ESI **The Erwin Schrödinger International Institute for Mathematical Physics**

Boltzmanngasse 9/2
A-1090 Vienna, Austria

ZVR 298034776

Bialgebras in Free Probability

February 1 - April 22, 2011

Workshop on "Random Matrix, Operator Algebra, and Mathematical Physics Aspects"

April 11 - 21, 2011

Schedule for week 1: April 11 - 15, 2011

organized by M. Aguiar, F. Lehner, R. Speicher, D. Voiculescu

- **Monday, April 11**

  10:00 – 10:50: T. Banica: **Probabilistic aspects of free quantum groups**

  10:50 – 11:30: Coffee

  11:30 – 12:20: S. Belinschi: **Convolution semigroups for operator valued distributions**
  Abstract: A well-known result of Nica and Speicher states that, unlike in classical probability, any probability measure μ on the real line embeds in a partial FREE convolution semigroup of probability measures $\{\mu^{\boxplus t} : t \geq 1\}$, starting at time t=1. In this talk we shall discuss an extension of this result to operator-valued distributions. The result will be viewed in the context of free infinite divisibility of operator-valued distributions. (This is joint work with M. Popa and V. Vinnikov and with M. Anshelevich, M. Fevrier and A. Nica.)

  14:15 – 15:05: S. Thorbjornsen: **Asymptotic expansions for GUE and Wishart random matrices**
  Abstract: For each n in $\mathbb{N}$ let X_n be a (suitably normalized) GUE random matrix, let $g\colon \mathbb{R} \to \mathbb{C}$ be a C^∞-function with all derivatives bounded and let tr_n denote the normalized trace on the $n \times n$ matrices. Reporting on joint work with Uffe Haagerup we describe an analytical approach to the derivation of an asymptotic expansion for the mean value $\mathbb{E}\{\mathrm{tr}_n(g(X_n))\}$, previously established by Ercolani and McLaughlin. Specifically we derive the expansion:

$$\mathbb{E}\big\{\mathrm{tr}_n(g(X_n))\big\} = \frac{1}{2\pi}\int_{-2}^{2} g(x)\sqrt{4-x^2}\,\mathrm{d}x + \sum_{j=1}^{k}\frac{\alpha_j(g)}{n^{2j}} + O(n^{-2k-2}),$$

  where k is an arbitrary positive integer. Considered as mappings of g, we describe the coefficients $\alpha_j(g)$, $j \in \mathbb{N}$, as distributions (in the sense of L. Schwarts). We derive a similar asymptotic expansion for the covariance $\mathsf{Cov}\{\mathrm{Tr}_n[f(X_n)], \mathrm{Tr}_n[g(X_n)]\}$, where f is a function of the same kind as g, and $\mathrm{Tr}_n = n\mathrm{tr}_n$. Special focus is drawn to the case where $g(x) = \frac{1}{\lambda - x}$ and $f(x) = \frac{1}{\mu - x}$ for λ, μ in $\mathbb{C} \setminus \mathbb{R}$. We finally describe similar results for Wishart matrices.

  15:05 – 15:30: Coffee

  15:30 – 16:20: P. Soltan: **An application of property (T) for discrete quantum groups**
  Abstract: A short introduction to property (T) for discrete quantum groups will be given. Using various equivalent descriptions of this property we will be able to solve some questions about existence of so called "exotic" completions of algebras of polynomials on compact quantum groups.

- **Tuesday, April 12**

  10:00 – 10:50: T. Banica: **Probabilistic aspects of free quantum groups**

  10:50 – 11:30: Coffee

  11:30 – 12:20: P. Di Francesco: **The Proof of the ASM-DPP Conjecture**
  Abstract: We prove a 28-year old conjecture by Mills-Robbins-Rumsey (1983) relating some refined enumerations of Alternating Sign Matrices (ASM) and Descending Plane Partitions (DPP). These are performed by reformulating the enumeration problems in terms of statistical models, namely the 6Vertex model for ASMs and Rhombus tilings/Dimers or Lattice Paths for DPPs. The conjecture then boils down to a determinant identity, which is proved by use of generating function techniques. Remarkably, the main player is the transfer matrix for discrete 1+1-dimensional Lorentzian quantum gravity, which generates random Lorentzian triangulations of the two-dimensional space-time. (This is joint work with Roger Behrend and Paul Zinn-Justin).

