International workshop

Higher structures emerging from renormalisation

October 12th to October 16th 2020

Titles and abstracts

Ajar Chandra (Imperial College, London)
Regularity and Coherence of Modelled Distributions

Abstract: I will discuss some recent work on a priori bounds for SDEs and SPDES that crucially leverages an algebraic insight into the regularity condition imposed on modelled distributions in the theory of regularity structures. This is based on one joint work with Augustin Moinat and Hendrik Weber and another joint work with Timothée Bonnefoi, Augustin Moinat, and Hendrik Weber. No prior knowledge of regularity structures is needed for this talk.

Ismaël Bailleul (Rennes 1 University, France)

Paracontrolled calculus and regularity structures

Abstract:
Paracontrolled calculus and regularity structures were given birth essentially at the same time. While the foundations of the theory of regularity structures are now fully clarified and can be popularized as a black box for solving a number of stochastic singular PDEs, paracontrolled calculus has not yet reached the same level of maturity. It happens nonetheless to be possible to compare the two set of tools and go from one world to another. This talk will be dedicated to explaining this intertwining.

Marc Bellon (Sorbonne University, Paris VI)

Ward-Schwinger-Dyson equations and their resurgent analysis

Abstract:
Ward-Schwinger-Dyson equations allow to obtain relations in quantum field theory directly on renormalised green functions. The solutions of their truncation at any order satisfy renormalisation group equations and presumably any symmetry associated to a gauge symmetry.

We indicate how they can be used to prove results on the resurgent nature of the various functions used to study quantum field theory, anomalous dimensions or propagators.

Yvain Bruned (Edinburgh University, UK)

Resonance based schemes for dispersive equations via decorated trees

Abstract:
In this talk, we will present a new numerical framework for dispersive equations based on algebraic methods for SPDEs. The main idea of the scheme is to embed the underlying resonance structure into the discretisation at low regularity. Using, a tailored decorated tree formalism, we control the nonlinear frequency interactions in the system up to arbitrary high order. We adapt SPDEs formalism to the context of dispersive PDEs by using a novel class of decorations {which encode the dominant frequencies}. The structure proposed in this paper is new and gives a variant of the Butcher-Connes-Kreimer Hopf algebra on decorated trees. This is a joint work with Katharina Schratz.

Damien Calaque (Montpellier University, France)

Deformation quantization with branes and coloured MZVs

Abstract:
In a recent work, Banks, Panzer and Pym have shown that the coefficients appearing in Kontsevich's celebrated deformation quantization formula are linear combinations of MZVs. We explain a generalization to the setting of deformation quantization in the presence of branes (in the sense of Cattaneo-Felder), where MZVs are replaced by coloured MZVs.

Viet Dang Nguyen (Lyon 1 University, France)

The spectral action principle on Lorentzian scattering spaces

Abstract:
The spectral action principle of Alain Connes is one of the cornerstones of the noncommutative geometry approach to the standard model, yet it is limited to the setting of compact Riemannian manifolds, which is incompatible with General Relativity. Generalizing the principle to the Lorentz signature has been a longstanding open problem. In the present work, we give a global definition of complex Feynman powers $(\square+m^2+i0)^{-s}$ on Lorentzian scattering spaces, and show that the restriction of their Schwartz kernel to the diagonal has a meromorphic continuation. When $d=4$, we show the pole at $s=1$ equals a generalized Wodzicki residue and is proportional to the Einstein-Hilbert action density, proving a spectral action principle in Lorentz signature. (This is joint work with Michal Wrochna.)

Joscha Diehl (Greifswald University, Germany)

Tropical quasisymmetric functions

Abstract:
The tropical (or min-plus) semiring appears in a plethora of areas, among them, algebraic geometry, optimization, and dequantization' in quantum physics. Motivated by time series analysis, we introduce the concept of quasisymmetric functions over the tropical (or, in fact, any commutative) semiring and investigate their algebraic setting. Joint work with Kurusch Ebrahimi-Fard (NTNU Trondheim) and Nikolas Tapia (WIAS Berlin).

