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Programme on

“Infinite-dimensional Riemannian geometry with applications to image matching and shape analysis”

January 7 – February 27, 2015

organized by

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Workshop on

“Riemannian geometry in shape analysis and computational anatomy ”

February 16 – 20, 2015

- **Monday, February 16, 2015**

9:30 – 10:30
Bayesian models in computational anatomy, neurodegenerative illnesses and Brain Clouds
Michael Miller
John Hopkins University

I will review several statistical estimation problems which arise in the computational anatomy setting, casting three problems of statistical estimation as examples of parameter estimation in the context of many templates: (i) disease estimation, (ii) segmentation and (iii) template estimation.

Then I will discuss one of the central questions in neurodegenerative diseases today, the staging of neurodegeneration in brain circuits associated to the network hypothesis. I will show results from Alzeimer’s and Huntington’s disease.

Along the way I will discuss progress with Susumu Mori on building several Brain Clouds associated to young subjects and aging subjects.

11:00 – 12:00
Freeform architecture and differential geometry
Helmut Pottmann
TU Wien and KAUST

Free forms constitute one of the major trends within contemporary architecture. While the modeling of freeform geometry with current tools is well understood, the actual fabrication is a challenge and raises numerous research problems many of which are of a mathematical nature. The speaker will report on recent progress in geometric computing for freeform architecture, with special emphasis on the close relation to differential geometry. Specific topics to be addressed include: meshes with planar quadrilateral faces and corresponding supporting structures, semi-discrete representations for structures from single curved panels, shape space exploration and the design of self supporting surfaces. The transfer of research into the architectural practice will be illustrated at hand of selected projects.

14:00 – 15:00
Approximation of diffeomorphisms with $U(n)$ and the Hilbert–Schmidt group
Henry Jacobs
Imperial College London

Diffeomorphic image registration can be approached in a variety of ways. In the large deformation diffeomorphic metric mapping (LDDMM) framework, one computes (sub-)Riemannian geodesics on the space of diffeomorphisms to match images. The Eulerian velocity field which generates such a diffeomorphism is generically “time”-dependent. Another approach is to generate diffeomorphisms with static velocity fields. This latter approach is computationally less challenging, but lacks the geometric interpretation of LDDMM. However, if one were to find a bi-invariant metric on the diffeomorphisms, then the use of static vector fields could serve as a special case of LDDMM. In this talk I will illustrate a numerical scheme in which diffeomorphism are modeled as elements of $U(n)$ and vector-fields members of $\mathfrak{u}(n)$, where n is a resolution parameter. The bi-invariant metric on $U(n)$ allows us to make the link between static vector-field registraion and LDDMM at the discrete level. Analytic obstructions prevent us from bringing this finding to the diffeomorphism group, and I will illustrate which terms explode as n goes to infinity. If there is time, I will present some ideas on how one could model the diffeomorphism group with the Hilbert–Schmidt group. The Hilbert–Schmidt group is an infinite-dimensional Lie group which does have a bi-invariant metric (by its definition) and can approximate diffeomorphism as integral operators. It helps to invoke a “pointless” and more algebraic perspective when making this transition.

15:00 – 16:00
On the usage of the Riemannian geometry framework for PDE constrained shape optimization
Volker Schulz
Universität Trier

Shape optimization is an important subject in many application fields – in particular if a shape is to be optimized with respect to a goal, which involves a process model in the form of differential equations. Examples of this are aerodynamic, thermoelastic, acoustic shape optimization problems. For this problem class, it is advantageous to use the so called shape calculus in order to obtain expressions for derivative information of the goal with respect to the shape. However, in contrast to nonlinear programming, there is not yet a complete structural understanding giving meaning to second order information and algorithmic convergence considerations. This talk gives motivating examples from the aforementioned application fields and explores the possibility to use the Riemannian geometry framework for shapes as a substitute for the vector space framework in traditional finite dimensional nonlinear programming.