  14:15 – 15:05: C. Donati: **Truncations of Haar distributed matrices and bivariate Brownian bridge**
  Abstract: Let U be a Haar distributed matrix on the unitary group or the orthogonal group of size N.
  Let $p, q \leq N$ and $U_{p,q}$ the left upper corner of U. We prove that after centering, the sequence of two-parameter processes

  $$W_{s,t}^{(N)} = Tr(U_{\lfloor Ns \rfloor, \lfloor Nt \rfloor} U_{\lfloor Ns \rfloor, \lfloor Nt \rfloor}^{*}),\ s,t \in [0,1]$$

  converges in distribution to the bivariate tied-down Brownian bridge.

  15:05 – 15:30: Coffee

  15:30 – 16:20: R. Vergnioux: **Path cocycles in quantum Cayley trees and L^2-cohomology**
  Abstract: I will report on my work on L^2-cohomology of universal discrete quantum groups. In particular I will present the notion of quantum Cayley graph, which is the geometrical tool of the study, and I will explain the strategy of the proof of the vanishing of the first L^2-Betti number in the orthogonal case.

  16:30 – 17:00: H. Cheballah: **Gog, Magog and Schützenberger Involution**
  Abstract: We describe an approach to finding a bijection between Alternating Sign Matrices (ASM) and Totally Symmetric Self-Complementary Plane Partitions (TSSCPP) which is based on the Schützenberger Involution.

- **Wednesday, April 13**

  10:00 – 10:50: D. Shlyakhtenko: **Planar algebras and free Probability**

  10:50 – 11:30: Coffee

  11:30 – 12:20: P. Zinn-Justin: **Planar algebras and Potts model on random lattice**
  Abstract: We discuss a recent proposal to use random matrix techniques in the context of planar algebras. We focus on a particular case of relevant matrix model, which turns to be equivalent to the Potts model on dynamical random lattices, and solve it. This is joint work with A. Guionnet, V. Jones and D. Shlyakhtenko.

  Free Afternoon

- **Thursday, April 14**

  10:00 – 10:50: T. Banica: **Probabilistic aspects of free quantum groups**

  10:50 – 11:30: Coffee

  11:30 – 12:20: S. Curran: **On the symmetric enveloping algebra of planar algebra subfactors**

Abstract: In a recent paper, Guionnet, Jones and Shlyakhtenko gave a diagrammatic method for constructing a subfactor, starting from a planar algebra. In this talk we will give a graphical description of Popa's symmetric enveloping algebras of these subfactors. As an application, we compute a free entropy dimension type quantity associated to these factors. This is based on joint work with D. Shlyakhtenko.

14:15 – 15:05: P. Biane: **Brownian motion on matrices**
Abstract: The motion of the eigenvalues of a matrix performing Brownian motion is a very interesting object. We describe it using a multidimensional generalization of Pitman's theorem, to provide a probabilistic interpretation of some convexity properties of the Duistermaat-Heckmann measure, whose Fourier transform is the HCIZ integral.

15:05 – 15:30: Coffee

15:30 – 16:20: M. Bozejko: **Deformed Fock spaces, Hecke operators and non-commutative Levy processes for generalized "anyonic" statistics (with E.Lytvynov and J.Wysoczanski)**
Abstract: We will present the following topics:

1. Fock spaces of Yang-Baxter type.
2. Hecke operators
3. Woronowicz-Pusz CAR operators $T(\mu)$ and connections with Muraki monotone Fock space.
4. "Anyonic" Fock space and Q-CCR relations

$$a(s)a^*(t) - q(s,t)a^*(t)a(s) = \delta(s,t),$$

   where $Q = q(s,t)$, $|q(s,t)| = 1$, and s, t are in a non-atomic measure space (T, σ), and a construction of Q-Wick product and Q-Levy processes.
5. Applications to Haagerup approximation property.

