

## Bialgebras in Free Probability

February 1 - April 22, 2011

Workshop on “Combinatorial, Bialgebra, and Analytic Aspects”

February 14 - 25, 2011

organized by M. Aguiar, F. Lehner, R. Speicher, D. Voiculescu

- **Tuesday, February 15**

**10:00 – 10:30:** Coffee

**10:30 – 11:20:** R. Speicher: **Combinatorial aspects of free probability 2**

**11:30 – 12:20:** R. Lenczewski: **Matricial  $R$ -transform**

*Abstract:* We show that addition of strongly matricially free random variables leads to the ‘matricial  $R$ -transform’ related to the associated convolution. It is a linear combination of Voiculescu’s  $R$ -transforms in free probability with coefficients given by internal units of the considered array of subalgebras. This allows us to view the associated linearization formula as the ‘matricial linearization property’ of the  $R$ -transform. Since strong matricial freeness unifies the main types of noncommutative independence, the matricial  $R$ -transform plays the role of a unified noncommutative analog of the logarithm of the Fourier transform for the main types of noncommutative independence.

**14:00 – 14:50:** H. Bercovici: **On sums and products in finite factors**

*Abstract:* We discuss the solution of the analogue of the Horn problem in finite factors. This is the problem of determining the allowed spectral behavior of a sum of two self-adjoint elements with given eigenvalues. We also discuss the multiplicative version of this problem about which much less is known.

**15:00 – 15:30:** Coffee

**15:30 – 16:20:** T. Hasebe: **On Cauchy distributions in non-commutative probability**

*Abstract:* Four independences are known in a non-commutative probability space, i.e., tensor, free, monotone and Boolean ones. For each independence, strictly stable distributions can be defined as analogues of the usual ones in probability theory. In general, strictly stable distributions depend on a choice of independence. However, strictly stable distributions with index one are Cauchy distributions in all the four independences. We will understand this coincidence by introducing generalized concepts of moments and cumulants.

**16:30 – 17:00:** M. Brannan: **Approximation properties for free orthogonal and free unitary quantum groups**

*Abstract:* The free orthogonal and free unitary quantum groups, denoted by  $O_N^+$  and  $U_N^+$  ( $N \geq 2$ ), were introduced by Shuzhou Wang in 1993. In recent years it has become increasingly apparent that these quantum groups share many deep connections with free probability theory. In particular, the associated reduced von Neumann algebras,  $L^\infty(O_N^+)$  and  $L^\infty(U_N^+)$ , turn out to share many structural properties with the free group factors  $L(\mathbb{F}_N)$ . In this talk, we will pursue this connection with  $L(\mathbb{F}_N)$  further, and show that  $L^\infty(O_N^+)$  and  $L^\infty(U_N^+)$  always have the Haagerup approximation property. Using this result together with some Haagerup-type inequalities obtained by Roland Vergnioux (J. Operator Theory, 2007), we also show that the reduced  $C^*$ -algebras  $C(O_N^+)$  and  $C(U_N^+)$  have the metric approximation property.