16:30 – 17:30
On the comparison of short processes
Jair Koiller
Fundação Getúlio Vargas

Time-varying computational anatomy (4DCA) is one of the current research themes in the area. Snapshots at given times are interpolated using geodesic splines; the problem arises of comparing two such sequences for classification purposes. In this presentation we consider a simple variant. A *short process* is interpreted as a tangent vector in the space of images, endowed with one of the Riemannian metrics used for shape analysis. We propose one way to compare two such short processes, namely, via a time-optimal path joining the tangent vectors under acceleration constraint. The transition time is then the proposed comparison measure. We hope that this idea, inspired by the classical Markov–Dubins problem in robotics (except that tangential acceleration is also allowed – which brings in a new twist to the problem), could be useful in some *morphokinetical* contexts. Along the way, we present general technical results about the underlying symplectic structures of control problems whose state space has a bundle structure. We discuss two (toy) finite dimensional examples using landmarks.

We thank Martins Bruveris for his generous help. This is joint work with Paula Balseiro and Alejandro Cabrera.

- **Tuesday, February 17, 2015**

9:00 – 10:00 and 10:00 – 11:00
Designing and learning Riemannian metrics in medical imaging
Laurent Risser and François-Xavier Vialard
Université Paul Sabatier and Université Paris–Dauphine

This talk presents a joint work with FX Vialard about variational strategies to learn spatially-varying metrics in the LDDMM framework. The spatially-varying metric model was proposed last year and can smooth shape deformations locally as well as globally. It can also take into account the direction of the deformations. Our talk will focus on an extension of last year's strategy to learn the spatially-varying metric parameters as well as its numerical behaviour. The key motivation of this new strategy was to make the learning step of spatially-varying metrics on 3D medical images computationally reasonable.

11:30 – 12:30
Bayesian mixed effect atlas estimation with a LDDMM deformation model
Stephanie Allassonnière
École Polytechnique

In this talk, I will present a new statistical model for atlas estimation where we have introduced a diffeomorphic constraint on the deformations. This generalizes the deformable template Bayesian mixed effect (BME) model. Our approach is built on a generic group of diffeomorphisms, that is parameterized by an arbitrary set of control point positions and momentum vectors. This enables to estimate the optimal positions of control points together with a template image and parameters of the deformation distribution which compose the atlas. We propose to use a stochastic version of the expectation-maximization (EM) algorithm where the simulation is performed using the anisotropic Metropolis adjusted Langevin algorithm (AMALA). We propose also an extension of the model including a sparsity constraint to select an optimal number of control points with relevant positions. Experiments are presented on the USPS database, on mandibles of mice and on 3D murine dendrite spine images.

14:30 – 15:30
Morphometry in neuroimaging
Stanley Durrleman
Institut du Cerveau et de la Moelle épinière

In this talk, I will review recent works aiming at developing alternative approaches for the study of anatomical data in neuroimaging, which build on the metric between shapes derived from large diffeomorphic deformations. The study of brain anatomy, which consists in an intricate network of structures raises specific computational and statistical issues. I will present recent contributions to estimate representative virtual models of brain anatomy, which account for co-variation among the structures. I will also present extensions to build virtual dynamical models of brain development or aging, which account for differences in the pace of shape changes across individuals.

15:30 – 16:15
A growth model in computational anatomy based on diffeomorphic matching
Irène Kaltenmark
ENS Cachan

The large deformation diffeomorphic metric mapping (LDDMM) framework has proved to be highly efficient for addressing the problem of modelling and analysis of the variability of populations of shapes, allowing for the direct comparison and quantization of diffeomorphic morphometric changes. However, the analysis of medical imaging data also requires the processing of more complex changes, which especially appear during growth or aging phenomena. The observed organisms are subject to transformations over the time which are no longer diffeomorphic, at least in a biological sense. One reason might be the creation over time of new material uncorrelated to the preexisting one. For this purpose, we offer to extend the LDDMM framework to address the problem of non diffeomorphic structural variations in longitudinal scenarios during a growth or degenerative process. We keep the geometric central concept of a group of deformations acting on a shape space. However, the shapes will be encoded by a new enriched mathematical object allowing an intrinsic evolution dissociated from external deformations. We focus on the specific case of the growth of animal horns.