Gerald Dunne (University of Connecticut, US)

Resurgent Asymptotics of Hopf Algebraic Dyson-Schwinger Equations

Abstract:
Hopf Algebraic methods in a wide class of quantum field theories lead to Dyson Schwinger equations expressed as systems of nonlinear differential equations. Formal solutions of these ODEs correspond to perturbative expansions, and methods of resurgent asymptotics can be used to decode associated non-perturbative information. I will discuss the basic ideas of resurgence in ODEs, and present some explicit examples in quantum field theory.

Loic Foissy (Université du Littoral, Calais, France)

Cointeracting bialgebras

Abstract:
Pairs of cointeracting bialgebras recently appears in the literatureof combinatorial Hopf algebras, with examples based on formal series,on trees (Calaque, Ebrahimi-Fard, Manchon), graphs (Manchon), posets...
We will give several results obtained on pairs of cointeracting bialgebras:actions on the group of characters, antipode, morphisms to quasi-symmetric functions...and we will give applications to Ehrhart polynomials and chromatic polynomials.

Alessandra Frabetti (Lyon 1 University, France)

Transport maps as direct connections on groupoids

Abstract:
In Martin Hairer's theory of Regularity Structures, transport maps relate local solutions of stochastic PDEs and allow to construct extended (global) solutions on a flat space or on

embedded submanifolds. In a work in progress with Sara Azzali, Youness Boutaib and Sylvie Paycha, we consider jets of sections of a vector bundle and interpretate the transport maps as direct connections on an associated Lie groupoid over the base manifold. This idea is coherent with Nicolai Teleman's generalization of the usual parallel transport induced by a linear connection on the vector bundle.

Klaus Fredenhagen (University of Hamburg, Germany)

Renormalization and C*-algebras

Abstract:
Renormalized quantum field theory is usually formulated in terms of formal power series with coefficients in algebras of unbounded operators. We investigate the possibility to formulate it instead in terms of unitary elements of a C*-algebra which is defined in terms of relations which reflect basic properties of the theory, including the condition of causality and the Lagrangian which determines the dynamics. The talk is based on joint work with Detlev Buchholz.

Nicolas Gilliers (NTNU, Trondheim, Norway)

Dendriform algebras in operator-valued probability theory

In this talk we explain the basics of operator-valued free non-commutative probability theory. Then, using operads and 2-monoidal categories, we will extend the dendriform algebras approach to free, boolean and monotone moment-cumulant relations to their operator-valued counterparts. I will end this talk with loose ends related to free multiplicative operator-valued convolution and potential use of dendriform algebras in this context.

Li Guo (Rutgers University, Newark, US)

Renormalization of quasisymmetric functions

Abstract: The Hopf algebra of quasisymmetric functions (QSym) has played a central role in a large class of combinatorial algebraic structures related to symmetric functions. A natural linear basis of QSym is the set of monomial quasisymmetric functions defined by compositions, that is, vectors of positive integers. Extending such a definition for weak compositions, that is, vectors of nonnegative integers, leads to divergent expressions. This difficulty was addressed by a formal regularization in a previous work with Jean-Yves Thibon and Houyi Yu. Here we apply the method of renormalization in the style of Connes and Kreimer and realize weak composition quasisymmetric functions as power series. The resulting Hopf algebra has the Hopf algebra of quasisymmetric functions as both a Hopf subalgebra and a Hopf quotient algebra. This is a joint work with Houyi Yu and Bin Zhang.

Antoine Hocquet (Technical University, Berlin)

Monoid-valued Sewing Lemmata and applications

Abstract:
I will present a generalisation of the multiplicative Sewing Lemma by Feyel, De La Pradelle and Mokobodzki '08, and explain how it relates to the existing variants in the literature, notably in the context of semi-groups. I will give a few examples that illustrate a concept of submultiplicative monoid, which is a natural framework in which this theorem can be used.

Dirk Kreimer (Humboldt University, Berlin)

Renormalization, Categories and Self-similarity

Abstract:
The talk will review the use of renormalization for and beyond perturbative quantum field theory. It aims at exhibiting universal combinatorial structure underlying any study of fixed-point equations and emphasizes connections to operads and category theory.

