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INFINITESIMAL RIGIDITY AND FLEXIBILITY AT NON-SYMMETRIC AFFINE CONNECTION SPACE

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Abstract

At the present work we consider infinitesimal deformations

$$f : x^i \to x^i + \varepsilon z^i(x^j)$$

of a space L_N with non-symmetric affine connection L^i_{jk}. Based on the non-symmetry of the connection, we use four kinds of covariant derivative to express the Lie derivative and the deformations. Rigidity of geometric objects (connection, tensors, curvature) is defined by virtue of Lie derivative.

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Key words: Lie derivative, rigidity , flexibility, non-symmetric affine connection space, infinitesimal deformation, curvature tensors

1 Introduction

Let us consider a space L_N of non-symmetric affine connection L^i_{jk} with the torsion tensor $T^i_{jk} = L^i_{jk} - L^i_{kj}$, at local coordinates $x^i \quad (i = 1, \ldots, N)$.

Notion of non-symmetric affine connection is used for the first time, for example, in [Eis1] (Eisenhart, 1927), [Hy] (Hayden, 1932), but use of non-symmetric connection became especially actual after appearance of the works of Einstein, relating to create the Unified Field Theory (UFT).

Einstein was not satisfied with his General Theory of Relativity (GTR, 1916), and from 1923. to the end of his life (1955), he worked on various variants of UFT. This theory had the aim to unite the gravitation theory, to which is related GTR, and the theory of electromagnetism.

Introducing different variants of his UFT, Einstein ([Ein1], 1945, [Ein2], 1946) uses a *complex basic tensor* g_{ij}, with symmetric real part and antisymmetric imaginary part with respect to i, j. Beginning with 1954, in the work [Ein3] Einstein uses *real non-symmetric basic tensor*. At that sense are also his works from this field up to the end of his life, and also the last work ([Ein5], 1955).

Remark that at UFT the symmetric part $g_{\underline{ij}}$ of the basic tensor g_{ij} is related to gravitation, and antisymmetric $g_{\underset{\vee}{ij}}$ to the electromagnetism. The same is valid for $\Gamma^i_{\underline{jk}}$ and $\Gamma^i_{\underset{\vee}{jk}}$. While at the Riemannian space (the space of GTR) the connection coefficients are expressed by virtue of g_{ij}, at Einstein's works from UFT (1950-1955) the connection between these magnitudes is determined by equations

$$g_{\underset{+-}{ij};m} \equiv g_{ij,m} - \Gamma^p_{im} g_{pj} - \Gamma^p_{mj} g_{ip} = 0, \quad (g_{ij,m} = \frac{\partial g_{ij}}{\partial x^m})$$

which is a system of $4^3 = 64$ equations with 64 unknowns.

Beginning with 1951. L. P. Eisenhart was in several works occupied with problems of spaces with non-symmetric basic tensor and non-symmetric connection. In the work [Eis2], 1951, Eisenhart defines a *generalized Riemannian space* GR_N, as a space of coordinates $x^i (i = 1, \ldots, N)$ with which is associated a non-symmetric tensor g_{ij}, and the connection coefficients are defined by equations

$$\Gamma_{i.jk} = \frac{1}{2}(g_{ji,k} - g_{jk,i} + g_{ik,j}), \quad \Gamma^i_{jk} = g^{\underline{ip}} \Gamma_{p.jk}.$$

In the work [Eis3], in 1952., Eisenhart obtains two curvature tensors at GR_N, using the fact that connection coefficients are non-symmetric.

As it is known, one can assign on differentiable manifold connection coefficients $L^i_{jk}(x^1, \ldots, x^N)$ independently of basic tensor, so that generally is

$$L^i_{jk}(x) \neq L^i_{kj}(x)$$

and then we have a **non-symmetric affine connection space** L_N.

There are different definitions of the notion of infinitesimal deformations. For example, K. Yano [Yano78] a deformed magnitude $\bar{A}(\bar{x})$ observes transmitted parallel from the from the point $\bar{x}$, obtained on the base of the transformation, at original point x and formes the difference between the obtained magnitudes at x.

At infinitesimal deformations one observes unchangeability of several geometric magnitudes, i.e. one looks for the conditions to be $\bar{A} = A$. When $\bar{A} = A$, we say that the magnitude A is unchangeable at infinitesimal deformation, i.e. the mentioned magnitude is **rigid**, on the contrary that magnitude is **non-rigid or flexible**.

In the case of the rigidity of the arc, one says that we have an **infinitesimal bending** of a manifold, particulary of a surface in E^3. The case of bending is specially important intensively is studied. In this case one observes changes of others magnitudes, and then we say that they are rigid or flexible. For example, the coefficients of the first quadrat form are rigid, and of the second one are flexible in the infinitesimal bending of a surface. In this matter there are many works from the geometers ([K-F], [Ef], [Vek], [A.D.Al], [IIK-S1], [IIK-S2]).

2 Lie derivative and infinitesimal deformations

At the beginning we are giving some basic facts according to [MVS2001], [RSt63], [VMS2003],[Min73].

Definition 2.1 A transformation $f : L_N \to L_N : x = (x^1, \dots, x^N) \equiv (x^i) \to \bar{x} = (\bar{x}^1, \dots, \bar{x}^N) \equiv (\bar{x}^i)$, where

$$\bar{x} = x + z(x)\varepsilon, \tag{2.1}$$

or in local coordinates

$$\bar{x}^i = x^i + z^i(x^j)\varepsilon, \quad i, j = 1, \dots, N, \tag{2.1$'$}$$

where ε is an infinitesimal, is called **infinitesimal deformation of a space** L_N, determined by the vector field $z = (z^i)$, which is called **infinitesimal deformation field** (2.1).

We denote with (i) local coordinate system in which the point x is endowed with coordinates x^i, and the point $\bar{x}$ with the coordinates $\bar{x}^i$. We will also introduce **a new coordinate system** (i'), corresponding to the point $x = (x^i)$ new coordinates

$$x^{i'} = \bar{x}^i, \tag{2.2}$$

i.e. as new coordinates $x^{i'}$ of the point $x = (x^i)$ we choose old coordinates (at the system (i)) of the point $\bar{x} = (\bar{x}^i)$. Namely, at the system (i') is $x = (x^{i'}) \underset{(2.2)}{=} (\bar{x}^i)$, where $\underset{(2.2)}{=}$ denotes "equal according to (2.2)".