16:45 – 17:30
Shape statistics with modular multi-scale diffeomorphic deformations
Barbara Gris
ENS Cachan

I will present a modular and multi-scale deformation model to construct an atlas of shapes. We developed a generic framework to build large deformations from base modules which combine and interact with each other to create the final deformation. Deformation modules act in a neighbourhood defined by a given scale which varies from a module to another so the developed framework is modular and multi scale. The contribution of each module or scale can be easily computed, allowing the computation of statistics at each scale and the interpretation of all local effects separately.

- **Wednesday, February 18, 2015**

9:00 – 10:00
Rate-invariant analysis of covariance trajectories with applications to video-based action recognition
Anuj Srivastava
Florida State University

Automated classification of actions in videos is performed by extracting relevant features, particularly covariance features, from image frames and studying longitudinal data relating to temporal evolutions of these features. A natural mathematical representation of activity videos is in form of parameterized trajectories on the covariance manifold, i.e. the set of symmetric, positive-definite matrices (SPDMs), endowed with a novel Riemannain structure (inherited from $SL(n)$ by identification with the quotient space of $SL(n)/SO(n)$). The variable execution-rate of actions dictates different parameterizations of resulting trajectories. Since action classification is invariant to execution rates, one requires metrics for comparing trajectories that are invariant to reparameterizations.

A recent paper represented trajectories using their transported square-root vector fields (TSRVFs), defined as parallel translations, of scaled-velocity vectors of trajectories, to a reference tangent space on the manifold. To avoid arbitrariness of reference point and to reduce distortion, we develop a more intrinsic approach where trajectories on manifold of SPDMs are represented by their TSRVFs at initial points of the trajectories, and analyzed as elements of a vector bundle on the manifold. Using a natural Riemannian metric, we compute geodesic paths and geodesic distances between trajectories in the quotient space of this vector bundle, with respect to the reparameterization group, thus making the resulting comparison reparameterization invariant. We demonstrate this framework on two applications involving videos: visual speech recognition or lip-reading and hand-gesture recognition.

This is joint work with Zhengwu Zhang, Eric Klassen and Huiling Le.

10:00 – 11:00
Precise matching of piecewise linear curves using the square root velocity function
Eric Klassen
Florida State University

The square root velocity function (SRVF) has proved to be an effective method for comparing curves in $\mathbb{R}^n$ modulo reparametrization. However, the choice of optimal parametrization has usually been found using dynamic programming, which can only provide an approximation of the correct result. In this talk we discuss an algorithm that finds the precise optimal parametrizations needed to compare two curves in the case that the curves are piecewise linear (PL). This is of special interest because the set of PL curves is dense in the complete space of all absolutely continuous curves with respect to the SRVF metric. We also demonstrate the implementation of this algorithm.

This is joint work with Daniel Robinson and Sayani Lahiri.

11:30 – 12:30
Deformable shape tracking and a new region-based Sobolev metric
Ganesh Sundaramoorthi
King Abdullah University of Science and Technology

In this talk, I will discuss the problem of determining the precise shape of a deforming object from a video. The problem is central to computer vision and remains a challenge due to the interactions of many different nuisances that generate infinite variations of the appearance and shape of the object in the image. Classical solutions to determining the precise shape of the object have primarily built on the idea of discriminating the image into foreground and background based on simple image statistics, but these methods are inadequate for objects with complex appearance in cluttered backgrounds. An alternative approach, which we propose, is to treat the tracking problem as one of matching, i.e., where the current estimate of the object is matched to the next image. This allows one to tailor advanced optical flow methods, that have been only recently proposed and are able to cope with complex object appearance, to the deforming shape tracking problem. I will discuss how to tailor these algorithms to shape tracking. I will then present a new region-based Sobolev shape metric, and associated optimization scheme that induces an automatic coarse-to-fine deformation behavior, natural for shape tracking, and show that it is beneficial over standard optical flow methods applied to shape tracking. I will show results on challenging video, and show state-of-the-art performance.

