Penrose Limits and Maximal Supersymmetry

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ABSTRACT. We show that the maximally supersymmetric pp-wave
of IIB superstring and M-theories can be obtained as a Penrose
limit of the supersymmetric AdS $\times S$ solutions. In addition we find
that in a certain large tension limit, the geometry seen by a brane
probe in an AdS $\times S$ background is either Minkowski space or a
maximally supersymmetric pp-wave.

CONTENTS

1. INTRODUCTION

It has recently been shown [1] that ten-dimensional type IIB su-
pergravity admits a maximally supersymmetric pp-wave background
analogous to the one discovered by Kowalski-Glikman [8] for eleven-
dimensional supergravity and discussed more recently in [5]. In both
cases, the geometry is given by a lorentzian symmetric space G/K with
solvable G, and the field strengths (the self-dual five-form in type IIB
and the four-form in eleven dimensions) are parallel and null; such solu-
tions were called Hpp-waves in [5]. The spacetime is geodesically com-
plete and the metric is that of a pp-wave, but the transverse geometry
is not asymptotically flat. The existence of these Hpp-wave solutions is
a little puzzling. The reason is that they are to be treated on the same
footing as the other maximally supersymmetric solutions: flat space
and solutions of the form AdS $\times S$; but whereas these latter solutions

play the role of asymptotic or near horizon limits of fundamental brane solutions, no such role was known for the former. This puzzle motivated the present work, in which we will show that these maximally super-symmetric Hpp-waves are obtained as *Penrose limits* of the maximally supersymmetric $\text{AdS} \times S$ solutions of eleven-dimensional and type IIB supergravity theories. In addition, we shall show that the geometry seen by a brane probe in an $\text{AdS} \times S$ spacetime in the weak coupling (equivalently *large tension*) limit is either a Minkowski space or a max-imally supersymmetric Hpp-wave.

In [12] Penrose showed that every spacetime has a limit which a neighbourhood of a null geodesic becomes a pp-wave spacetime. Al-though his paper focused on the case of four-dimensional spacetimes, he pointed out that this persists in higher dimensions. More recently, in [7] Güven extended Penrose's work to supergravity backgrounds in ten and eleven dimensions. He achieved this by extending the limiting procedure to the other fields present in the supergravity theories. (This limit had already appeared in the context of world-sheet sigma-model actions and 'contractions' of WZW models in [13, 11, 14, 15]). The limiting solutions are characterised by having a pp-wave-like geometry and null fluxes. We will review this briefly in Section 2.

In Section 3 we exhibit the maximally supersymmetric Hpp-waves [8, 5, 1] of eleven-dimensional and IIB supergravity as Penrose limits of the maximally supersymmetric $\text{AdS} \times S$ solutions. Similar methods also yield the Hpp-wave solutions to five- and six-dimensional supergravities obtained recently in [9], as well as Hpp-wave solutions in supergravity theories in various dimensions. These will appear in a forthcoming paper [2] where we present a systematic and comprehensive discussion of the Penrose limit in string theory. We also show that the Penrose limit provides a natural explanation for the structure of the group of isometries of the maximally supersymmetric Hpp-waves.

In Section 4 we explore the worldvolume dynamics of a brane probe in a spacetime under the Penrose limit. There are different Penrose limits for each spacetime which depend on the choice of null geodesic. The effect of the limit is to blow up a neighbourhood of the geodesic. The Penrose limit also blows up the induced worldvolume of branes. This can be seen by placing a brane probe in a spacetime. A Penrose limit can be taken in which the brane tension is scaled and at the same time a spacetime coordinate transformation is performed. There is a limit in which the tension of the probe goes to infinity and the spacetime in the neighbourhood of a null geodesic goes to the Penrose limit.

2. Penrose limit of supergravity theories

In this section we will briefly review the Penrose limit as described by Güven for backgrounds of ten- and eleven-dimensional supergravities.