Antti Kupiainen (University of Helsinki, Finland)

Integrability of Liouville Conformal Field Theory

Abstract: A. Polyakov introduced Liouville Conformal Field theory (LCFT) in 1981 as a way to put a natural measure on the set of Riemannian metrics over a two dimensional manifold. Ever since, the work of Polyakov has echoed in various branches of physics and mathematics, ranging from string theory to probability theory and geometry.
In the context of 2D quantum gravity models, Polyakov's approach is conjecturally equivalent to the scaling limit of Random Planar Maps and through the Alday-Gaiotto-Tachikava correspondence LCFT is conjecturally related to certain 4D Yang-Mills theories.
Through the work of Dorn,Otto, Zamolodchikov and Zamolodchikov and Teschner LCFT is believed to be to a certain extent integrable.
I will review a probabilistic construction of LCFT developed together with David, Rhodes and Vargas and recent proofs concerning the integrability of LCFT:

-The proof in a joint work with Rhodes and Vargas of the DOZZ formula
(Annals of Mathematics, 81-166,191 (2020)
-The proof in a joint work with Guillarmou, Rhodes and Vargas of the
bootstrap conjecture for LCFT (arXiv:2005.11530).

Franz Lehner (University of Graz, Austria)

Cumulants, Hausdorff Series and Noncommutative Quasisymmetric Functions

Abstract: Classical, free and Boolean independence is characterized by the vanishing of the associated mixed cumulants. This is not the case for monotone independence, the reason being the failure of exchangeability which is replaced by the weaker property of spreadability. In joint work with T. Hasebe we could show that in this generalized setting mixed cumulants can be described in terms of Goldberg coefficients.
This development has a parallel in the transition from symmetric functions to quasisymmetric functions and indeed Novelli and Thibon pointed out that the cumulant identities can be interpreted as identities satisfied by Eulerian idempotents in the Hopf algebra WQSym of word quasi-symmetric functions. We report on our joint effort to understand this correspondence.

Claudia Malvenuto (University La Sapienza, Rome)

Primitive elements in the Poirier-Reutenauer Hopf Algebra of tableaux

Abstract: The theories of character of symmetric group and of symmetric functions make use of the combinatorics of Young tableaux, such as the Robinson-Schensted algorithm, Schuetzenberger's "jeu de taquin", and evacuation.
In 1995 Poirier and Reutenauer introduced some algebraic structures, different from the plactic monoid, which induce some products and coproducts of tableaux, with homomorphisms. Their starting point are the two dual Hopf algebras of permutations, introduced by Malvenuto and Reutenauer in 1995.
In 2006 Aguiar and Sottile studied in more detail the so called Malvenuto-Reutenauer algebra of permutations : among other things, they introduce a new basis, by Moebius inversion in the poset of weak Bruhat order, that allows them to describe the primitive elements of the Hopf algebra of permutations.
Using this method, we determine the primitive elements of the Poirier-Reutenauer algebra of tableaux, using a partial order on tableaux defined by Taskin.
(Joint work with Christophe Reutenauer)

Dominique Manchon (Auvergne University, Clermont-Ferrand, France)

Families of algebraic structures

Abstract:
Algebraic structures may come into families, where each operation at hand is replaced by a family of operations indexed by some parameter set, which often bears a semigroup structure. The characteristic relations of the structure at stake (e.g. associativity relation, of Jacobi identity, of pre-Lie relation, etc.) persist together with a play on the parameters involving the semigroup structure. The first example, namely Rota-Baxter family algebras, comes from the momentum scheme in Quantum Field Theory. I will introduce Rota-Baxter families, then address other family structures (dendriform, duplicial, pre-Lie,...), and finally give a general account of family algebras over a finitely presented linear operad, this operad together with its presentation naturally defining an algebraic structure on the set of parameters. Based on recent joint works with Loïc Foissy, Xing Gao and Yuanyuan Zhang.

Antoine Mouzard (Université de Rennes 1, France)

Quasilinear singular SPDEs and paracontrolled calculus

Abstract:
We show how to refine the toolbox of paracontrolled calculus to make sense of a number of quasilinear singular PDEs, including the quasilinear versions of the 3-dimensional (gPAM) and (1+1)-dimensional (gKPZ) equations.