Definition 2.2. Coordinate transformation we get based on punctual transformation $f : x \to \bar{x}$, getting for the new coordinates of the point x the old coordinates of its transform $\bar{x}$, is called **dragging along punctual transformation**. New coordinates $x^{i'} = \bar{x}^i$ of the point $\bar{x}$ are called **dragged along coordinates.**

In the case of infinitesimal deformation (2.1$'$) coordinate transformation

$$x^{i'} = \bar{x}^i = x^i + z^i(x^1, \dots, x^N)\varepsilon \tag{2.3}$$

is called **dragging along** by $z^i\varepsilon$.

Let us consider a geometric object $\mathcal{A}$ with respect to the system (i) at the point $x = (x^i) \in L_N$, denoting this with $\mathcal{A}(i, x)$.

Definition 2.3 The point $\bar{x}$ is said to be **deformed point** of the point x, if (2.1) holds. Geometric object $\bar{\mathcal{A}}(i, x)$ is **deformed object** $\mathcal{A}(i, x)$ with respect to deformation (2.1), if its value at the system (i'), at the point x is equal to the value of the object $\mathcal{A}$ at the system (i) at the point $\bar{x}$, i.e. if

$$\bar{\mathcal{A}}(i', x) = \mathcal{A}(i, \bar{x}). \tag{2.4}$$

Remark 2.1. In this study of infinitesimal deformations according to $(2.1')$ quantities of an order higher then the first with respect to ε are neglected.

We will now define some important notions of the theory of infinitesimal deformations, following from (2.1): Lie differential and Lie derivative, and in further considerations we will find them for some geometric objects.

Definition 2.4 The magnitude $\mathcal{DA}$, the difference between deformed object $\bar{\mathcal{A}}$ and initial object $\mathcal{A}$ at the same coordinate system and at the same point with respect to $(2.1')$, i.e.

$$\mathcal{DA} = \bar{\mathcal{A}}(i, x) - \mathcal{A}(i, x), \tag{2.5}$$

is called **Lie difference (Lie differential)**, and the magnitude

$$\mathcal{L}_z\mathcal{A} = \lim_{\varepsilon\to 0}\frac{\mathcal{DA}}{\varepsilon} = \lim_{\varepsilon\to 0}\frac{\bar{\mathcal{A}}(i, x) - \mathcal{A}(i, x)}{\varepsilon} \tag{2.5'}$$

is **Lie derivative** of geometric object $\mathcal{A}(i, x)$ with respect to the vector field $z = (z^i(x^j))$.

Using the relation (2.5), for deformed object $\bar{\mathcal{A}}(i, x)$ we have

$$\bar{\mathcal{A}}(i, x) = \mathcal{A}(i, x) + \mathcal{DA}, \tag{2.5''}$$

and thus we can express $\bar{\mathcal{A}}$, finding previously $\mathcal{DA}$.

Definition 2.5 Geometric object $\mathcal{A}$ is **rigid** with respect to infinitesimal deformations (2.1) if there exist nontrivial field of deformations z for which $\mathcal{L}_z A = 0$, i.e. $\bar{\mathcal{A}} = \mathcal{A}$. Geometric object $\mathcal{A}$ is **flexible** if $\mathcal{L}_z A \neq 0$ for non-trivial field of deformation z.

The known main cases are:

2.1. According to (1.5) we have $\mathcal{D}x^i = \bar{x}^i - x^i$, i.e. for the **coordinates** we have

$$\mathcal{D}x^i = z^i(x^j)\varepsilon, \tag{2.6}$$

from where

$$\mathcal{L}_z x^i = z^i(x^j). \tag{2.6'}$$

2.2. For the **scalar function** $\varphi(x) \equiv \varphi(x^1, \dots, x^N)$ we have

$$\mathcal{D}\varphi(x) = \varphi_{,p} z^p(x)\varepsilon = \mathcal{L}_z\varphi(x)\varepsilon, \quad (\varphi_{,p} = \partial\varphi/\partial x^p), \tag{2.7}$$

i.e. Lie derivative of the scalar function is derivative of this function in direction of the vector field z.

2.3. For a **tensor of the kind** (u, v) we get

$$\begin{aligned}\mathcal{D}t^{i_1\dots i_u}_{j_1\dots j_v} &= [t^{i_1\dots i_u}_{j_1\dots j_v,p} z^p - \sum_{\alpha=1}^{u} z^{i_\alpha}_{,p} \binom{p}{i_\alpha} t^{i_1\dots i_u}_{j_1\dots j_v} + \sum_{\beta=1}^{v} z^p_{,j_\beta} \binom{j_\beta}{p} t^{i_1\dots i_u}_{j_1\dots j_v}]\varepsilon \\ &= \mathcal{L}_z t^{i_1\dots i_u}_{j_1\dots j_v}\varepsilon,\end{aligned} \tag{2.8}$$

where we denoted

$$\binom{p}{i_\alpha} t^{i_1\dots i_u}_{j_1\dots j_v} = t^{i_1\dots i_{\alpha-1} p i_{\alpha+1}\dots i_u}_{j_1\dots j_v}, \quad \binom{j_\beta}{p} t^{i_1\dots i_u}_{j_1\dots j_v} = t^{i_1\dots i_u}_{j_1\dots j_{\beta-1} p j_{\beta+1}\dots j_v}. \tag{2.9}$$

2.4. For the **vector** dx^i we have

$$\mathcal{D}(dx^i) = \mathcal{L}_z(dx^i) = 0. \tag{2.10}$$

2.5. In the same way, as for the tensors, for the **connection coefficients** we have

$$\mathcal{D}L^i_{jk} = (L^i_{jk,p} z^p + z^i_{,jk} - z^i_{,p} L^p_{jk} + z^p_{,j} L^i_{pk} + z^p_{,k} L^i_{jp})\varepsilon = \mathcal{L}_z L^i_{jk}\varepsilon. \tag{2.11}$$

Definition 2.6 Infinitesimal deformation (2.1) is **affine colineation** or **projective infinitesimal deformation** if the Lie derivative of connection coefficients is zero ($\mathcal{L}_z L^i_{jk} = 0$).

In case of infinitesimal deformation with zero Lie derivative of curvature ($\mathcal{L}R^i_{jk} = 0$) we have **curvature colineation**.