Let (M, g) be a lorentzian spacetime. According to [12, 7] in a neighbourhood of a segment of a null geodesic γ containing no conjugate points, it is possible to introduce local coordinates U, V, Y^i such that the metric takes the form

$$g = dV \left(dU + \alpha dV + \sum_i \beta_i dY^i \right) + \sum_{i,j} C_{ij} dY^i dY^j \ , \qquad (1)$$

where α, β_i and C_{ij} are functions of all the coordinates, and where C_{ij} is a symmetric positive-definite matrix. The coordinate system breaks down as soon as $\det C = 0$, signalling the existence of a conjugate point. The coordinate U is the affine parameter along a congruence of null geodesics labelled by V and Y^i. The geodesic γ is the one for which $V = 0 = Y^i$.

In ten- and eleven-dimensional supergravity theories there are other fields besides the metric, such as the dilaton Φ, gauge potentials or more generally p-form potentials A_p with $(p+1)$-form field strengths. The gauge potentials are defined up to gauge transformations $A_p \mapsto A_p + d\Lambda_{p-1}$ in such a way that the field strength $F_{p+1} = dA_p$ is gauge invariant. It is possible to use this gauge freedom in order to gauge away some of the components of the p-form potentials. Indeed, one can choose a gauge locally in which

$$i(\partial/\partial U)A = 0 \ , \qquad (2)$$

or in components

$$A_{U i_1 i_2 \ldots i_{p-1}} = A_{UV i_1 i_2 \ldots i_{p-2}} = 0 \ .$$

Similar results apply for field strengths with intereaction terms, $F = dA + \ldots$. The starting point of the Penrose limit is the data (M, g, Φ, A_p) defined in a neighbourhood of a conjugate-point-free segment of a null geodesic γ where g and A_p take the forms (1) and (2), respectively.

We now introduce a positive real constant $\Omega > 0$ and rescale the coordinates as follows

$$U = u \ , \qquad V = \Omega^2 v \qquad \text{and} \qquad Y^i = \Omega y^i \ . \qquad (3)$$

Acting with this diffeomorphism on the tensor fields of the theory we obtain an Ω-dependent family of fields $g(\Omega)$, $\Phi(\Omega)$ and $A_p(\Omega)$. The coordinate and gauge choices (1) and (2) ensure that the following *Penrose limit* [12] (as extended by Güven [7] to fields other than the metric) is well-defined:

$$\begin{aligned} \bar{g} &= \lim_{\Omega \to 0} \Omega^{-2} g(\Omega) \\ \bar{\Phi} &= \lim_{\Omega \to 0} \Phi(\Omega) \\ \bar{A}_p &= \lim_{\Omega \to 0} \Omega^{-p} A_p(\Omega) \ . \end{aligned} \qquad (4)$$

By virtue of (3) the limiting fields only depend on the coordinate u, which is the affine parameter along the null geodesic. The resulting expression for the metric is of the form

$$\bar{g} = dudv + \sum_{i,j} \bar{C}_{ij}(u)dy^i dy^j \ . \tag{5}$$

The gauge potentials $\bar{A}_p$ only have components in the transverse directions y^i,

$$i(\partial/\partial u)\bar{A}_p = 0 = i(\partial/\partial v)\bar{A}_p \ ,$$

and the field strengths $\bar{F}_{p+1}$ are therefore of the form

$$\bar{F}_{p+1} = du \wedge \bar{A}_p(u)' \ , \tag{6}$$

where $'$ denotes d/du. Note that $\bar{F}_{p+1}$ is null.

As the supergravity actions transform homogeneously [7] under the scaling (4), $(\bar{g}, \bar{\Phi}, \bar{A}_p)$ will be a solution to the supergravity equations of motion whenever (g, Φ, A_p) is. These and other hereditary properties (in the sense of [6]) of Penrose limits will be discussed in detail in [2].