- **Thursday, February 19, 2015**

  9:00 – 10:00
  On an extension of PCA on manifolds
  Xavier Pennec
  INRIA Sophia Antipolis

  I will adress in this talk the problem of principal component analysis (PCA) on Riemannian manifolds. Tangent PCA is good for analyzing data which are sufficiently centered around a central value (unimodal or Gaussian-like data). However, this is often not sufficient for multi-modal or large support distributions (e.g. uniform on closed compact subspaces like circles). Instead of a covariance matrix analysis, principal geodesic analysis (PGA) is proposing to minimize the distance to "geodesic subspaces"which are spanned by the geodesics going through the mean with a tangent vector is a given linear sub-space of the tangent space. This definition is intrinsic and allows to build a flag (sequences of embedded subspaces) of principal geodesic subspaces which is consistent with a forward component analysis approach: we build iteratively the components from dimension 0 (the mean point) to 1D (a geodesic) and higher dimensions by iteratively selecting the direction in the tangent space at the mean that optimally reduce the square distance of data point to the geodesic subspace. However, the price to pay is that the mean always belong to geodesic subspaces even when the mean is not part of the support of the distribution. In this paper we propose a new and more general type of family of subspaces in manifolds. We show that our definition is defining locally a submanifold of controlled dimension and that it generalizes the geodesic subspaces. Like PGA, it allows the construction of a forward flag of approximations of data points which contains the Frechet mean. However, unlike PGA, it also allows the construction of backward flags which may not contain the mean.

  10:00 – 11:00
  Shape and image invariants under Lie group actions
  Stephen Marsland
  Massey University

  Lie group methods play a fundamental role in many aspects of computer vision and image processing, including object recognition, pattern matching, feature detection, tracking, shape analysis, tomography, and geometric smoothing. We consider the setting in which a Lie group G acts on a space M of objects such as points, curves, or images, and convenient methods of working in M/G are sought. One such method is based on the theory of invariants, i.e., on the theory of G-invariant functions on M, which has been extensively developed from mathematical, computer science, and engineering points of view. When the objects in M are planar, the most-studied groups are the Euclidean, affine, similarity, and projective groups. As well as introducing these methods I will describe our work on invariants of planar objects under the Möbius group $\mathrm{PSL}(2, \mathbb{C})$ which acts on the Riemann sphere $\overline{\mathbb{C}} = \mathbb{C} \cup \{\infty\}$ by

$$\phi\colon \overline{\mathbb{C}} \to \overline{\mathbb{C}}, \quad \phi(z) = \frac{az+b}{cz+d}, \quad a, b, c, d \in \mathbb{C}, \quad ad - bc \neq 0.$$

It is a 6-dimensional Lie group, forming the identity component of the inversive group, the group generated by the Möbius transformations and a reflection. In addition to its importance on fundamental grounds—it is one of the very few classes of Lie groups that act on the plane, it crops up in numerous branches of geometry and analysis, it is the smallest nonlinear planar group that contains the direct similarities, and it is the set of biholomorphic maps of the Riemann sphere. It also has direct applications in image processing.

This is joint work with Robert McLachlan.