In the case of $\mathcal{L}_z g_{ij} = 0$, infinitesimal deformation is **infinitesimal motion**, and for $\mathcal{L}_z g_{ij} = \sigma(x) g_{ij}$, we have **infinitesimal conformal deformation** or in the case $\sigma(x) = const.$- homothety.

3 The Lie derivative and rigidity of a tensor

3.1. Definition 3.1 A **tensor** $t^{i_1\dots i_u}_{j_1\dots j_v}$ **is rigid** with respect to a given infinitesimal deformation field z if $\mathcal{L}_z t^{i_1\dots i_u}_{j_1\dots j_v}=0$

Considering of the rigidity of a tensor requires consideration of their Lie derivatives. Based on non-symmetry of the connection, at L_N we can consider two types of covariant derivatives for a vector and four types for general tensor. So, denoting by $\underset{\theta}{|}(\theta=1,\dots,4)$ derivative of the type θ, we have ([Min73], [Min77], [Min79]):

$$(3.1a-d)\quad t^{i_1\dots i_u}_{j_1\dots j_v\underset{\substack{1\\2\\3\\4}}{|}m}=t^{i_1\dots i_u}_{j_1\dots j_v,m}+\sum_{\alpha=1}^{u}L^{i_\alpha}_{\substack{pm\\mp\\pm\\mp}}\binom{p}{i_\alpha}t^{i_1\dots i_u}_{j_1\dots j_v}-\sum_{\beta=1}^{v}L^{p}_{\substack{j_\beta m\\mj_\beta\\mj_\beta\\j_\beta m}}\binom{j_\beta}{p}t^{i_1\dots i_u}_{j_1\dots j_v}.$$

Generally, the next theorem is in the force:

Theorem 3.1 *Lie derivative of a tensor* $t^{i_1\dots i_u}_{j_1\dots j_v}$ *of the type* (u,v) *is a tensor of the same type and can be presented in the following four ways*

$$(3.2a,b)\quad \begin{aligned}&\mathcal{L}_z t^{i_1\dots i_u}_{j_1\dots j_v}=\underset{\theta}{\mathcal{L}}_z t^{i_1\dots i_u}_{j_1\dots j_v}\\ &\equiv t^{i_1\dots i_u}_{j_1\dots j_v\underset{\theta}{|}p}z^p-\sum_{\alpha=1}^{u}z^{i_\alpha}_{\ \underset{\theta}{|}p}\binom{p}{i_\alpha}t^{\cdots}_{\cdots}+\sum_{\beta=1}^{v}z^{p}_{\ \underset{\theta}{|}j_\beta}\binom{j_\beta}{p}t^{\cdots}_{\cdots}\\ &+(-1)^{\theta-1}\sum_{\alpha=1}^{u}T^{i_\alpha}_{ps}\binom{s}{i_\alpha}t^{\cdots}_{\cdots}z^p+(-1)^{\theta-1}\sum_{\beta=1}^{v}T^{s}_{j_\beta p}\binom{j_\beta}{s}t^{\cdots}_{\cdots}z^p,\\ &\theta=1,2;\end{aligned}$$

$$(3.3a,b)\quad \begin{aligned}&\mathcal{L}_z t^{i_1\dots i_u}_{j_1\dots j_v}=\underset{\theta}{\mathcal{L}}_z t^{i_1\dots i_u}_{j_1\dots j_v}\\ &\equiv t^{i_1\dots i_u}_{j_1\dots j_v\underset{\theta}{|}p}z^p-\sum_{\alpha=1}^{u}z^{i_\alpha}_{\ \underset{\theta}{|}p}\binom{p}{i_\alpha}t^{\cdots}_{\cdots}+\sum_{\beta=1}^{v}z^{p}_{\ \underset{\theta}{|}j_\beta}\binom{j_\beta}{p}t^{\cdots}_{\cdots}\\ &+(-1)^{\theta-1}\sum_{\alpha=1}^{u}T^{i_\alpha}_{ps}\binom{s}{i_\alpha}t^{\cdots}_{\cdots}z^p,\quad \theta=3,4,\end{aligned}$$

where $\underset{\theta}{\mathcal{L}}_z$ *denotes that Lie derivative* $\mathcal{L}_z$ *is expressed by covariant derivative of the type* θ, $(\underset{\theta}{|}),\quad \theta=1,\dots,4.\Box$

The proof is given in [VMS 2003].

Corollary 3.1. For the space $\overset{0}{L}_N$ of **symmetric connection** $\overset{0}{L}{}^i_{jk}\quad (T^i_{jk}=0)$

we have

$$\begin{aligned}\mathcal{L}_z t^{i_1\dots i_u}_{j_1\dots j_v} &= \overset{0}{\mathcal{L}}_z t^{i_1\dots i_u}_{j_1\dots j_v}\\ &\equiv t^{i_1\dots i_u}_{j_1\dots j_v;p} z^p - \sum_{\alpha=1}^{u} z^{i_\alpha}_{;p}\binom{p}{i_\alpha} t^{i_1\dots i_u}_{j_1\dots j_v} + \sum_{\beta=1}^{v} z^p_{;j_\beta}\binom{j_\beta}{p} t^{i_1\dots i_u}_{j_1\dots j_v},\end{aligned} \tag{3.4}$$

because in that case all 4 types of covariant derivatives reduce to one, which we denote by semicolon (;).

3.2. So, Lie derivative of a tensor at a space of symmetric affine connection can be obtained as a special case from the formulae for Lie derivative at a space of non-symmetric affine connection. We will investigate the way of **presenting the Lie derivative of a tensor by covariant derivative with respect to symmetrical part $\underset{0}{L}{}^i_{jk}$ of non-symmetric connection L^i_{jk}**. Let us consider a space L_N of non-symmetric affine connection L^i_{jk} and let be

$$\underset{0}{L}{}^i_{jk} = \frac{1}{2}(L^i_{jk} + L^i_{kj}), \quad T^i_{jk} = L^i_{jk} - L^i_{kj}. \tag{3.5}$$