The above expression for $\bar{g}$ is that of a pp-wave in Rosen coordinates. It is possible to change to Brinkman (also called harmonic) coordinates in such a way that the resulting metric takes the form

$$\bar{g} = 2dx^+ dx^- + \left(\sum_{i,j} A_{ij}(x^-)x^i x^j \right)(dx^-)^2 + \sum_i dx^i dx^i \ . \tag{7}$$

When A_{ij} is constant this metric describes a lorentzian symmetric Cahen–Wallach space [3]. Such spaces include the maximally supersymmetric Hpp-waves of eleven-dimensional [8, 5] and IIB supergravity [1], namely

$$g_{11} = 2dx^+ dx^- - \left(\sum_{i,j=1}^{3} \delta_{ij}x^i x^j + \tfrac{1}{4} \sum_{i,j=4}^{9} \delta_{ij}x^i x^j \right)(dx^-)^2 + \sum_{i=1}^{9} dx^i dx^i \tag{8}$$

$$g_{\mathrm{IIB}} = 2dx^+ dx^- - \sum_{i,j=1}^{8} \delta_{ij}x^i x^j (dx^-)^2 + \sum_{i=1}^{8} dx^i dx^i \tag{9}$$

(up to an overall scaling of A_{ij} by a real positive constant which can always be absorbed into a scaling of (x^+, x^-)). It is this observation which gives rise to the investigation reported here and in [2].

The explicit change of variables which takes the metric from Rosen to Brinkman form is given by

$$u = 2x^- \qquad v = x^+ - \tfrac{1}{2}\sum_{i,j} M_{ij}(x^-)x^i x^j \qquad y^i = \sum_j Q^i_j(x^-)x^j \ ,$$

where Q^i_j is an invertible matrix satisfying (a $'$ now denotes d/dx^-)

$$C_{ij}Q^i_k Q^j_l = \delta_{kl} \qquad \text{and} \qquad C_{ij}\left(Q'^i_j Q^j_l - Q'^i_k Q^j_l\right) = 0 \ , \tag{10}$$

and
$$M_{ij} = C_{kl}Q'^{k}_{i}Q^{l}_{j} \ ,$$
which is symmetric by virtue of the second equation in (10). This equation guarantees that the limiting metric $\bar{g}$ has the form (7). The relation between C_{ij} and A_{ij} is
$$A_{ij} = -[C_{kl}Q'^{l}_{j}]'Q^{k}_{i} \ .$$

It is possible to rewrite the field strengths $\bar{F}_{p+1}$ given in (6) in terms of Brinkman coordinates, to arrive at the following expression:

$$\bar{F}_{p+1} = \sum_{i_k,j_k} \frac{d}{dx^-}\bar{A}_{i_1 i_2 ... i_p}(2x^-)Q^{i_1}_{j_1}Q^{i_2}_{j_2}\cdots Q^{i_p}_{j_p}$$

$$\times \, dx^- \wedge dx^{j_1} \wedge dx^{j_2} \wedge \cdots \wedge dx^{j_p} \ .$$

3. Penrose limit of AdS $\times S$ solutions

In this section we exhibit the maximally supersymmetric Hpp-wave solutions to eleven-dimensional and IIB supergravity as Penrose limits of AdS $\times S$ supergravity solutions.

3.1. The metrics. The near horizon geometry of the M2-, M5- and D3-brane solutions is of the form $\mathrm{AdS}_{p+2} \times S^{D-p-2}$ where the values of p and D corresponding to each of the above branes are listed in Table 1 along with the ratio $\rho := R_{\mathrm{AdS}_{p+2}}/R_{S^{D-p-2}}$ of the radii of curvature of the two factors.

Brane	p	D	ρ
M2	2	11	$\frac{1}{2}$
D3	3	10	1
M5	5	11	2

Table 1. Dimensions and radii of curvature

The metric for anti-de Sitter space AdS_{p+2} with radius of curvature R_{AdS} can be written as

$$g_{\mathrm{AdS}} = R^2_{\mathrm{AdS}}\left[-d\tau^2 + (\sin\tau)^2\left(\frac{dr^2}{1+r^2} + r^2 d\Omega^2_p\right)\right] \ , \qquad (11)$$

where $d\Omega^2_p$ is the p-sphere metric.

Similarly, we write the round metric on the n-sphere S^n with radius of curvature R_S as

$$g_S = R^2_S\left[d\psi^2 + (\sin\psi)^2 d\Omega^2_{n-1}\right] \ , \qquad (12)$$

where $d\Omega^2_{n-1}$ is the metric on the equatorial $(n-1)$-sphere and ψ is the colatitude.

