ESI **The Erwin Schrödinger International Institute for Mathematical Physics**

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Bialgebras in Free Probability

February 1 - April 22, 2011

Workshop on "Random Matrix, Operator Algebra, and Mathematical Physics Aspects"

April 11 - 21, 2011

Schedule for week 2: April 18 - 21, 2011

organized by M. Aguiar, F. Lehner, R. Speicher, D. Voiculescu

- **Monday, April 18**

  10:00 – 10:50: S. Woronowicz: **Simplified $E(2)$ quantum group**

  10:50 – 11:30: Coffee

  11:30 – 12:20: F. Benaych-Georges: **Finite rank perturbations of random matrices and free probability theory**
  Abstract: Let us consider a random Hermitian matrix X which empirical eigenvalue distribution tends to a limit distribution as the dimension tends to infinity and such that the extreme eigenvalues tend to the bounds of the support of the limit distribution (it is for example the case when the X is a Wigner matrix: in this case, the limit distribution is the semi-circle law). We shall add a perturbation to X, and thus consider X +P, under the hypothesis is that the rank of P stays bounded as the dimension tends to infinity and that the eigenspaces of X and P are in generic position one to each other (it is for example the case when X is distributed according to the GUE).

  Then, a natural question arises: how are the eigenvalues and the eigenvectors of X perturbed by the addition of P ?

  This question had first been asked, for a quite close model, by Johnstone, and been solved, in a several particular cases, by Baik, Ben Arous, Pch, Fral, Capitaine and Donati-Martin.

  We shall give a general answer, uncovering a remarkable phase transition phenomenon: the limit of the extreme eigenvalues of the perturbed matrix differs from the original matrix if and only if the eigenvalues of the perturbing matrix are above a certain critical threshold. We also examine the consequences of this eigenvalue phase transition on the associated eigenvectors and generalize our results to examine the case of multiplicative perturbations or of additive perturbations for the singular values of rectangular matrices.

  14:15 – 15:05: D. Voiculescu: **Around the free Riemann sphere and duality for infinitesimal bialgebras**

  15:05 – 15:30: Coffee

  15:30 – 16:20: U. Franz: **Symmetries of Levy processes on compact quantum groups**

  16:30 – 17:00: J. D. Williams: **Decomposition and Tightness in Free Probability**
  Abstract: I will present some recent results on a 'prime' decomposition for free probability. Time permitting, I will speak about the tightness phenomenon for divisors of a given probability measure that underlie the proof of this Theorem.

- **Tuesday, April 19**

  10:00 – 10:50: M. Junge: **Martingales with continuous time and application to brownian motion and dilation**
  Abstract: I will briefly discuss a general characterization of Levy concerning brownian motion and how minimal knowledge of stochastic integration (in the noncommutative context) is used towards a classification of certain classes. Another related appearance is the construction of brownian motions from martingales with continuous time parameter following an idea of Doob in the classical case. This construction is useful in construction a free brownian motion driving force behind a semigroup of completely positive unital selfadjoint maps on an arbitrary von Neumann algebra.

  10:50 – 11:30: Coffee

  11:30 – 12:20: M. Capitaine: **Free subordination property and deformed matricial models**
  Abstract: We will show how the subordination function related to the free additive resp. multiplicative convolution allows to describe the eigenstructure of large additive resp. multiplicative spiked deformations of classical matricial models.

  14:15 – 15:05: D. Voiculescu: **Around the free Riemann sphere and duality for infinitesimal bialgebras**

  15:05 – 15:30: Coffee

  15:30 – 16:20: K. Ueda: **On free product von Neumann algebras**
  Abstract: I'll report my recent works (arXiv:1011.5017, arXiv:1101.4991) on arbitrary free product von Neumann algebras.

  16:30 – 17:00: N. Blitvic: **Chords, Norms, and q-Commutation Relations**
  Abstract: The q-commutation relations, represented on the q-Fock space of Bo.zjeko and Speicher, interpolate between the classical commutation relations and the classical anti-commutation relations. In this setting, one can construct the q-semicircular and q-circular operators, acting as deformations of the classical Gaussian and complex Gaussian random variables, respectively.

  Considering the moments of the q-semicircular and q-circular, we contrast the combinatorial structure of the two operators and provide some new characterizations of their moments. As a surprising consequence, the $2n$-norms of the q-circular turn out to be significantly less well behaved (in a certain analytic sense) than those of q-semicircular. In addition, connecting these moments to several essential combinatorial objects appearing in the classical work of Touchard and Riordan and the recent work of Corteel and Williams provides new indication of the structural depth of the of the q-commutative framework.

