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Saddle points of stringy action

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Abstract

We show that Einstein-Yang-Mills-dilaton theory has a countable family of static globally regular solutions which are purely magnetic but uncharged. The discrete spectrum of masses of these solutions is bounded from above by the mass of extremal Gibbons-Maeda solution. Linear stability analysis shows that all solutions are unstable.

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1 Introduction

In a seminal paper [1] Bartnik and Mckinnon (BM) have discovered a countable family of globally regular static spherically symmetric solutions of the Einstein-Yang-Mills (EYM) equations. Later on a rigorous proof of existence of BM solutions was given by Smoller et al. [2]. Recently, Sudarsky and Wald (SW) proposed a heuristic argument [3], in the spirit of Morse theory, which explains existence and properties of BM solutions. The SW argument exploits the existence of topologically nonequivalent multiple vacua in the $SU(2)$-YM theory (which is related to the fact that $\pi_3(SU(2)) \simeq Z$) and is essentially insensitive to the concrete form of the coupling which suggests that there should exist solutions similar to BM solutions in other theories involving YM field. Indeed, such solutions were found in YM-dilaton theory [4,5] and remarkable parallels between these solutions and BM solutions were observed [4].

The YM-dilaton theory and the Einstein-YM theory may be embedded in a single Einstein-YM-dilaton theory governed by the action

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{G}R - 2(\nabla\phi)^2 - e^{-2a\phi}F^2\right] \ , \tag{1}$$

where R is a scalar curvature, ϕ is a dilaton, F is a Yang-Mills curvature, G is Newton's constant and a is the dilaton coupling constant. This theory is characterized by a dimensionless parameter $\alpha = G/a^2$. When $\alpha = 0$ the action (1) reduces to the YM-dilaton theory. When $\alpha \to \infty$ the action (1) becomes the Einstein-YM theory (plus trivial kinetic term for the scalar field). Finally, the case $\alpha = 1$ corresponds to the low-energy action of heterotic string theory.

The aim of this paper is to show that the theory defined by the action (1) has (for all values of α) static spherically symmetric globally regular solutions with the following properties:

a) there exist a countable family of solutions X_n $(n \in N)$,

b) the total mass m_n increases with n and is bounded from above,

c) the solution X_n has exactly n unstable modes.

The solutions X_n depend continously on α and interpolate smoothly between the YM-dilaton solutions ($\alpha = 0$) and BM solutions ($\alpha = \infty$). In the limit $n \to \infty$ the solution $X_n(\alpha)$ tends to the abelian extremal charged dilatonic black hole [6,7], whose mass therefore provides un upper bound for the spectrum m_n.

The paper is organized as follows. In the next Section the field equations are derived and some scaling properties are discussed. In Section 3 the explicit abelian solutions are described. In Section 4 the numerical non-abelian solutions are presented and their qualitative properties are discussed. Section 5 is devoted to linear stability analysis.

2 Field equations

We are interested in static spherically symmetric configurations. It is convenient to parametrize the metric in the following way

$$ds^2 = -A^2 N dt^2 + N^{-1} dr^2 + r^2 (d\vartheta^2 + \sin^2 \vartheta d\varphi^2) \ , \tag{2}$$

where A and N are functions of r.

We assume that the electric part of the YM field vanishes (actually this is not a restriction because one can show, following the argument given in [7], that there are no globally regular static solutions with nonzero electric field). The purely magnetic spherically symmetric $SU(2)$ YM connection can be written, in the abelian gauge, as [8]

$$eA = w\tau_1 d\vartheta + (\cot\vartheta\tau_3 + w\tau_2)\sin\vartheta d\varphi \ , \tag{3}$$

where τ_i $(i = 1, 2, 3)$ are Pauli matrices and w is a function of r. The corresponding YM curvature $F = dA + eA \wedge A$ is given by

$$eF = w'\tau_1 dr \wedge d\vartheta + w'\tau_2 dr \wedge \sin\vartheta d\varphi - (1 - w^2)\tau_3 d\vartheta \wedge \sin\vartheta d\varphi \ , \tag{4}$$

where prime denotes derivative with respect to r.