11:30 – 12:15
Statistics on Lie groups: can we obtain a consistent framework with pseudo-Riemannian metrics?
Nina Miolane
INRIA Sophia Antipolis

Models in computational anatomy often require to estimate continuous transformations of the 3D space, i.e. elements of a Lie group. This demands a consistent framework for statistics on Lie groups. It is known that there is no fully consistent (bi-invariant) Riemannian metric on most Lie groups. We investigated the existence of bi-invariant pseudo-Riemannian metrics and we present an algorithm to compute such a pseudo-metric on a Lie group if it exists. Unfortunately results show that most Lie groups do not possess a bi-invariant pseudo-metric in general, although the class of such groups is larger than for the Riemannian case.

15:00 – 16:00
Scaling dimension: a promising new parameter for morphometrics of organisms
Fred Bookstein
University of Washington

"Differences between medical images are encoded by diffeomorphisms," the call for this programme reads, and so the search for a robust statistical method for extraction of features from those diffeomorphisms in particular settings is one challenging thrust of our community's work with this model. My presentation will import some ideas from current work in morphometrics toward this goal. In this connection you might want to think of morphometrics as affording a convenient finite-dimensional simplification of a family of diffeomorphisms suitable for linear multivariate statistical analysis over scientific samples.

Quantitative biological reports often take the form of a contrast between the *global* descriptors of shape change that generally characterize processes of growth or evolution and the *local* descriptors corresponding more often to disease processes or other vicissitudes of postnatal animal life. Recently the biomathematicians have turned to morphometrics for help with one particular approach to this assignment, the search for numerical parameters of *integration*, the extent (whatever it may be) to which global descriptors dominate in this connection. The thin-plate spline formalism that links geometric and statistical aspects of work with biologically plausible samples of diffeomorphisms permits the computation of one helpful parameter along these lines, a *scaling dimension* summarizing the fall of geometric signal amplitude with scale.

I will show examples of these analyses corresponding to the three regions of interest in current organismal biological applications (integrated, selfsimilar, disintegrated). This trichotomy is reminiscent of the distinctions among "colors" of noise in the signal-processing literature. The figure below presents a sample of nine from the middle category the self-similar transformations. (In other words, none of these examples have any local features, in spite of what your own eye is telling you.) As applications thus far have typically been not mathematical but biological, I seek your advice in moving this work back toward the biomathematical side, including the possibility of a limiting formalism for these coefficients in the passage from finite approximations to the full Riemannian manifold, the extension to three-dimensional data, and the ties, if any, with the more medical side of our work with image diffeomorphisms.