The metric on $\mathrm{AdS}_{p+2} \times S^{D-p-2}$ is then $g = g_{\mathrm{AdS}} + g_S$, which is given by

$$R^{-2}g = \rho^2 \left[-d\tau^2 + (\sin\tau)^2 \left(\frac{dr^2}{1+r^2} + r^2 d\Omega_p^2 \right) \right]$$
$$+ d\psi^2 + (\sin\psi)^2 d\Omega_{D-p-3}^2 \ ,$$

where we have introduced the ratio ρ defined above and where R is the radius of curvature of the sphere. Let us now change coordinates in the (ψ, τ) plane to

$$u = \psi + \rho\tau \qquad v = \psi - \rho\tau \ , \tag{13}$$

in terms of which, the metric g becomes

$$R^{-2}g = dudv + \rho^2 \sin((u-v)/2\rho)^2 \left(\frac{dr^2}{1+r^2} + r^2 d\Omega_p^2 \right)$$
$$+ \sin((u+v)/2)^2 d\Omega_{D-p-3}^2 \ .$$

We now take the Penrose limit along the null geodesic parametrised by u. In practice this consists in dropping the dependence on coordinates other than u. Doing so we find

$$R^{-2}\bar{g} = dudv + \rho^2 \sin(u/2\rho)^2 ds^2(\mathbb{E}^{p+1}) + (\sin(u/2))^2 ds^2(\mathbb{E}^{D-p-3}) \ , \tag{14}$$

which is the metric of a Cahen–Wallach symmetric space in Rosen coordinates (compare with [4] for the $d = 11$ solution).

To see this let us introduce coordinates y^a for $a = 1, \ldots, D - 2$ in such a way that the metric (14) becomes

$$R^{-2}\bar{g} = dudv + \sum_{a=1}^{D-2} \frac{(\sin(\lambda_a u))^2}{(2\lambda_a)^2} dy^a dy^a \ ,$$

where

$$\lambda_a = \begin{cases} 1/2\rho & a = 1, \ldots, p+1 \\ 1/2 & a = p+2, \ldots, D-2 \ . \end{cases} \tag{15}$$

We change coordinates to (x^+, x^-, x^a) where

$$x^- = u/2 \ , \quad x^+ = v - \frac{1}{4}\sum_a y^a y^a \frac{\sin(2\lambda_a u)}{2\lambda_a} \ , \quad x^a = y^a \frac{\sin(\lambda_a u)}{2\lambda_a} \ , \tag{16}$$

in such a way that the metric now becomes

$$R^{-2}\bar{g} = 2dx^+ dx^- - 4 \left(\sum_a \lambda_a^2 x^a x^a \right) (dx^-)^2 + \sum_a dx^a dx^a \ , \tag{17}$$

which we recognise as a Cahen–Wallach metric (7) whose matrix A_{ij} is constant and diagonal with negative eigenvalues $\{-\lambda_a^2\}$. For λ_a given as in (15) we obtain, if $\rho = \frac{1}{2}$ or $\rho = 2$, precisely the metrics of the maximally supersymmetric Hpp-waves of eleven-dimensional supergravity

(8) discovered in [8] (see also [5]), and if $\rho = 1$ the maximally super-symmetric Hpp-wave of IIB supergravity (9) discovered in [1]. The two cases $\rho = \frac{1}{2}$ and $\rho = 2$ are isometric—an explicit diffeomorphism being given by

$$x^- \mapsto \tfrac{1}{2}x^-$$
$$x^+ \mapsto 2x^+$$
$$(x^1, \ldots, x^6, x^7, \ldots, x^9) \mapsto (x^4, \cdots, x^9, x^1, \ldots, x^3) \ .$$

3.2. **The p-forms.** The near horizon geometries of the M2, D3 and M5 brane solutions carry fluxes with respect to $(D - p - 2)$-form field strengths. Let us consider the M-branes first. The 7-form in the $\mathrm{AdS}_4 \times S^7$ solution is given by the Hodge dual of the 4-form

$$F_4 = \sqrt{6|s|}\,\mathrm{dvol}(\mathrm{AdS}_4) \ ,$$

whereas the 4-form in the near horizon geometry of the M5 brane is given by

$$F_4 = \sqrt{6|s|}\,\mathrm{dvol}(S^4) \ ,$$

where s is the scalar curvature of the supergravity solution. The scalar curvature of the solution is $1/8$ of the scalar curvature of the four-dimensional factor, which for a four-dimensional space form is (in absolute value) $12R^{-2}$, where R is the radius of curvature. Therefore, in terms of the radii of curvature, the above 4-forms can be written as

$$F_4 = 3R_{\mathrm{AdS}}^{-1}\,\mathrm{dvol}(\mathrm{AdS}_4) \ ,$$

and

$$F_4 = 3R_S^{-1}\,\mathrm{dvol}(S^4) \ ,$$

respectively.