For these Anzatze and for $\phi = \phi(r)$ the action (1) gives the lagrangian (where $S = 16\pi \int L dt$)

$$L = \int_0^\infty A(m' - U) dr \ \ , \tag{5}$$

where the mass function $m(r)$ is defined by $N = 1 - 2Gm/r$ and

$$U = \frac{1}{2} r^2 N \phi'^2 + \frac{1}{e^2} e^{-2a\phi} \left[N w'^2 + \frac{(1 - w^2)^2}{2r^2} \right] \ , \tag{6}$$

Hereafter we put $e = a = 1$ which means that we choose a/e as the unit of length and $1/ea$ as the unit of energy. Then the system is characterized by one dimensionless parameter $\alpha = G/a^2$. Variation of L with respect of m, A, w, and ϕ yields the field equations

$$m' = \alpha U \ , \tag{7}$$

$$A' = \alpha A(r\phi'^2 + \frac{2}{r}e^{-2\phi}w'^2) \ , \tag{8}$$

$$(ANe^{-2\phi}w')' + \frac{1}{r^2}Ae^{-2\phi}w(1-w^2) = 0 \ , \tag{9}$$

$$(r^2AN\phi')' + 2Ae^{-2\phi}\left[Nw'^2 + \frac{(1-w^2)^2}{2r^2}\right] = 0. \tag{10}$$

Note that L has a characteristic for general relativity "pure constraint" form, namely the integrand in Eq.(5) is the hamiltonian constraint, Eq.(6), multiplied by the lapse function A. Actually L is not differentiable because the variation of L gives an unwanted surface term at infinity. To remedy this one has to add to L the Regge-Teitelboim correction term, $-A(\infty)m(\infty)$. Then $L' = L - A(\infty)m(\infty)$ has a well-defined functional derivative. It is convenient to define the energy functional

$$E = -L' = \int_0^\infty (AU - A'm)dr \ . \tag{11}$$

On shell E is equal to the total mass $m(\infty)$ (assuming the boundary condition $A(\infty) = 1$ i.e. that t is proper time at spatial infinity).

Eqs.(7)-(10) have a scaling symmetry: if w, ϕ, N and A are solutions so are

$$\begin{aligned}
&w_\lambda(r) = w(e^\lambda r) \ , \\
&\phi_\lambda(r) = \phi(e^\lambda r) + \lambda \\
&N_\lambda(r) = N(e^\lambda r) \qquad \left[m_\lambda(r) = e^{-\lambda}m(e^\lambda r)\right] \ , \\
&A_\lambda(r) = A(e^\lambda r).
\end{aligned} \tag{12}$$

Under this transformation the energy functional scales as follows

$$E_\lambda = e^{-\lambda}E \ . \tag{13}$$

The existence of this scaling symmetry does not exclude globally regular solutions because for the variation induced by the transformation (12) the energy is not extremized since $\delta\phi(\infty)$ is nonzero. Hereafter, we will assume that all solutions satisfy $\phi(\infty) = 0$, which can always be set by the transformation (12). This choice sets the scale of energy in the theory.

3 Abelian solutions

In the $U(1)$ sector of the theory (i.e. in Einstein-Maxwell-dilaton theory) there are no static globally regular solutions but there are known explicit black hole solutions of Eqs.(7)-(10). The uncharged solution is Schwarzschild

$$w = \pm 1 \, , \qquad \phi = 0 \, , \qquad m = const \, , \qquad A = 1. \tag{14}$$

Charged black hole solutions were found by Gibbons and Maeda [6] (and later rediscovered in [7]). In the so called extremal limit the area of the horizon of these charged dilatonic black holes goes to zero. This singular extremal soltion will play an important role in our discussion of non-abelian solutions. It has very simple form in isotropic coordinates

$$ds^2 = -e^{-2\alpha\phi}dt^2 + e^{2\alpha\phi}(d\rho^2 + \rho^2 d\vartheta^2 + \rho^2 \sin^2\vartheta d\varphi^2) \tag{15}$$

where

$$\phi = \frac{1}{1+\alpha} \ln\left(1 + \frac{\sqrt{1+\alpha}}{\rho}\right) \tag{16}$$

and $w = 0$, so the YM curvature is

$$F = -\tau_3 \; d\vartheta \wedge \sin\vartheta d\varphi \, , \tag{17}$$

which corresponds to the Dirac magnetic monopole with magnetic charge $1/e$. For $\alpha = 0$ the solution (15) reduces to the dilatonic magnetic monopole discussed in [4]. The total mass is $m(\infty) = 1/\sqrt{1+\alpha}$.

