

Bialgebras in Free Probability

February 1 - April 22, 2011

Workshop on “Combinatorial, Bialgebra, and Analytic Aspects”

February 14 - 25, 2011

organized by M. Aguiar, F. Lehner, R. Speicher, D. Voiculescu

- **Wednesday, February 16**

10:00 – 10:30: Coffee

10:30 – 11:20: J-L. Loday: **Generalized Hopf algebras and operads**

Abstract: A classical result of Hopf and Borel says that a Hopf algebra which is commutative, cocommutative and conilpotent is necessarily free as a commutative algebra. There is a similar theorem for associative algebras, but one has to modify the Hopf compatibility relation by the so-called “unital infinitesimal compatibility relation”. We will show how to generalize these structure theorems to other settings involving different kinds of types of algebras, that is of operads.

11:30 – 12:20: Piotr Śniady: **Free cumulants in representation theory**

Abstract: Characters of the symmetric groups can be calculated by a combinatorial formula which involves summation over some graphs drawn on two-dimensional surfaces. The leading term of the character is related to the summands which are planar (graphs drawn on a sphere); this leading term turns out to be a free cumulant of the representation. Surprisingly, the sub-leading terms can be expressed as a polynomial function of the free cumulants (called Kerov polynomial). Similar combinatorial objects show up in the calculation of some matrix integrals; is it possible to extend Kerov polynomials to the domain of the random matrix theory?

14:15 – 15:00: C. Köstler: **Noncommutative independence in the infinite braid and symmetric group**

Abstract: We introduce in an elementary setting to the recent merge of a noncommutative de Finetti type result with representations of the infinite braid and symmetric group which allows to derive factorization properties from symmetries. We explain some of the main ideas of this approach and discuss a constructive procedure to use in applications. Finally we illustrate the method by applying it to the theory of group characters.

15:05 – 15:30: Coffee

15:30 – 16:20: F. Götze: **Asymptotic Expansions in the Free Central Limit Theorem**

Abstract: We show asymptotic approximations of first and second order in the Central Limit Theorem of Free Probability. For the n -fold free convolution we establish error bounds of order $o(n^{-1/2})$ and $o(n^{-1})$ depending e.g. on the existence of three or four moments. The expansion results obtained are valid under minimal moment assumptions when compared to the classical case.

This is joint work with G. Chistyakov.

- **Thursday, February 17**

10:00 – 10:30: Coffee

10:30 – 11:20: R. Speicher: **Combinatorial aspects of free probability 3**

11:30 – 12:20: F. Lehner: **The normal law, free probability, and a Hopf algebra of rooted binary trees**

Abstract: In joint work with S. Belinschi, M. Bożejko and R. Speicher we proved some time ago the somewhat strange fact that the classical normal distribution is freely infinitely divisible. This fact is equivalent to the positive definiteness of the sequence of free cumulants of the normal distribution. In this talk we review combinatorial aspects of this sequence, which counts the number of connected pair partitions and has been studied by Touchard, Riordan and others previously. We present other combinatorial interpretations, from computer science to Hopf algebras of rooted binary trees which appear in the context of renormalization theory. In spite of the seeming simplicity of the sequence, there still is no direct combinatorial proof of its positive definiteness.

14:15 – 15:00: C. Mazza: **B-Series, Schwinger-Dyson Equations And Wigner Processes**

Abstract: We consider series indexed by rooted trees, which are relevant for some models in quantum field theory. We established a link between the combinatorics of renormalization and the so-called Butcher's group. This group is composed of B-series, which are in fact numerical solutions of ordinary differential equations associated to methods. The Butcher's group is also the character group of Kreimer's algebra of rooted trees. We will focus on some combinatorial aspects of B-series, and derive associated Schwinger-Dyson equations in some special cases. We will also exhibit a special family of B-series which are related to traces of products of semi-circular elements.

15:05 – 15:30: Coffee

15:30 – 16:20: T. Cabanal-Duvillard: **A generalization of a result of Marchenko & Pastur, providing a family of Bercovici-Pata bijections**

Abstract: In 1967, Marchenko and Pastur have determined the limit of the spectral law of sums of weighted rank one projectors. These limit distributions have been identified afterwards as free compound Poisson laws. In 2005, the authors have extended Marchenko-Pastur approach to any infinitely divisible law, giving a matricial realization of the Bercovici-Pata bijection. In this talk, another generalization will be presented, providing the limit distribution when the rank-one projectors are chosen with much fewer assumptions than in Marchenko-Pastur paper.

16:30 – 17:00: C. Sattler: **Free log-normal distribution and confluent hypergeometric series**

Abstract: Bercovici and Voiculescu (1992) introduced measures supported on the positive, real line \mathbb{R}^+ and the torus \mathbb{T} , which are free analogues of the log-normal distribution and the rolled-up normal distribution, respectively. These measures appear in the free multiplicative central limit theorem, the free multiplicative brownian motion (Biane, 1997), and as limit distribution of some unitary random matrices (Biane, 1997).

We compute the free cumulants of these measures. This leads to some nice formulas for Kummer's confluent hypergeometric series ${}_1F_1(a; b; z)$. Connections between hypergeometric series ${}_1F_1(1 \pm n; 2; z)$ and noncrossing partitions $NC(n)$ are obtained. We also discuss the density function and the support of these measures on the torus \mathbb{T} .

- **Friday, February 18**

10:00 – 10:30: Coffee

10:30 – 11:20: R. Speicher: **Combinatorial aspects of free probability 4**

11:30 – 12:20: J. Mingo: **Second Order Freeness and Wigner Ensembles**

Abstract: Twenty years ago Voiculescu showed that self-adjoint Gaussian random matrices and deterministic matrices were asymptotically free. This was later extended to Wigner random matrices. Roland Speicher and I introduced a theory of second order freeness to do for fluctuation moments what freeness does for moments. Self-adjoint Gaussian random matrices and deterministic random matrices are asymptotically free of second order. With Wigner matrices this fails, but an interesting formula involving conditional expectations can still be proved. This is joint work with Roland Speicher.

14:15 – 15:00: K. Dykema: **An application of asymptotic freeness to soficity of groups**

Abstract: After recalling some results about asymptotic freeness of random matrices and showing how they can be used to give lower bounds on free entropy dimension in amalgamated free products, we will examine an analogous discrete situation, and show how these can be used to prove soficity of certain amalgamated free products of groups.

15:05 – 15:30: Coffee

15:30 – 16:20: M. Dołęga : **Colorings of bipartite graphs and polynomial functions on the set of Young diagrams**

Abstract: Polynomial functions (in the sense of Kerov and Olshanski) on the set of Young diagrams are functions which have a prominent role in the asymptotic representation theory of a permutation groups. We will discuss some examples concerning normalized characters and free cumulants. We will also show how to construct a function on the set of Young diagrams from a given bipartite graph and when these kind of functions are polynomial functions. We will give a connection of our result with Jack symmetric functions and some conjectures of Lassalle. Our method involves a differential calculus on the set of Young diagrams and combinatorics of bipartite graphs which will be also discussed. This is a joint work with Piotr Śniady.

All lectures take place in the ESI Boltzmann Lecture Hall