Next consider the case of the $\mathrm{AdS}_7 \times S^4$ solution M5 brane. The metric g_S of the 4-sphere of radius of curvature R is

$$R^{-2}g_S = d\psi^2 + (\sin\psi)^2 d\Omega_3^2 \ ,$$

where $d\Omega_3^2$ is the round metric on the equatorial 3-sphere. The corresponding volume form is then

$$\mathrm{dvol}(S^4) = R^4(\sin\psi)^3 d\psi \wedge \mathrm{dvol}(S^3) \ ,$$

whence the 4-form is

$$F_4 = 3R^3(\sin\psi)^3 d\psi \wedge \mathrm{dvol}(S^3) \ .$$

Taking the Penrose limit along the geodesic with affine parameter $u = \psi + 2\tau$, we find

$$\bar{F}_4 = \tfrac{3}{2}R^3(\sin(u/2))^3 du \wedge dy^7 \wedge dy^8 \wedge dy^9$$

in Rosen coordinates. Changing to Brinkman coordinates as in (16), we obtain

$$R^{-3}\bar{F}_4 = 3dx^- \wedge dx^7 \wedge dx^8 \wedge dx^9 \ ,$$

which agrees with the expression for the 4-form in the maximally super-symmetric Hpp-wave solution of eleven-dimensional supergravity [8, 5]. The same holds (after a change of variables) for the 4-form in the $\mathrm{AdS}_4 \times S^7$ solution.

Finally we consider the $\mathrm{AdS}_5 \times S^5$ solution. In our conventions, the self-dual 5-form in the $\mathrm{AdS}_5 \times S^5$ solution is given by

$$F_5 = \tfrac{1}{2} R^{-1} \left(\mathrm{dvol}(\mathrm{AdS}_5) + \mathrm{dvol}(S^5) \right) \ ,$$

where R is the radius of curvature of both anti-de Sitter spacetime and the sphere. In terms of the coordinates in which we wrote the metrics, we have

$$F_5 = \tfrac{1}{2} R^4 \left(\frac{r^3}{\sqrt{1+r^2}} (\sin \tau)^4 d\tau \wedge dr \wedge \mathrm{dvol}(S^3) + (\sin \psi)^4 d\psi \wedge \mathrm{dvol}(S^4) \right) \ .$$

Taking the Penrose limit along the geodesic parametrised by $u = \psi + \tau$, we obtain

$$\bar{F}_5 = \tfrac{1}{4} R^4 (\sin{(u/2)})^4 du \wedge \left(dy^1 \wedge dy^2 \wedge dy^3 \wedge dy^4 + dy^5 \wedge dy^6 \wedge dy^7 \wedge dy^8 \right)$$

in Rosen coordinates. Using (16) to go to Brinkman coordinates, we obtain

$$R^{-4} \bar{F}_5 = \tfrac{1}{2} dx^- \wedge \left(dx^1 \wedge dx^2 \wedge dx^3 \wedge dx^4 + dx^5 \wedge dx^6 \wedge dx^7 \wedge dx^8 \right) \ ,$$

which, together with (9), agrees with the maximally supersymmetric Hpp-wave discovered in [1].

In summary, we conclude that the maximally supersymmetric Hpp-waves of [8, 5] and [1] appear as Penrose limits of maximally super-symmetric $\mathrm{AdS} \times S$ spacetimes. Moreover this is not an accident. In fact, it can be shown [2] that there are only two possible Penrose limits of these $\mathrm{AdS} \times S$ solutions: generically one obtains the maximally supersymmetric Hpp-waves, but one can also get flat space for very special null geodesics, namely those which have no velocity component tangent to the sphere.