- **Wednesday, April 20**

  10:00 – 10:50: H. Maassen: **Entanglement of Werner states: greatest cross norm and immanant inequalities**
  Abstact: We discuss the greatest cross norm on multiple tensor products of state spaces as a measure of entanglement of quantum states. In particular the completely symmetric (or "Werner") states on $B(H^{\otimes k})$ are expressed in terms of Littlewood's immanants of Gram matrices. Immanant inequalities such as those of Schur and Lieb provide bounds on these Werner states.

  10:50 – 11:30: Coffee

  11:30 – 12:20: G. Tucci: **Random Vandermonde Matrices and Covariance Estimates**
  Abstract: The talk will consist of two parts. In the first part we will center on the limit eigenvalue distribution of random Vandermonde matrices with unit magnitude complex entries. The phases of the entries are chosen independently and identically distributed from the interval $[-\pi, \pi]$. Various types of distribution for the phase are considered and we establish the limit eigenvalue distribution in a wide range of cases. We also provide a combinatorial and analytic formula

for the sequence of moments. The rate of growth of the maximum and minimum eigenvalue is examined.

In the second part, we will discuss a new approach to the estimation of covariance estimates. The estimation of a covariance matrix from insufficient data is one of the most common problems in multivariate statistics. More specifically, assume we have a set of n independent identically distributed measurements of an m dimensional random vector where $n < m$. The maximum likelihood estimate is the sample covariance matrix but in the case $n < m$ this estimate is singular, and therefore it is a fundamentally bad estimate. In this part we will discuss a new approach to this problem where we use random matrices techniques and free probability.

14:15 – 15:05: D. Voiculescu: **Around the free Riemann sphere and duality for infinitesimal bialgebras**

15:05 – 15:30: Coffee

15:30 – 16:20: A. Tikhomirov: **Limit theorems for spectrum of products of large random matrices**

- **Thursday, April 21**

10:00 – 10:50: B. Collins: **Random matrices, representations of $GL(n)$ and free probability of higher order**
Abstract: We study random matrices whose entries are in the enveloping Lie algebra of $GL(n)$, and show that under suitable conditions, their moments and their fluctuations have the same behaviour as unitarily invariant random matrices. As an application, we generalize previous results about the asymptotic behaviour of representations of $GL(n)$ and obtain new results about their fluctuations. This is joint work with Piotr Sniady.

10:50 – 11:30: Coffee

11:30 – 12:20: C. Koestler: **Noncommutative independence from characters of the infinite symmetric group**
Abstract: Recently we have found a new operator algebraic proof of Thoma's theorem which characterizes the extremal characters of the infinite symmetric group. We give an outline of the underlying ideas of our approach and address in particular spectral properties of certain mean ergodic averages of cycles.

14:15 – 15:05: A. Nica: **Convolution powers in operator-valued framework**
Abstract: I will present a recent joint work with Michael Anshelevich, Serban Belinschi and Maxime Fevrier, concerning convolution powers in the framework of an operator-valued non-commutative probability space over a C^*-algebra B. We show how for a B-valued distribution μ one can define convolution powers $\mu^{\boxplus\eta}$ (for free additive convolution) and $\mu^{\uplus\eta}$ (for Boolean convolution) where the exponent η is a suitable positive map from B to B, instead of being just a non-negative real number. We show moreover how these two types of convolution powers can be combined into an "evolution" semigroup related to the Boolean Bercovici-Pata bijection, and we prove some basic properties for this semigroup.

The talk will focus on combinatorial aspects of the B-valued convolution powers, and will complement the talk given by Serban Belinschi (who will discuss some of their analytic aspects).

15:05 – 15:30: Coffee

15:30 – 16:20: N. Demni: **Kanter random variable and positive free stable laws**

16:30 – 17:00: M. Anshelevich: **Two-state free Brownian motions**
Abstract: A familiar phenomenon in free probability is that many purely algebraic constructions and notions extend to the von Neumann algebra context. This is already the case for the notion of free independence itself. I will show that such behavior need not hold in the two-state free probability theory. Specifically, I will construct a large family of processes which, in the

algebraic setting, deserve to be called two-state free Brownian motions. However, in the von Neumann algebra setting, among all these processes, only a one-parameter family exists.

All lectures take place in the ESI Boltzmann Lecture Hall