Then

$$L^i_{jk} = \underset{0}{L}{}^i_{jk} + \frac{1}{2} T^i_{jk}. \tag{3.6}$$

The magnitudes $\underset{0}{L}{}^i_{jk}$ are the coefficients of symmetric connection associated to the connection L^i_{jk}, and T^i_{jk} are the components of torsion tensor of connection L^i_{jk}. If we denote with $\underset{0}{\mathcal{L}}_z t^{i_1\dots i_u}_{j_1\dots j_v}$ the expression as on the right side at (3.4), but formed by means of $\underset{0}{L}{}^i_{jk}$from (3.5) (instead of $\overset{0}{L}{}^i_{jk}$), we have the next theorem [VMS 2003]:

Theorem 3.2 *In the non-symmetric connection space L_N Lie derivative of tensor $t^{i_1\dots i_u}_{j_1\dots j_v}$ can be expressed as*

$$\begin{aligned}\mathcal{L}_z t^{i_1\dots i_u}_{j_1\dots j_v} &= \underset{\theta}{\mathcal{L}}_z t^{i_1\dots i_u}_{j_1\dots j_v} = \underset{0}{\mathcal{L}}_z t^{i_1\dots i_u}_{j_1\dots j_v}\\ &\equiv t^{i_1\dots i_u}_{j_1\dots j_v;p} z^p - \sum_{\alpha=1}^{u} z^{i_\alpha}_{;p}\binom{p}{i_\alpha} t^{i_1\dots i_u}_{j_1\dots j_v} + \sum_{\beta=1}^{v} z^p_{;j_\beta}\binom{j_\beta}{p} t^{i_1\dots i_u}_{j_1\dots j_v},\end{aligned} \tag{3.7}$$

where the semicolon (;) *denotes covariant derivative with respect to symmetric part $\underset{0}{L}{}^i_{jk}$ of the connection L^i_{jk}.* □

3.3 Comparing (3.2, 3) and (3.7), we can see that Lie derivative of a tensor at L_N can simpler be given by means of (3.7), i.e. with respect to covariant derivative formed by symmetrical part $\underset{0}{L}{}^i_{jk}$ of non-symmetrical connection L^i_{jk}.

If we use **at the same time different kinds of covariant derivative** at the right side at $(3.2, 3)$ with respect to L^i_{jk}, we can write this equations in the more condensed form (analogously to (3.7)). In connection with this the next theorem is in the force:

Theorem 3.3 *The Lie derivative of a tensor of the type* (u, v) *can be expressed using covariant derivatives with respect to non-symmetric connection* L^i_{jk} *in the next way*

$$(3.8a-d)\quad \mathcal{L}_z t^{i_1\ldots i_u}_{j_1\ldots j_v} = t^{i_1\ldots i_u}_{j_1\ldots j_v\underset{\lambda}{|}p} z^p - \sum_{\alpha=1}^{u} z^{i_\alpha}_{\underset{\mu}{|}p} \binom{p}{i_\alpha} t^{i_1\ldots i_u}_{j_1\ldots j_v} + \sum_{\beta=1}^{v} z^p_{\underset{\nu}{|}j_\beta} \binom{j_\beta}{p} t^{i_1\ldots i_u}_{j_1\ldots j_v},$$

where $(\lambda, \mu, \nu) \in \{(1, 2, 2), (2, 1, 1), (3, 4, 3), (4, 3, 4)\}$.

Proof: We will prove only the second case, the others can be proved analogously. Let us start from (3.2b). We have

$$z^{i_\alpha}_{\underset{2}{|}p} \binom{p}{i_\alpha} t^{i_1\ldots i_u}_{j_1\ldots j_v} = (z^{i_\alpha}_{,p} + L^{i_\alpha}_{ps} z^s) \binom{p}{i_\alpha} t^{i_1\ldots i_u}_{j_1\ldots j_v} = (z^{i_\alpha}_{,p} + L^{i_\alpha}_{sp} z^s - L^{i_\alpha}_{sp} z^s$$
$$+ L^{i_\alpha}_{ps} z^s) \binom{p}{i_\alpha} t^{i_1\ldots i_u}_{j_1\ldots j_v} = z^{i_\alpha}_{\underset{1}{|}p} \binom{p}{i_\alpha} t^{i_1\ldots i_u}_{j_1\ldots j_v} + T^{i_\alpha}_{ps} z^s \binom{p}{i_\alpha} t^{i_1\ldots i_u}_{j_1\ldots j_v}$$

and analogously

$$z^p_{\underset{2}{|}j_\beta} \binom{j_\beta}{p} t^{i_1\ldots i_u}_{j_1\ldots j_v} = z^p_{\underset{1}{|}j_\beta} \binom{j_\beta}{p} t^{i_1\ldots i_u}_{j_1\ldots j_v} + T^p_{s j_\beta} \binom{j_\beta}{p} t^{i_1\ldots i_u}_{j_1\ldots j_v} z^s.$$

Substituting this at (3.2b) it follows that

$$\mathcal{L}_z t^{i_1\ldots i_u}_{j_1\ldots j_v}$$
$$= t^{i_1\ldots i_u}_{j_1\ldots j_v\underset{2}{|}p} z^p - \sum_{\alpha=1}^{u} [z^{i_\alpha}_{\underset{1}{|}p} \binom{p}{i_\alpha} t^{i_1\ldots i_u}_{j_1\ldots j_v} + T^{i_\alpha}_{ps} z^s \binom{p}{i_\alpha} t^{i_1\ldots i_u}_{j_1\ldots j_v}] + \sum_{\beta=1}^{v} [z^p_{\underset{1}{|}j_\beta} \binom{j_\beta}{p} t^{i_1\ldots i_u}_{j_1\ldots j_v}$$
$$- T^s_{p j_\beta} \binom{j_\beta}{s} t^{i_1\ldots i_u}_{j_1\ldots j_v} z^p] + \sum_{\alpha=1}^{u} T^{i_\alpha}_{sp} \binom{s}{i_\alpha} t^{i_1\ldots i_u}_{j_1\ldots j_v} z^p - \sum_{\beta=1}^{v} T^s_{j_\beta p} \binom{j_\beta}{s} t^{i_1\ldots i_u}_{j_1\ldots j_v} z^p,$$

from where we obtain (3.8) for $(\lambda, \mu, \nu) = (2, 1, 1)$.□

¿From exposed the next theorem is valid:

Theorem 3.4 *A necessary and sufficient condition for the tensor field* $t^{i_1\cdots_u}_{j_1\ldots j_v}$ *to be rigid with respect to infinitesimal deformations* (2.1) *is the annulment of the right side in any of the equations* $(2.8), (3.2, 3, 4, 7, 8)$. □

For the example, based on (3.7) it follows that a necessary and sufficient condition the tensor field $t^{i_1\cdots_u}_{j_1\ldots j_v}$ to be rigid is

$$(3.9)\qquad t^{i_1\cdots_u}_{j_1\ldots j_v;p} z^p = \sum_{\alpha=1}^{u} z^{i_\alpha}_{;p} \binom{p}{i_\alpha} t^{i_1\cdots_u}_{j_1\ldots j_v} - \sum_{\beta=1}^{v} z^p_{;j_\beta} \binom{j_\beta}{p} t^{i_1\cdots_u}_{j_1\ldots j_v},$$

what gives a manner for an expression of covariant derivative in the direction of deformation vector z, that derivative being taken by virtue of symmetric part $\underset{0}{L}$ of the connection L.