Kasia Rejzner (University of York, UK)

A geometrical interpretation of unitary Master Ward Identity

Abstract:
Recently, Buchholz and Fredenhagen proposed a formulation for interacting quantum field theory (QFT) using the C*-algebra generated by unitaries interpreted as local S-matrices and by relations corresponding to causality and dynamics; the latter implemented by the unitary Schwinger-Dyson equation. Unitary Master Ward Identity is a natural generalization of the unitary Schwinger-Dyson equation to the situation, where non-trivial local symmetries are present. This talk is based on recent results in collaboration with Brunetti, Duetsch, Fredenhagen and Hawkins.

Vincent Rivasseau (University Paris Sud)

Three roads from tensors models to continuous geometry

Abstract:
I shall describe the three roads from tensors models to continuous geometry: the road of the double scaling followed by multiples scalings, the road of the renomalization group (by breaking the propagator symmetry), and the road that I currently work on, that of fields theories on random trees.

Enrico Russo (Sorbonne University, Paris VI)

Renormalisation through WSD equations

Abstract:
We introduce Ward-Schwinger-Dyson equations in the case of phi^3 QFT model in 6 dimensions. The aim of our techniques is to overcome the overlapping divergences appearing in the usual Schwinger-Dyson equation for the propagator. The main characteristics of this system are to be formulated purely in terms of renormalized quantities and to give solutions satisfying renormalization group equations. We show how WSD are intertwined with Renormalisation group, and how the latter is an integral part of any efficient resolution scheme of this system. It is our belief that this method can be generalized to the case of gauge fields.

Alexander Schmeding (University of Bergen, Norway)

Lie groups of Hopf algebra characters

Abstract:
Character groups of Hopf algebras have provided a convenient setting to formulate renormalisation procedures. Already in the earliest works by Connes and Kreimer on these topics, certain differential equations were formulated and solved on these groups. However, it was not until much later, that the global analytic structure of these groups was described. In this talk we will review the (infinite-dimensional) Lie group structure of character groups of Hopf algebras and certain subgroups. We will then report on their relation to renormalisation and recent developments and applications of these groups.

Leila Schneps (CNRS, Paris)

A dimorphic description of the Grothendieck-Teichmüller Lie algebra

Abstract:
It is a long-standing conjecture that the Grothendieck-Teichmüller Lie algebra grt is isomorphic to the double shuffle Lie algebra, which is itself conjectured to be dual to the Q-vector space of multiple zeta values. The comparison between these two Lie algebras is difficult due to the difference in their definitions. In this talk I will give a new equivalent definition of grt which is much closer in nature to the description of couble shuffle, and use it to recover Furusho's important theorem proving that grt is included in the double shuffle Lie algebra. The formulas in the new definition of grt are interesting and in particular heavily rely on an algebraic form of renormalisation.

Nikolas Tapia (WIAS, Berlin)

Unified signature cumulants and generalized Magnus expansions

Abstract: Classical cumulants linearise properties of random variables normally encoded by moments. They are recursively defined through their generating function, as the logarithm of the corresponding characteristic function. For càdlàg semimartingales, the Marcus signature lift plays a similar role. Signature cumulants, defined as logarithm of expected signatures, are seen to satisfy a fundamental functional relation. This equation, in a deterministic setting, contains Hausdorff's differential equation, which itself underlies Magnus' expansion. The (commutative) case of multivariate cumulants arises as another special case and yields a new Riccati-type relation valid for general semimartingales. Here, the accompanying expansion provide a new view on recent "diamond" and "martingale cumulants" (Alos et al '17, Lacoin et al '19., Friz et al. '20) expansions. Some concrete examples are given.

Béatrice de Tilière (Paris-Dauphine University)

Elliptic dimers and genus 1 Harnack curves

Abstract:
We consider the dimer model on a bipartite periodic graph with elliptic weights introduced by Fock. The spectral curves of such models are in bijection with the set of all genus 1 Harnack curves. We prove an explicit and local expression for the two-parameter family of ergodic Gibbs measures and for the slope of the measures. This is joint work with Cédric Boutillier and David Cimasoni.