3.3. Symmetries.
One intriguing feature of the maximally supersymmetric Hpp-waves is that their symmetry (super)algebra has the same dimension as that of the $\mathrm{AdS} \times S$ solutions. It was observed in [1] that the symmetry (super)algebras of these classes of solutions are related essentially by a contraction, and this in turn suggests a limiting procedure relating the two classes of solutions. This limiting procedure is none other than the Penrose limit, as we will now show. We will be brief, leaving the details to our forthcoming paper [2] which also includes a more general discussion of what happens to isometries and supersymmetries under the Penrose limit.

Let ξ be a Killing vector of the $\mathrm{AdS} \times S$ solution and let us change coordinates to those considered above that are adapted to a null geodesic. After the change of variables (3), the vector field acquires a dependence on the scaling parameter Ω. Let us make this manifest by writing it as $\xi(\Omega)$. For all $\Omega > 0$, $\xi(\Omega)$ is a Killing vector with respect

to the rescaled metric $g(\Omega)$; hence in the limit, $\bar{\xi} = \lim_{\Omega \to 0} \Omega^\Delta \xi(\Omega)$, where Δ is chosen so that the above limit exists and is non-zero, is a Killing vector with respect to the limiting metric $\bar{g}$. An argument originally due to Geroch [6] and described in more detail in the present context in [2] shows that all Killing vectors are preserved, although the algebraic structure is generally contracted, due to the fact that different Killing vectors may have to be rescaled with different values of Δ. A similar argument shows that the Killing spinors of a supergravity solution are also preserved. It is important to notice, however, that the Penrose limit might admit additional (super)symmetries not present in the original solution.

Concretely, consider the Penrose limit of $\mathrm{AdS}_{p+2} \times S^{D-p-2}$ with radii of curvature $R_{\mathrm{AdS}} = \rho R$ and $R_S = R$, respectively. The Penrose limit along the null geodesic considered above is an Hpp-wave metric of the form (7) where the matrix A_{ij} is constant and has two eigenvalues with ratio ρ and multiplicities $p+1$ and $D-p-3$. Under the Penrose limit, the $\mathfrak{so}(2, p+1) \oplus \mathfrak{so}(D-p-1)$ isometry algebra of $\mathrm{AdS}_{p+2} \times S^{D-p-2}$ gets modified in the following way. The $\mathfrak{so}(2, p+1)$ factor gets contracted to $\mathfrak{h}(p+1) \rtimes \mathfrak{so}(p+1)$, where $\mathfrak{h}(p+1)$ is a Heisenberg algebra with $2p+3$ generators and $\mathfrak{so}(p+1)$ acts on the creation and annihilation operators in the natural way (i.e., they transform as vectors). Similarly the $\mathfrak{so}(D-p-1)$ factor contracts to $\mathfrak{h}(D-p-3) \rtimes \mathfrak{so}(D-p-3)$. The central element in both Heisenberg algebras coincide. This means that there are two Killing vectors $\xi_1(\Omega)$ and $\xi_2(\Omega)$ such that they agree to leading order in Ω and hence coincide in the limit. Consider instead the linear combinations $\xi_\pm(\Omega) = \xi_1(\Omega) \pm \xi_2(\Omega)$. These vector fields must be rescaled differently for their limits to exist and be non-zero: $\xi_+(\Omega)$ becomes in the limit the common central element of the combined Heisenberg algebra $\mathfrak{h}(D-2)$, whereas $\xi_-(\Omega)$ becomes an outer automorphism commuting with $\mathfrak{so}(p+1) \oplus \mathfrak{so}(D-p-3)$. In Brinkman coordinates, $\bar{\xi}_\pm$ are realised as $\partial/\partial x^\pm$. We see, therefore, that the isometry algebra $\mathfrak{so}(2, p+1) \oplus \mathfrak{so}(D-p-1)$ of $\mathrm{AdS}_{p+2} \times S^{D-p-2}$ contracts to a semidirect product

$$\mathfrak{h}(D-2) \rtimes \left(\mathfrak{so}(p+1) \oplus \mathfrak{so}(D-p-3) \oplus \mathbb{R} \right) .$$

If the ratio ρ of the radii of curvature is equal to 1, there is an additional enhancement of the isometry, and the subalgebra $\mathfrak{so}(p+1) \oplus \mathfrak{so}(D-p-3)$ is enlarged to the full $\mathfrak{so}(D-2)$. This however has no counterpart in the original metric.