4 The Lie derivative and the rigidity of the connection

4.1 On the base of (2.11) for the Lie derivative of the connection we have

$$\mathcal{L}_z L^i_{jk} = z^i_{,jk} + L^i_{jk,p} z^p - z^i_{,p} L^p_{jk} + z^p_{,j} L^i_{pk} + z^p_{,k} L^i_{jp}. \tag{4.1}$$

As it was proved at $[MVS2001]$ Lie derivative can be written in the next way

$$\mathcal{L}_z L^i_{jk} = z^i_{\underset{1}{|}jk} + \underset{1}{R}^i_{jkp} z^p + T^i_{jp,k} z^p + L^s_{jk} T^i_{ps} z^p + L^i_{sk} T^s_{jp} z^p + T^i_{jp} z^p_{,k}, \tag{4.2}$$

$$\mathcal{L}_z L^i_{jk} = \underset{1}{\mathcal{L}}_z L^i_{jk} \equiv z^i_{\underset{1}{|}jk} + \underset{1}{R}^i_{jkp} z^p + (T^i_{jp} z^p)_{\underset{1}{|}k}, \tag{4.2$'$}$$

$$\begin{aligned} \mathcal{L}_z L^i_{jk} = \underset{2}{\mathcal{L}}_z L^i_{jk} &= z^i_{\underset{2}{|}jk} + \underset{2}{R}^i_{jkp} z^p + T^i_{pj\underset{2}{|}k} z^p + T^i_{pk} z^p_{\underset{2}{|}j} \\ &+ T^p_{kj} z^i_{\underset{2}{|}p} + T^i_{jk\underset{2}{|}p} z^p + (T^i_{sj} T^s_{kp} + T^i_{sk} T^s_{pj} + T^i_{sp} T^s_{jk}) z^p, \end{aligned} \tag{4.3}$$

$$\mathcal{L}_z L^i_{jk} = \underset{3}{\mathcal{L}}_z L^i_{jk} \equiv z^i_{\underset{3}{|}jk} + \underset{3}{R}^i_{jkp} z^p - T^p_{jk} z^i_{\underset{3}{|}p} + T^i_{jp} z^p_{\underset{3}{|}k}, \tag{4.4}$$

$$\mathcal{L}_z L^i_{jk} = \underset{4}{\mathcal{L}}_z L^i_{jk} \equiv z^i_{\underset{4}{|}jk} + \underset{4}{R}^i_{jkp} z^p + (T^i_{pj\underset{4}{|}k} + T^i_{sj} T^s_{pk} + T^i_{sk} T^s_{pj}) z^p + T^i_{pk} z^p_{\underset{4}{|}j}, \tag{4.5}$$

where [Min73], [Min77], [Min79]

$$\underset{1}{R}^i_{jkp} = L^i_{jk,p} - L^i_{jp,k} + L^s_{jk} L^i_{sp} - L^s_{jp} L^i_{sk} \tag{4.6}$$

$$\underset{2}{R}^i_{jkp} = L^i_{kj,p} - L^i_{pj,k} + L^s_{kj} L^i_{ps} - L^s_{pj} L^i_{ks} \tag{4.7}$$

$$\underset{3}{R}^i_{jkp} = L^i_{jk,p} - L^i_{pj,k} + L^s_{jk} L^i_{ps} - L^s_{pj} L^i_{sk} + L^s_{pk} T^i_{sj} \tag{4.8}$$

$$\underset{4}{R}^i_{jkp} = L^i_{jk,p} - L^i_{pj,k} + L^s_{jk} L^i_{ps} - L^s_{pj} L^i_{sk} + L^s_{kp} T^i_{sj} \tag{4.9}$$

are curvature tensors of the space L_N.

4.2. We have proved at Theorem 3.3. that the Lie derivative of a tensor can be expressed more concise by using several types of covariant derivatives at L_N simultaneously. It is the same case for the Lie derivative of the connection. Namely, the next theorem is in force [VMS 2003]:

Theorem 4.1 *The Lie derivative of non-symmetric connection* L^i_{jk} *is a tensor of the type* $(1,2)$ *and can be expressed with respect to covariant derivatives by equations* $(4.2.-5)$, *as well as by*

$$\mathcal{L}_z L^i_{jk} = z^i_{\underset{2}{|}j\underset{1}{|}k} + \underset{1}{R}{}^i_{jkp} z^p. \tag{4.10}$$

$$\mathcal{L}_z L^i_{jk} = z^i_{\underset{1}{|}k\underset{2}{|}j} + \underset{2}{R}{}^i_{kjp} z^p. \tag{4.11}$$

□

4.3. Comparing (3.4) and (3.7), we can see that the Lie derivative of a tensor at the space $\overset{0}{L}_N$ of symmetric connection $\overset{0}{L}{}^i_{jk}$ and Lie derivative at the space L_N of non-symmetric connection L^i_{jk} are expressed in the same way: with respect to given symmetric connection $\overset{0}{L}{}^i_{jk}$ in the first case, and in the second with respect to the symmetric part $\underset{0}{L}{}^i_{jk}$ of non-symmetric connection L^i_{jk}.