4 Non-abelian solutions

In order to construct solutions which are globally regular we have to impose the boundary conditions which ensure regularity a$r = 0$ and asymptotic flatness. The asymptotic solutions to Eqs.(7)-(10) satisfying these requirements are

$$\begin{aligned} w &= 1 - br^2 + O(r^4) \, , \\ \phi &= c - 2e^{-2c}b^2r^2 + O(r^4) \, , \\ N &= 1 - 4\alpha e^{-2c}b^2r^2 + O(r^4) \, , \\ A &= 1 + 4\alpha e^{-2c}b^2r^2 + O(r^4) \, . \end{aligned} \tag{18}$$

near $r=0$, and

$$\begin{aligned}
&\pm w = 1 - \tfrac{d}{r} + O(\tfrac{1}{r^2})\ , \\
&\phi = f + \tfrac{g}{r} + O(\tfrac{1}{r^4})\ , \\
&N = 1 - \tfrac{2\alpha m(\infty)}{r} + O(\tfrac{1}{r^2}\ , \\
&A = 1 - \tfrac{\alpha g^2}{2r^2} + O(\tfrac{1}{r^4}.
\end{aligned} \tag{19}$$

near $r=\infty$.

Here b, c, d, f, g, and $m(\infty)$ are arbitrary constants. All higher order terms in the above expansions are uniquely determined, through recurrence relations, by b and c in (16), and d, f, g and $m(\infty)$ in (17).

Note that, for the asymptotic behaviour (17), the radial magnetic curvature, $B = \tau_3(1-w^2)/r^2$, falls-off as $1/r^3$, and therefore all globally regular solutions have zero YM magnetic charge.

It is very easy to show that the solutions satisfying above boundary conditions, if they exist, have the function w oscillating around zero between -1 and 1 and monotonically deacreasing ϕ.

Now, let us assume that near $r=0$ there exist a family of local solutions defined by the expansion (16). Note that this is a nontrivial statement because the point $r=0$ is a singular point of Eqs.(7)-(10), so the formal power-series expansion (16) may have, in principle, a zero radius of convergence. Due to the scaling symmetry (12) the initial value c is irrelevant so effectively this is a one-parameter family parametrized by b. For generic b the solution will not satisfy the asymptotic conditions (17) (in fact, the solution may even become singular at some finite distance). The standard numerical strategy, called the shooting method, is to find initial value b for which the local solution extends to a global solution with the asymptotic behaviour (17). For generic orbits with $b < b_\infty(\alpha)$ a function w oscillates finite number of times in the region between $w=-1$ and $w=1$ and then goes to $\pm\infty$. For $b > b_\infty(\alpha)$ all orbits become singular at a finite distance (in a sense that w' becomes infinite).

The numerical results strongly indicate that for all values of α there exist a countable family of initial values b_n $(n \in N)$ determining globally regular solutions. Here the index n labels the number of nodes of the function w. In Table 1 are displayed the initial values and masses of the first five solutions for $\alpha = 1$. The initial value c is determined by the condition that the dilaton vanishes at infinity i.e. $f = 0$. The functions w, ϕ and N are graphed in Figs.1-3.