In summary, the Penrose limit provides a natural explanation for the isometry structure of the maximally supersymmetric Hpp-waves discussed in [1]. Finding such an explanation was one of the motivations for the present work.

4. WORLDVOLUME DYNAMICS AND PENROSE LIMITS

To investigate the effect of the Penrose limit on the dynamics of brane probes, we consider the action of a typical p-brane with tension T_p propagating in a AdS $\times S$ background. Here we shall focus on the bosonic part of brane probe actions without worldvolume fields of Born-Infeld type. The analysis which follows can be extended to include Born-Infeld fields and this will be presented in [2]. Under these assumptions, the action of a p-brane probe in a background (e.g., AdS $\times S$) with non-vanishing form-field potential C is

$$I_p[g, C; \kappa_p] = T_p \left(\int_D d^{p+1}\sigma \sqrt{g} + \int_D C \right) ,$$

where $T_p = 1/\kappa_p$ is the p-brane tension and D is a region of worldvolume. The first term in the action is the usual induced volume term and the second is a Wess–Zumino term; we do not distinguish between the induced metric and form-gauge potential and the associated spacetime objects g and C, respectively. It is clear from the context which is which.

To implement the Penrose limit for the probe action above, we first rescale $\kappa_p \to \Omega^{p+1}\kappa_p$, obtaining

$$I[g, C; \Omega^{p+1}\kappa_p] = I[\Omega^{-2}g, \Omega^{-(p+1)}C; \kappa_p] .$$

Next, adopting appropriate coordinates for the Penrose limit and performing the coordinate transformation (3), we can write

$$I[\Omega^{-2}g, \Omega^{-(p+1)}C; \kappa_p] = I[\Omega^{-2}g(\Omega), \Omega^{-(p+1)}C(\Omega); \kappa_p] .$$

Now for $\Omega \ll 1$ we can expand the probe action in a power series in Ω as

$$I[\Omega^{-2}g, \Omega^{-(p+1)}C; \kappa_p] = I[\bar{g}, \bar{C}; \kappa_p] + O(\Omega) ,$$

where $(\bar{g}, \bar{C})$ are the metric and gauge-form potential at the Penrose limit. Now we have seen that there are two types of Penrose limits for AdS $\times S$ spacetimes, either the maximally supersymmetric Hpp-wave or Minkowski spacetime. This limit takes worldvolume solutions of probes in AdS $\times S$ to worldvolume solutions of probes in the Penrose limit of AdS $\times S$, which is either an Hpp-wave spacetime or Minkowski space, depending on the choice of null geodesic.

A physical interpretation of the Penrose limit is as a particular large tension limit of p-branes in a given spacetime, arising from making the Ω-dependent change of coordinates (3) and the rescaling of the tension $T_p \to T_p/\Omega^{p+1}$. The theory can then be developed as a perturbation series in Ω, and there will be a different perturbation series for different choices of null geodesic. Taking the limit $\Omega \to 0$ then reduces the brane action to that of a brane in a spacetime which is the Penrose limit of the original spacetime, and this will depend on the choice of null geodesic [2]. In this limit $\kappa_p \to 0$, so it is a weak coupling limit from the

persepctive of the p-brane. For branes in $\mathrm{AdS} \times S$, the limit arising in this way is Minkowski spacetime or the maximally supersymmetric Hpp-wave, depending on the choice of geodesic.

In particular we find that the IIB string in $\mathrm{AdS}_5 \times S^5$ has two large tension limits of this kind: the string in ten-dimensional Minkowski spacetime and the string in the Hpp-wave solution found in [1]. It is worth mentioning that in both of these limits the IIB superstring Green–Schwarz action does not exhibit any interactions after gauge fixing (see [10] for the case of the Hpp-wave) despite the presence of Ramond-Ramond field and can be quantised exactly.

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