An analogous problem can be considered in the case of a connection (that is not a tensor). At the space $\overset{0}{L}_N$ of symmetric connection $\overset{0}{L}{}^i_{jk}$, by reason of $T^i_{jk} = 0$ all the cases of expresses for the Lie derivative considered before, reduce to

$$\mathcal{L}_z \overset{0}{L}{}^i_{jk} = z^i_{;jk} + \overset{0}{R}{}^i_{jkp} z^p, \tag{4.12}$$

where $\overset{0}{R}{}^i_{jkp}$ is curvature tensor, generated by $\overset{0}{L}{}^i_{jk}$. Let us examine a space L_N of non-symmetric affine connection L^i_{jk}, where $\underset{0}{L}{}^i_{jk}, T^i_{jk}$ are given by (3.5).

The main purpose is to express $\mathcal{L}_z L^i_{jk}$ $(4.2')$ by covariant derivatives with respect to $\underset{0}{L}{}^i_{jk}$, and $\underset{1}{R}{}^i_{jkp}$ by $\underset{0}{R}{}^i_{jkp}$, formed by $\underset{0}{L}{}^i_{jk}$. We have [VMS 2003]:

$$\mathcal{L}_z L^i_{jk} = \underset{0}{\mathcal{L}}_z L^i_{jk} \equiv z^i_{;jk} + \underset{0}{R}{}^i_{jkp} z^p + \frac{1}{2} \underset{0}{\mathcal{L}}_z T^i_{jk}. \tag{4.13}$$

$$\mathcal{L}_z \underset{0}{L}{}^i_{jk} = \underset{0}{\mathcal{L}}_z \underset{0}{L}{}^i_{jk} = z^i_{;jk} + \underset{0}{R}{}^i_{jkp} z^p. \tag{4.14}$$

Based on the pointed out facts it follows

Theorem 4.2 *Lie derivative of non-symmetric connection* L^i_{jk} *can be given by the equation* (4.13), *where covariant derivative denoted by ; and curvature tensor*

$\underset{0}{R}{}^i_{jkp}$ *are formed with respect to symmetric part* $\underset{0}{L}{}^i_{jk}$ *of the connection* L^i_{jk}*, and* $\underset{0}{\mathcal{L}}_z T^i_{jk}$ *is expressed according to* (3.7) *with respect to* $\underset{0}{L}{}^i_{jk}$*. The Lie derivative of symmetric part of connection is given according to* (4.14) *i.e. it is in the same form as for symmetric connection (equation* (4.12)*).*□

In relation with the rigidity of the connection, from exposed above it follows
Theorem 4.3*For the rigidity of a non-symmetric connection* L^i_{jk} *with respect to infinitesimal deformation* (2.1)*, a necessary and sufficient condition is an annulment of the right side in any of the equations* $(4.1-5, 10, 11)$. □

All these conditions are equivalent as they signify the annulment of the Lie derivative of the connection. E.g., from (4.10) the cited rigidity condition reduces to

$$z^i_{\underset{2}{|}j\underset{1}{|}k} = \underset{1}{R}{}^i_{jpk} z^p.$$

In the case of symmetric connection this condition reduces to

$$z^i_{;jk} = R^i_{jpk} z^p,$$

where is taken into consideration the skew symmetry of curvature tensor with respect to the last two indices.

5 Infinitesimal deformation of curvature tensors

At [Min 73], [Min 77], [Min 79] are obtained at all 12 curvature tensors in L_N, and at [Min 79] is proved that 5 of them are independent. They are $(4.6-9)$ and

$$\begin{aligned}(5.1)\qquad \underset{5}{R}{}^i_{jmn} = \frac{1}{2}(&L^i_{jm,n} + L^i_{mj,n} - L^i_{jn,m} - L^i_{nj,m} \\ &+ L^p_{jm}L^i_{pn} + L^p_{mj}L^i_{np} - L^p_{jn}L^i_{mp} - L^p_{nj}L^i_{pm}).\end{aligned}$$

Denoting by semicolon (;) covariant derivative with respect to symmetric connection $\underset{0}{L}$, then according to [Min79], we have

$$(5.2)\qquad \underset{1}{R}{}^i_{jmn} = \underset{o}{R}{}^i_{jmn} + \frac{1}{2}T^i_{jm;n} - \frac{1}{2}T^i_{jn;m} + \frac{1}{4}T^p_{jm}T^i_{pn} - \frac{1}{4}T^p_{jn}T^i_{pm},$$

$$(5.3)\qquad \underset{2}{R}{}^i_{jmn} = \underset{0}{R}{}^i_{jmn} - \frac{1}{2}T^i_{jm;n} + \frac{1}{2}T^i_{jn;m} + \frac{1}{4}T^p_{jm}T^i_{pn} - \frac{1}{4}T^p_{jn}T^i_{pm},$$

$$(5.4)\quad \underset{3}{R}{}^i_{jmn} = \underset{0}{R}{}^i_{jmn} + \frac{1}{2}T^i_{jm;n} + \frac{1}{2}T^i_{jn;m} - \frac{1}{4}T^p_{jm}T^i_{pn} + \frac{1}{4}T^p_{jn}T^i_{pm} - \frac{1}{2}T^p_{mn}T^i_{pj},$$

$$(5.5)\quad \underset{4}{R}{}^i_{jmn} = \underset{0}{R}{}^i_{jmn} + \frac{1}{2}T^i_{jm;n} + \frac{1}{2}T^i_{jn;m} - \frac{1}{4}T^p_{jm}T^i_{pn} + \frac{1}{4}T^p_{jn}T^i_{pm} + \frac{1}{2}T^p_{mn}T^i_{pj}$$

$$(5.6)\qquad \underset{5}{R}{}^i_{jmn} = \underset{0}{R}{}^i_{jmn} + \frac{1}{4}T^p_{jm}T^i_{pn} + \frac{1}{4}T^p_{jn}T^i_{pm}.$$

At $(5.2-6)$ all the addends at the right side are tensors.

We will consider infinitesimal deformations of cited five curvature tensors.