Table 1: Initial data (b, c) and masses of the first five globally regular solutions for $\alpha = 1$.

n	b	c	m_∞
1	0.1666666	0.9312	
2	0.2318001	1.7925	
3	0.24686	2.6919	
4	0.249483	3.5974	
5	0.249915	4.5007	

The solutions display three characteristic regions. The energy density is concentrated in the inner core region $r < R_1$, where R_1 is approximately the location of the first zero of w. This region decreases with n and shrinks to zero as $n \to \infty$. In the second region, $R_1 < r < R_2$, where R_2 is approximately the location of the last but one zero of w, the function w slowly oscillates around $w = 0$ with a very small amplitude. In this region the solution is very well approximated by the singular abelian solution This region extends to infinity as $n \to \infty$. Finally, in the asymptotic region $r > R_2$, the function w goes monotonically to $w = \pm 1$ (hence the YM magnetic charge is gradually screened) and for $r \to \infty$ the solution tends to the vacuum ($w = \pm 1$, $\phi = 0$). The metric coefficient N has one minimum aproximately at R_1. For $n \to \infty$ $N_{min} \to 1/(1 + \alpha)^2$.

a one-parameter family of field configurations defined by

$$\begin{aligned} w_\beta(r) &= w(\beta r) \ , \\ \phi_\beta(r) &= \phi(\beta r) \ . \end{aligned} \tag{20}$$

For this family the energy (9) is given by

$$E[w_\beta, \phi_\beta] = \beta^{-1} I_1 + \beta \, I_2 \tag{21}$$

where

$$I_1 = \frac{1}{2} \int_0^\infty r^2 \phi'^2 dr \ , \tag{22}$$

$$I_2 = \int_0^\infty e^{-2\phi} \left[w'^2 + \frac{(1-w^2)^2}{2r^2} \right] dr \ . \tag{23}$$

Since the energy is extremized at $\beta = 1$, it follows from (26) that on-shell

$$I_1 = I_2 \ . \tag{24}$$

Next, integrating eq.(8) one gets $D = -2I_2$ (D is the dilaton charge defined in Section 2), and therefore eq.(29) yields

$$E = -D \ . \tag{25}$$

Thus the energy of a static solution can be read-off from the monopole term of the asymptotic expansion (23) of the dilaton field. This is a remarkable property which reminds very much the situation in general relativity and shows a relation between the dilaton field and the conformal degree of freedom of the metric.

Secondly, the most striking analogy between our solutions and the BM solutions is their spectrum of energies (see Table 1 and compare with Table I in ref.[14]). In both cases the energies increase with n and are bounded from above by $E = 1$. This cannot be a coincidence, but what distinguishes this particular value of energy which provides the common upper bound? The answer is remarkably simple. The limiting X_∞ solutions (whose energies give upper bounds) of our family and the BM family saturate the Bogomol'nyi inequalities in the abelian sectors of respective theories and therefore their energies are equal to the unit magnetic charge. To see this, consider first the dilatonic solutions. As was discussed above, the second region $R_1 < r < R_2$, covers the whole space as $n \to \infty$, since in this limit $R_1 \to 0$ and $R_2 \to \infty$. As n grows the amplitude of oscillations of the function w decreases and goes to zero as $n \to \infty$. Thus, for $n \to \infty$ the solution X_n tends (nonuniformly) to the (singular) abelian magnetic monopole described in Section 3:

$$X_\infty = (\ w = 0,\ \phi = \ln(1 + \frac{1}{r})\ ) \ . \tag{26}$$

As I have shown in Section 3, in the $U(1)$ sector of the YM-dilaton theory, the static solutions satisfy the Bogomol'nyi inequality $E \geq Q$. The limiting solution X_∞ saturates the bound in the Q=1 sector.

The behaviour of the BM solutions (in isotropic coordinates) is similar: as $n \to \infty$ the YM field tends to the abelian magnetic monopole while the metric develops a horizon and becomes the extremal Reissner-Nordstrom black hole solution with unit magnetic charge. Thus

$$X_{\infty}^{BM} = \Big(w = 0,\ ds^2 = -e^{-2U}dt^2 + e^{2U}(dx^2 + dy^2 + dz^2)\Big) \ , \tag{27}$$

where

$$U = \ln(1 + \frac{1}{r}) \ . \tag{28}$$

It is well known that this solution saturates the Bogomol'nyi inequality in Einstein-Maxwell theory [15]. Actually, the limiting solutions (31) and (32) can be mapped one into another by the duality transformation $U \leftrightarrow \phi$ and $\alpha \leftrightarrow 1/\alpha$ in the abelian sector of the theory defined by the action (1).

