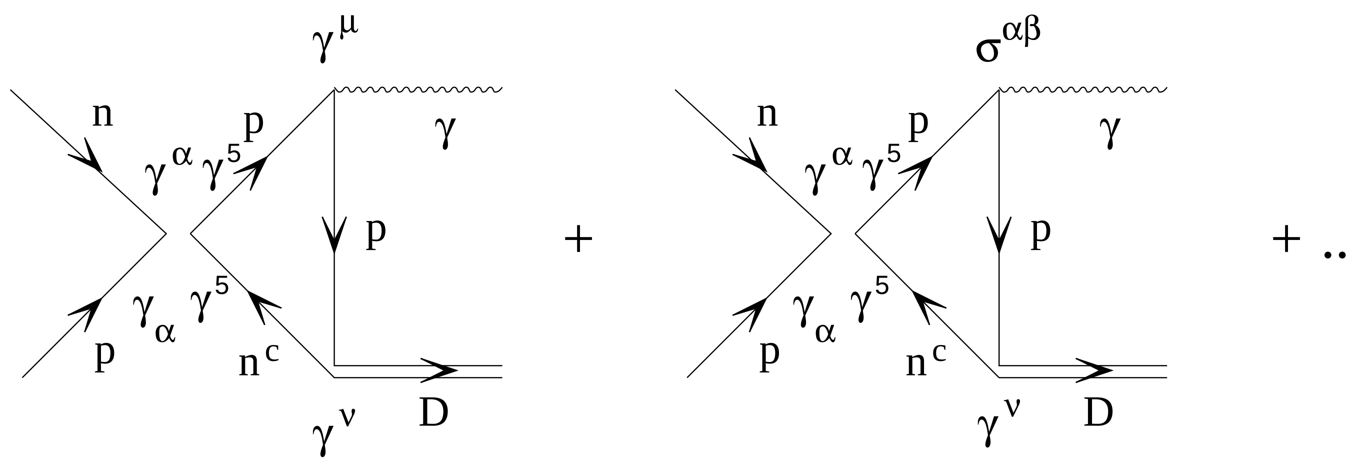
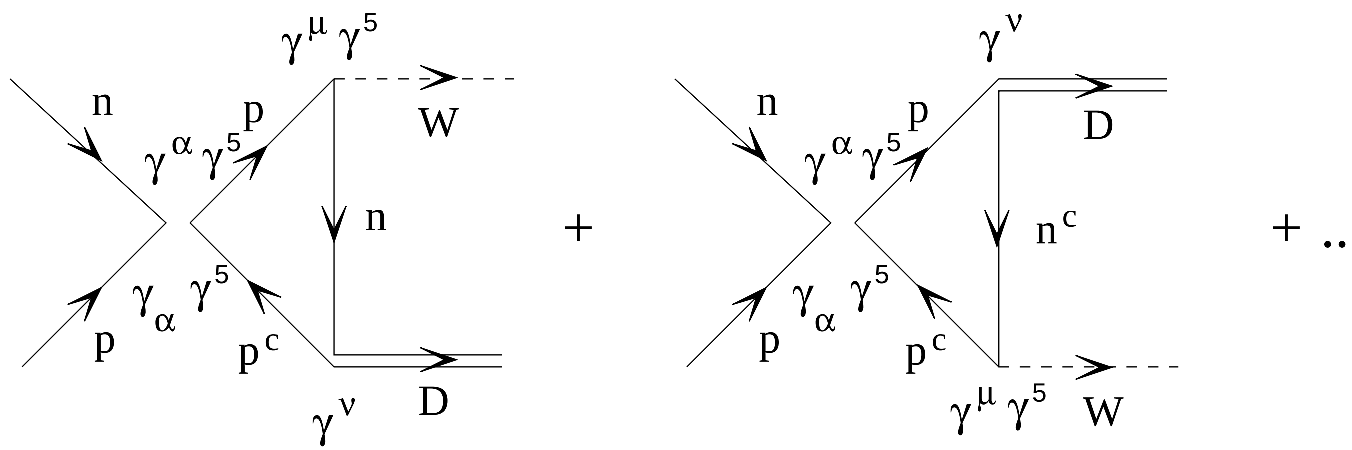
Fig. 3



a b

Fig. 4



a   b

Fig. 5