5.1 Infinitesimal deformation of curvature tensor $\underset{1}{R}$

According to (5.1) for deformed first curvature tensor at L_N i.e. for $\underset{1}{\bar{R}}$, we have

$$(5.7)\qquad \underset{1}{\bar{R}}{}^i_{jmn}(x) = \bar{L}^i_{jm,n} - \bar{L}^i_{jn,m} + \bar{L}^p_{jm}\bar{L}^i_{pn} - \bar{L}^p_{jn}\bar{L}^i_{pm},$$

and with respect to $\bar{L}^i_{jm}(x) = L^i_{jm}(x) + \mathcal{D}L^i_{jm}$, one obtains

$$\begin{aligned}\underset{1}{\bar{R}}{}^i_{jmn} &= (L^i_{jm} + \mathcal{D}L^i_{jm})_{,n} - (L^i_{jn} + \mathcal{D}L^i_{jn})_{,m}\\ &+ (L^p_{jm} + \mathcal{D}L^p_{jm})(L^i_{pn} + \mathcal{D}L^i_{pn}) - (L^p_{jn} + \mathcal{D}L^p_{jn})(L^i_{pm} + \mathcal{D}L^i_{pm}).\end{aligned}$$

Developing this and omitting the members of the form $\mathcal{D}L^{\cdots}_{\cdots}\cdot\mathcal{D}L^{\cdots}_{\cdots}$, as they include $(\varepsilon)^2$, we have

$$(5.8)\qquad \begin{aligned}\underset{1}{\bar{R}}{}^i_{jmn} &= L^i_{jm,n} + (\mathcal{D}L^i_{jm})_{,n} - L^i_{jn,m} - (\mathcal{D}L^i_{jn})_{,m} + L^p_{jm}L^i_{pn}\\ &+ L^p_{jm}\mathcal{D}L^i_{pn} + (\mathcal{D}L^p_{jm})L^i_{pn} - L^p_{jn}L^i_{pm} - L^p_{jn}\mathcal{D}L^i_{pm} - (\mathcal{D}L^p_{jn})L^i_{pm}.\end{aligned}$$

As $\mathcal{D}L^i_{jm}$ is a tensor, we can consider covariant derivative:

$$(\mathcal{D}L^i_{jm})_{\underset{1}{|}n} = (\mathcal{D}L^i_{jm})_{,n} + L^i_{pn}\mathcal{D}L^p_{jm} - L^p_{jn}\mathcal{D}L^i_{pm} - L^p_{mn}\mathcal{D}L^i_{jp}$$

wherefrom

$$(5.9)\qquad (\mathcal{D}L^i_{jm})_{,n} + L^i_{pn}\mathcal{D}L^p_{jm} = (\mathcal{D}L^i_{jm})_{\underset{1}{|}n} + L^p_{jn}\mathcal{D}L^i_{pm} + L^p_{mn}\mathcal{D}L^i_{jp}$$

and in the same manner

$$(5.9')\qquad (\mathcal{D}L^i_{jn})_{,m} + L^i_{pm}\mathcal{D}L^p_{jn} = (\mathcal{D}L^i_{jn})_{\underset{1}{|}m} + L^p_{jm}\mathcal{D}L^i_{pn} + L^p_{nm}\mathcal{D}L^i_{jp}$$

If we have in mind $(4.6), (5.9, 9')$, the equation (5.8) becomes

$$(5.10)\qquad \underset{1}{\bar{R}}{}^i_{jmn} = \underset{1}{R}{}^i_{jmn} + (\mathcal{D}L^i_{jm})_{\underset{1}{|}n} - (\mathcal{D}L^i_{jn})_{\underset{1}{|}m} + T^p_{mn}\mathcal{D}L^i_{jp}.$$

From here Lie derivative of curvature tensor is

$$(5.11)\qquad \mathcal{L}_z\underset{1}{R}{}^i_{jmn} = (\mathcal{L}_zL^i_{jm})_{\underset{1}{|}n} - (\mathcal{L}_zL^i_{jn})_{\underset{1}{|}m} + T^p_{mn}\mathcal{L}_zL^i_{jp}.$$

We can also start from (5.2), and then we have

$$\begin{aligned}\bar{\underset{1}{R}}{}^i_{jmn} &= \bar{\underset{0}{R}}{}^i_{jmn} + \frac{1}{2}\bar{T}^i_{jm;n} - \frac{1}{2}\bar{T}^i_{jn;m} + \frac{1}{4}\bar{T}^p_{jm}\bar{T}^i_{pn} - \frac{1}{4}\bar{T}^p_{jn}\bar{T}^i_{pm}\\ &= \underset{0}{R}{}^i_{jmn} + \mathcal{D}\underset{0}{R}{}^i_{jmn} + \frac{1}{2}(T^i_{jm} + \mathcal{D}T^i_{jm})_{;n} - \frac{1}{2}(T^i_{jn} + \mathcal{D}T^i_{jn})_{;m}\\ &+ \frac{1}{4}(T^p_{jm} + \mathcal{D}T^p_{jm})(T^i_{pn} + \mathcal{D}T^i_{pn}) - \frac{1}{4}(T^p_{jn} + \mathcal{D}T^p_{jn})(T^i_{pm} + \mathcal{D}T^i_{pm}).\end{aligned}$$

Omitting the members containing $\mathcal{D}T \cdot \mathcal{D}T$, we get

$$\begin{aligned}\bar{\underset{1}{R}}{}^i_{jmn} &= \underset{0}{R}{}^i_{jmn} + \mathcal{D}\underset{0}{R}{}^i_{jmn} + \frac{1}{2}T^i_{jm;n} + \frac{1}{2}(\mathcal{D}T^i_{jm})_{;n}\\ &- \frac{1}{2}T^i_{jn;m} - \frac{1}{2}(\mathcal{D}T^i_{jn})_{;m} + \frac{1}{4}(T^p_{jm}T^i_{pn} + T^p_{jm}\mathcal{D}T^i_{pn}\\ &+ \mathcal{D}T^p_{jm}T^i_{pn} - T^p_{jn}T^i_{pm} - T^p_{jn}\mathcal{D}T^i_{pm} - \mathcal{D}T^p_{jn}T^i_{pm}).\end{aligned}$$

So, with respect to (5.2), one obtains

$$\begin{aligned}\bar{\underset{1}{R}}{}^i_{jmn} &= \underset{1}{R}{}^i_{jmn} + \mathcal{D}\underset{0}{R}{}^i_{jmn} + \frac{1}{2}(\mathcal{D}T^i_{jm})_{;n} - \frac{1}{2}(\mathcal{D}T^i_{jn})_{;m}\\ &+ \frac{1}{4}\mathcal{D}(T^p_{jm}T^i_{pn} - T^p_{jn}T^i_{pm}),\end{aligned} \tag{5.12}$$

where with ; is denoted covariant derivative with respect to $\underset{0}{L}$ (3.5), and (dividing with ε):

$$\begin{aligned}\mathcal{L}_z\underset{1}{R}{}^i_{jmn} &= \mathcal{L}_z\underset{0}{R}{}^i_{jmn} + \frac{1}{2}(\mathcal{L}_zT^i_{jm})_{;n} - \frac{1}{2}(\mathcal{L}_zT^i_{jn})_{;m}\\ &+ \frac{1}{4}\mathcal{L}_z(T^p_{jm}T^i_{pn} - T^p_{jn}T^i_{pm}).\end{aligned} \tag{5.11'}$$