¿From the content of the last two paragraphs it is clear that to prove rigorously that the energy is bounded from above by one, one needs in the YM-dilaton theory (and in the Einstein-Yang-Mills theory in the case of BM solutions) a sort of Bogomol'nyi inequality with *reversed* sign, $E \leq Q$, which is saturated by the limiting abelian solution. Unfortunately, I wasn't able to find such an inequality. It would be probably easier, but also much less interesting, to find an upper bound which is not sharp (for the BM solutions that was done in ref.[14]). Also, it is not difficult to obtain not strict bounds on initial parameters. For example, multiplying eq.(8) by ϕ, integrating by parts and combining the result with eq.(29) yields the identity

$$\int_0^\infty (\phi - 1)e^{-2\phi} \left[w'^2 + \frac{(1-w^2)^2}{2r^2} \right] dr = 0 \ , \tag{29}$$

which implies that $\phi(0) \geq 1$.

5 Stability analysis

In this Section I address the issue of linear stability of the static solutions described above. To that purpose one has to analyse the time evolution of linear perturbations about the equilibrium configuration. I will assume that the time-dependent solutions remain spherically symmetric and the YM field stays within the ansatz (6). This is sufficient to demonstrate instability because unstable modes appear already in this class of perturbations. The spherically symmetric evolution equations are

$$-(e^{-2\phi}\dot{w})^{\cdot} + (e^{-2\phi}w')' + \frac{1}{r^2}e^{-2\phi}w(1-w^2) = 0 \ , \tag{30}$$

$$-r^2\ddot{\phi} + (r^2\phi')' + 2e^{-2\phi}\left[w'^2 + \frac{(1-w^2)^2}{2r^2}\right] = 0\ , \tag{31}$$

where dot denotes differentiation with respect to time t.

Now, I take the perturbed fields: $w(r) + \delta w(r,t)$, and $\phi(r) + \delta\phi(r,t)$, where $(w(r), f(r))$ is a static solution, and insert them into the eqs. (35),(36). Linearizing and assuming the harmonic time-dependence for the perturbations: $\delta w(r,t) = e^{i\sigma t}\xi(r)$ and $\delta\phi(r,t) = e^{i\sigma t}\psi(r)$, one obtains an eigenvalue problem

$$-\xi'' + 2\phi'\xi' + 2w'\psi' - \frac{1}{r^2}(1-3w^2)\xi = \sigma^2\xi\ , \tag{32}$$

$$-(r^2\psi')' - 4e^{-2\phi}\left[w'\xi' - \frac{1}{r^2}w(1-w^2)\xi - (w'^2 + \frac{(1-w^2)^2}{2r^2})\psi\right] = \sigma^2 r^2\psi\ . \tag{33}$$

It is easy to check that if the perturbations satisfy the boundary conditions

$$\begin{aligned} &\xi(0) = 0 \qquad\qquad \psi(0) = const\ , \\ &\xi(\infty) = 0 \qquad\qquad \psi(\infty) = 0\ , \end{aligned} \tag{34}$$

then the above system is self-adjoint, hence eigenvalues σ^2 are real. Instability manifests itself in the presence of at least one negative eigenvalue.

To solve the eigenvalue equations (37),(38) with the boundary conditions (39), is a straighforward but tedious numerical problem. I have done that for several lowest-energy static solutions. It turns out that the solution X_1 has exactly one unstable mode of frequency $\sigma^2 \simeq -0.0225$. Each succesive static solution picks up one additional unstable mode (I have checked this up to $n = 4$). This is consistent with the fact that the limiting solution X_∞, given by (31), has infinitely many unstable modes. To prove this, consider the perturbations of X_∞ with $\delta\phi = 0$. Then, eq.(37) reads

$$-\xi'' - \frac{2}{r(1+r)}\xi' - \frac{1}{r^2}\xi = \sigma^2\xi\ . \tag{35}$$

This equation has infinitely many negative modes because the zero energy solution satisfying $\xi(0) = 0$, has infinitely many nodes as can be seen easily from the asymptotic solution.

