Abstract: Elliptic operators have proved to be a powerful tool for relating geometry on manifolds to global topological properties. A large number of geometric structures can be described in terms of filtered manifolds, contact structures arguably being among the best known. Interesting differential operators associated with filtered manifolds tend to be hypoelliptic rather than elliptic. To study these operators, the classical Heisenberg calculus on contact manifolds has recently been generalized to all filtered manifolds. In this talk we will present a Rockland type theorem, characterizing the existence of a parametrix in this calculus. As a first application of this result, we will discuss the heat kernel asymptotics for Rockland differential operators on general filtered manifolds, and address related statements, including the McKean–Singer index formula and Weyl’s law. We will also discuss applications to a particular geometry in five dimensions, related to the exceptional Lie group G2, which has been a motivating example in our investigations. This talk is based on joint work with Shantanu Dave.

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