16:30 – 17:00: N. Alekseev: **Genus expansion for some ensembles of random matrices**
Abstract: There exists the famous genus expansion of the moments of the eigenvalue distribution of GUE. Namely, we consider a random square $N \times N$ Hermitian Gaussian matrix H with complex entries. It turns out that the k^{th} moment of the eigenvalue distribution of the matrix H, which equals to $\frac{1}{N}\,\mathbf{E}\,\mathrm{Tr}\,H^{2k}$, has a nice topological combinatorial interpretation

$$\mathbf{E}\,\mathrm{Tr}\,H^{2k} = N^{k+1}\sum_{g=0}^{[k/2]} T(k,g)\frac{1}{N^{2g}},$$

where $T(k,g)$ is the number of ways to glue pairwise all the edges of a $2k$-gon so as to produce a surface of a given genus g (see Haagerup and Thorbjornsen, "'Random Matrices with Complex Gaussian Entries"'). Also in this paper the authors consider the complex Wishart case and obtain the asymptotic of $\mathrm{Tr}(XX^*)^k$. In the talk we discuss these theorems and consider generalizations for the real Wishart case and for the case of the singular value distribution of the product of two (or more) independent matrices.

- **Friday, April 15**

  10:00 – 10:50: T. Banica: **Probabilistic aspects of free quantum groups**

  10:50 – 11:30: Coffee

  11:30 – 12:20: M. Nowak: **Multiplication law and S-transform for non-hermitian random matrices**
  Abstract: We derive the multiplication law for free non-hermitian matrices using the planar diagrammatic technique. We define the corresponding non-hermitian S transform being a natural generalization of the Voiculescu S transform. Using examples we show how to use this law to determine the complex eigenvalue density for matrices given as free products of nonhermitian ensembles.

  14:15 – 15:05: Y. Dabrowski: **Applications of free SDEs to von Neumann algebras of q-gaussian variables**
  Abstract: We will explain two kinds of applications of free stochastic differential equations to von Neumann algebras $M_{q,N}$ generated by N q-Gaussian variables (for small q). First, we can use them to compute microstate free entropy dimension of N q-Gaussian variables if $|q|N < 1$ and $|q|N^{1/2} < 0.13$. Its value is identically $\delta_0(X_1, ..., X_N) = N$. In fact we can show q-Gaussian variables have finite Fisher information in this range of q, even though the computation of microstate free entropy dimension is not a consequence and also involves various almost coassociative derivations. Second, we can show that for small q, $M_{q,N}$ have complete metric approximation property (and as a consequence of a result of Popa and Ozawa, they are thus strongly solid). We will discuss how a better understanding of our general dilation results for Markov semigroups (solving variants of free SDEs) could improve the range of q for this second kind of statements.

  15:05 – 15:30: Coffee

  15:30 – 16:20: F. Radulescu: **A quantum double for Hecke operators, Ramanujan Petersson Conjectures and Free Probability**

  16:30 – 17:00: C. Male: **The norm of polynomials in random and deterministic matrices**
  Abstract: In this talk, I will present a strengthened version of the Asymptotic freeness of Gaussian and deterministic matrices. Let $\mathbf{X}_N = (X_1^{(N)}, \dots, X_p^{(N)})$ be a family of $N \times N$ independent, normalized random matrices from the Gaussian Unitary Ensemble. We state sufficient conditions on matrices $\mathbf{Y}_N = (Y_1^{(N)}, \dots, Y_q^{(N)})$, possibly random but independent of $\mathbf{X}_N$, for which the operator norm of $P(\mathbf{X}_N, \mathbf{Y}_N, \mathbf{Y}_N^*)$ converges almost surely for all polynomials P. The method of the proof is based on recent works of Haagerup and Thorbjornsen, where the case $\mathbf{Y}_N = 0$ has been studied.

All lectures take place in the ESI Boltzmann Lecture Hall