The result that the solution X_n has n unstable modes fits very nicely to the interpretation of solutions. In particular, for the interpretation of the solution X_1 as a sphaleron, it is crucial that it has exactly one unstable mode. However, remember that I have considered a restricted class of perturbations, and by doing so some directions of instability might have been supressed.

If there are additional unstable modes outside the ansatz (which I doubt), the interpretation of solutions given by Sudarsky and Wald would have to be revised.

6 Sudarsky and Wald's argument

Sudarsky and Wald have recently proposed a heuristic argument which "explains" the existence and instability of the BM solutions of the Einstein-YM equations. This argument is, in principle, applicable to other theories involving the $SU(2)$-YM field, which are not scale invariant and possess a stable solution. Below I outline the SW argument in application to the $SU(2)$-YM-dilaton theory.

Let $\tilde{\Gamma}$ be a space of all functions (A_i, ϕ), defined over R^3, for which the energy E is finite. Let $\overline{\Gamma}$ be a subspace of $\tilde{\Gamma}$ with $\phi(\infty) = 0$. The static solutions are extrema of energy on $\overline{\Gamma}$. One such extremum is the vacuum solution $(A_i, \phi = 0)$ for which energy has a global minimum $E = 0$. Now, the key feature of the $SU(2)$-YM group is the presence of "large gauge transformations" i.e. topologically inequivalent cross-sections of the YM-bundle, classified by the homotopy group $\pi_3(SU(2)) \simeq Z$. Thus, the energy functional E has a countable set of disconnected global minima corresponding to the trivial vacuum $(A_i, \phi = 0)$ and all large gauge transformations of it. To avoid complications with the group of small gauge transformations G, it is convenient to pass from $\overline{\Gamma}$ to the space of gauge orbits $\Gamma = \overline{\Gamma}/G$.

Now, to apply Morse theory methods in Banach spaces, one needs a sort of compactness condition (like the Palais-Smale condition). A convenient way of implementing such a condition (which is here simply assumed to hold), is to introduce on Γ a Riemannian metric G_{AB} (upper case latin letters denote indices of tensor fields on Γ), such that the flow generated by the vector field $M^A = -G^{AB}\nabla_B E$ carries each point of Γ to a critical point of E. As discussed above, there exist a countable set of global minima of E. Since this set is disconnected, the flow defined by M^A cannot carry all points of Γ to global minima (or local minima if any exist), because this would contradict the connectedness of Γ. Thus, the set, $\Gamma_1 \subset \Gamma$, of points which do not flow to local minima, must contain at least one critical point of E. A critical point on Γ_1 with smallest energy E_1 is a saddle point on Γ with exactly one unstable direction. This is believed to account for the existence of the solution X_1.

Actually, there is a countable set of local minima of E restricted to Γ_1, namely X_1 and all large gauge transformations of it. Hence, one can repeat the above argument, replacing Γ by Γ_1 (and assuming that Γ_1 is connected), to predict the existence of a submanifold $\Gamma_2 \subset \Gamma_1$ with a point X_2, whose energy E_2 minimizes E restricted to Γ_2. The point X_2 is an extremum of E on Γ, which has the 2-dimensional space of unstable directions. This is believed to account for the existence of the second static solution X_2. All higher n solutions are predicted by the repetition of this argument.

It seems very unlikely that the SW argument in its present form can be converted into a rigorous proof. However, the same argument can be made for spherically symmetric connections. Then, the powerful methods of equivariant Morse theory are available, and in fact these methods were succesfully applied in related problems [16]. In my opinion this is a very promising direction for future research.

Another possibility of proving rigorously the existence of numerical solutions found in this paper is to apply the methods of the dynamical systems theory. This approach was used recently by Smoller and his collaborators [17,18], who proved the existence of the BM family of solutions to the Einstein-YM equations. A similar proof should be possible for the YM-dilaton equations although it might be more difficult, because here the corresponding (nonautonomous) dynamical system is four-dimensional whereas in the Einstein-YM case it is three-dimensional.

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Figure captions

Fig.1 The function w for the first three globally regular solutions.

Fig.2 The function ϕ for the first three globally regular solutions.