Josef Teichmann (ETH Zürich)

Randomized signature methods and Reservoir Computing

Abstract: We connect the paradigm of reservoir computing with well-known methods from the theory of rough paths to represent path space functionals. The construction yields efficient techniques for learning singular dynamics from observations.

Bruno Vallette (Sorbonne University, Paris XIII)

Operadic renormalisation group

Abstract:
The operadic interpretation of the Batalin—Vilkovisky formalism given by Mnev and Merkulov opened the doors to the operadic calculus in renormalisation theory. These authors showed that the passage from an action to an effective action is nothing but the application of the homotopy transfer theorem of unimodular Lie bialgebras, which is a structure encoded by a wheeled properad. In some sense, this is not a surprise since the homotopy transfer theorem relies on some homological perturbation method. Dotsenko--Shadrin--Vallette proved that the homotopy transfer theorem can be obtained by the action of seminal elements of the deformation group obtained by integrating operadic convolution pre-Lie algberas. In this talk, I will explain how to integrate, with graph exponentials, properadic convolution algbras which encode the deformation theory of types of bialgebras. The action of some seminal elements of this new deformation group gives the homotopy transfer theorem for these types of bialgebras, so this new group of symmetries is a universal renormalisation group. This is a joint work with Ricardo Campos.

Yannic Vargas (IVIC, Venezuela)

Algebraic structure of the Hopf algebra of double posets

Abstract: A Hopf algebra of double posets was introduced by Claudia Malvenuto and Christophe Reutenauer in 2011, motivated by the study of pictures of tableaux as defined by Zelevinsky. Starting from the correspondence between top-cones in the braid arrangement and partial orders, we investigate several properties of the Hopf algebra of double posets as the image of a Hopf monoid (via the Fock functor). In particular, we obtain a non-cancellative formula for the antipode and a description of the primitive space.

Michał Wrochna (University of Cergy-Pontoise, France)

Why do we need de Sitter space?

Abstract:
The folklore knowledge is that Quantum Field Theory is (or should be) better behaved on de Sitter space than on its much better known cousin Minkowski space. The main reason are better infrared properties, which also imply better behaviour of classical non-linear fields. On top of that, de Sitter space has a remarkable asymptotic structure which underpins dS/CFT duality and survives under rather dramatic perturbations of the metric. This talk is intended as an introduction to asymptotically de Sitter spaces and as a survey of recent developments in QFT in this setting, with a special emphasis on propagators and their analytic properties.

Raimar Wulkenhaar (University of Münster, Germany)

Blobbed topological recursion of the quartic analogue of the Kontsevich model

Abstract:
We provide strong evidence for the conjecture that the analogue of Kontsevich's matrix Airy function, with the cubic potential Tr(\Phi^3) replaced by a quartic term Tr(\Phi^4), obeys the blobbed topological recursion of Borot and Shadrin. We identify in the quartic Kontsevich model three families of correlation functions for which we establish interwoven loop equations. One family consists of symmetric meromorphic differential forms \omega_{g,n} labelled by genus and number of marked points of a complex curve. We reduce the solution of all loop equations to a straightforward but lengthy evaluation of residues. In all evaluated cases, the $\omega_{g,n} consist of a part with poles at ramification points which satisfies the universal formula of topological recursion, and of a part holomorphic at ramification points for which we provide an explicit residue formula.

Lorenzo Zambotti (Sorbonne University, Paris VII)

A Microlocal Approach to Renormalization in Stochastic PDEs

Abstract:
We present a novel framework for the study of a large class of non-linear stochastic PDEs, which is inspired by the algebraic approach to quantum field theory. The main merit is that, by realizing random fields within a suitable algebra of functional-valued distributions, we are able to use techniques proper of microlocal analysis which allow us to discuss renormalization and its associated freedom without resorting to any regularization scheme and to the subtraction of infinities.
This talk is based on the article https://arxiv.org/pdf/2009.07640.pdf, joint work with C. Dappiaggi, N. Drago and P. Rinaldi.