5.2 Infinitesimal deformation of curvature tensors $\underset{2}{R}, \dots, \underset{5}{R}$

By analogical procedure one obtains the Lie derivative for curvature tensors $\underset{2}{R}, \dots, \underset{5}{R}$. In that manner is

$$\mathcal{L}_z\underset{2}{R}{}^i_{jmn} = (\mathcal{L}_zL^i_{mj})_{\underset{2}{|}n} - (\mathcal{L}_zL^i_{nj})_{\underset{2}{|}m} + T^p_{nm}\mathcal{L}_zL^i_{pj}. \tag{5.13}$$

and

$$\begin{aligned}\mathcal{L}_z\underset{2}{R}{}^i_{jmn} =&\mathcal{L}_z\underset{0}{R}{}^i_{jmn} + \frac{1}{2}(\mathcal{L}_zT^i_{mj})_{;n} - \frac{1}{2}(\mathcal{L}_zT^i_{nj})_{;m}\\ &+ \frac{1}{4}\mathcal{L}_z(T^p_{jm}T^i_{pn} - T^p_{jn}T^i_{pm}),\end{aligned} \tag{5.13'}$$

and

(5.14)
$$\begin{aligned}\mathcal{L}_z \underset{3}{R}{}^i_{jmn} &= (\mathcal{L}_z L^i_{jm})_{\underset{4}{|}n} - (\mathcal{L}_z L^i_{nj})_{\underset{3}{|}m}\\ &+ T^p_{jm}\mathcal{L}_z L^i_{np} + T^p_{jn}\mathcal{L}_z L^i_{pm} + L^p_{mn}\mathcal{L}_z T^i_{jp} + \mathcal{L}_z(T^i_{pj}L^p_{nm}),\end{aligned}$$

(5.14′)
$$\begin{aligned}\mathcal{L}_z \underset{3}{R}{}^i_{jmn} &= \mathcal{L}_z \underset{0}{R}{}^i_{jmn} + \frac{1}{2}(\mathcal{L}_z T^i_{jm})_{;n} + \frac{1}{2}(\mathcal{L}_z T^i_{jn})_{;m}\\ &+ \frac{1}{4}\mathcal{L}_z(T^i_{pm}T^p_{jn} - T^i_{pn}T^p_{jm} - 2T^i_{pj}T^p_{mn}),\end{aligned}$$

(5.15)
$$\begin{aligned}\mathcal{L}_z \underset{4}{R}{}^i_{jmn} &= (\mathcal{L}_z L^i_{jm})_{\underset{4}{|}n} - (\mathcal{L}_z L^i_{nj})_{\underset{3}{|}m}\\ &+ T^p_{jm}\mathcal{L}_z L^i_{np} + T^p_{jn}\mathcal{L}_z L^i_{pm} + L^p_{mn}\mathcal{L}_z T^i_{jp} + \mathcal{L}_z(T^i_{pj}L^p_{mn}),\end{aligned}$$

and

(5.15′)
$$\begin{aligned}\mathcal{L}_z \underset{4}{R}{}^i_{jmn} &= \mathcal{L}_z \underset{0}{R}{}^i_{jmn} + \frac{1}{2}(\mathcal{L}_z T^i_{jm})_{;n} + \frac{1}{2}(\mathcal{L}_z T^i_{jn})_{;m}\\ &+ \frac{1}{4}\mathcal{L}_z(T^i_{pm}T^p_{jn} - T^i_{pn}T^p_{jm} + 2T^i_{pj}T^p_{mn}),\end{aligned}$$

(5.16)
$$\mathcal{L}_z \underset{5}{R}{}^i_{jmn} = \frac{1}{2}[(\mathcal{L}_z L^i_{jm})_{\underset{3}{|}n} - (\mathcal{L}_z L^i_{nj})_{\underset{3}{|}m} + (\mathcal{L}_z L^i_{mj})_{\underset{4}{|}n} - (\mathcal{L}_z L^i_{jn})_{\underset{4}{|}m}],$$

and

(5.16′)
$$\mathcal{L}_z \underset{5}{R}{}^i_{jmn} = \mathcal{L}_z \underset{0}{R}{}^i_{jmn} + \frac{1}{4}\mathcal{L}_z(T^p_{jm}T^i_{pn} + T^p_{jn}T^i_{pm}).$$

Based on exposed, related to infinitesimal deformations and rigidity of curvature tensors in L_N, the following theorems are valid:

Theorem 5.1*Lie derivatives of the curvature tensors* $\underset{1}{R}, \ldots, \underset{5}{R}$ *in the space* L_N *of non-symmetric affine connection* L^i_{jk} *can be expressed by the equations* $(5.11, 11', 13, 13') - (5.16, 16')$.

Theorem 5.2*If with respect to infinitesimal deformation* (2.1) *the connection is rigid, i.e.* $\mathcal{L}_z L^i_{jk} = 0$, *then all curvature tensors* $\underset{\theta}{R}, \theta = 1, \ldots, 5$, *are rigid, that is* $\mathcal{L}_z \underset{\theta}{R}{}^i_{jmn} = 0$, *with respect to that deformation. Conversely, e.g. from* $\mathcal{L}_z \underset{1}{R}{}^i_{jmn} = 0$, *it follows*

(5.17)
$$(\mathcal{L}_z L^i_{jm})_{\underset{1}{|}n} = (\mathcal{L}_z L^i_{jn})_{\underset{1}{|}m} - T^p_{mn}\mathcal{L}_z L^i_{jp}.$$

and similar relations for the rest curvature tensors. In the case of symmetric connection (T=0) the equation (5.17) *and corresponding equations for the remaining curvature tensors reduce to*

$$(\mathcal{L}_z L^i_{jm})_{;n} = (\mathcal{L}_z L^i_{jn})_{;m}.$$

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