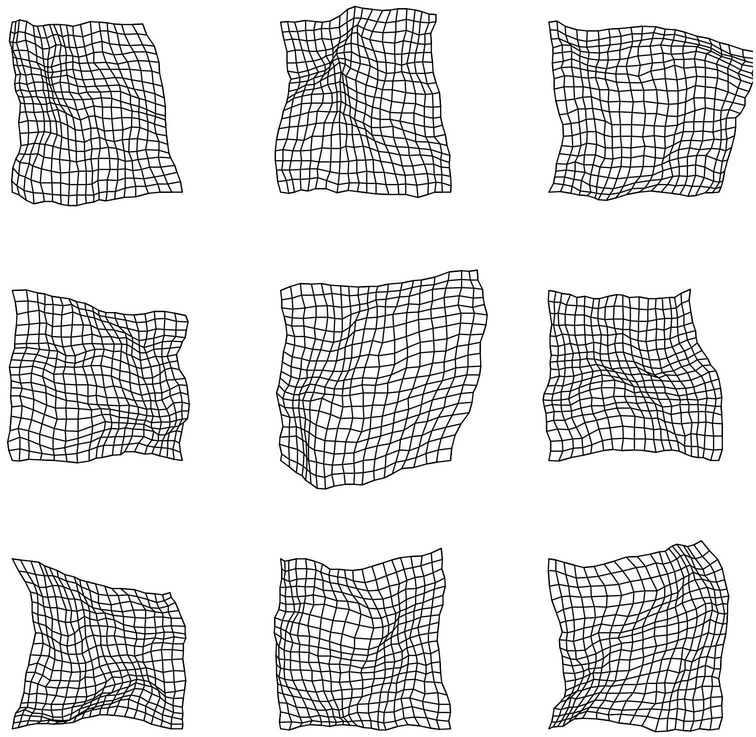

Figure. Nine realizations of a purely self-similar deformation process on a 20x20 square grid. This family of deformations provides a natural boundary between two interesting classes of empirical possibilities: the integrated transformations, where the amplitude of residuals from trends falls faster than linearly with the scale of the trend, and the disintegrated transformations, where the fall is slower than linear.

16:30 – 17:30
A stochastic algorithm finding generalized means on compact manifolds
Marc Arnaudon
Institut de Mathématiques de Bordeaux

A stochastic algorithm is proposed, finding the set of generalized means associated to a probability measure on a compact Riemannian manifold M and a continuous cost function on the product of M by itself. Generalized means include p-means for $p > 0$, computed with any continuous distance function, not necessarily the Riemannian distance. They also include means for lengths computed from Finsler metrics, or for divergences. The algorithm is fed sequentially with independent random variables Y_n distributed according to the probability measure on the manifold and this is the only knowledge of this measure required. It evolves like a Brownian motion between the times it jumps in direction of the Y_n. Its principle is based on simulated annealing and homogenization, so that temperature and approximations schemes must be tuned up. The proof relies on the investigation of the evolution of a time-inhomogeneous L^2 functional and on the corresponding spectral gap estimates due to Holley, Kusuoka and Stroock.

- **Friday, February 20, 2015**

9:00 – 10:00
Matching of geometric-functional structures via a BV-energy
Giacomo Nardi
University of Orléans

We present some numerical and theoretical results about a new matching model for surfaces (with boundary) equipped with a signal (fshapes). This kind of models are needed in medical imaging because of the new developments in non-invasive acquisition techniques (fMRI, OCT), which allow to get geometric-functional data for several diseases.

We consider a matching energy taking into account a BV-penalty for the signal and a varifold-type attachment term. From a theoretical point of view, we point out the main issues rising from the discretization of the problem. This leads to the definition of a suitable class of triangulations converging towards the original surface. In such a setting we prove a Gamma-convergence result of the discrete problems towards the continuous one. Numerically, we compare the BV-penalty with the L^2-and H^1-penalties. In particular, we point out that the choice of a BV-penalty improves the matching result.

This is a joint work with B. Charlier and A. Trouvé.

10:00 – 11:00
Diffusion processes and dimensionality reduction on manifolds with affine connections
Stefan Sommer
Københavns Universitet

The Eells-Elworthy-Malliavin construction of Brownian motion links stochastic differential equations on manifolds with Euclidean space SDEs. Based on this construction, we use diffusion processes with anisotropic generators to construct a framework for likelihood estimation of template and covariance structure from manifold valued data. The procedure avoids the linearisation that arise when first estimating a mean or template before performing PCA in the tangent space of the mean. We identify the most probable paths reaching sampled data as extremal paths in the frame bundle equipped with a sub-Riemannian metric, and we use this to derive evolution equations. The paths that generally do not project to geodesics are used for estimating the likelihood of the model, and we discuss how mixing a lifted Riemannian metric and the covariance cometric allows for efficient numerical implementations.

11:30 – 12:30
Stochastic multisymplectic variational integrators for $SE(3)$-strands
François Demoures
Imperial College London

We analyse a problem in shape analysis, in which the increments in time and space variables are both stochastic. In particular, this induces random perturbations to deformations which belong to a Lie group G identified as a subgroup of $\mathrm{Diff}(M)$. The general framework of this study is field theory and its corresponding discrete mechanics, which affords a different perspective and a new solution approach to this tricky problem. We believe this new type of stochastic variational integrator can be applied to image processing problems such as image matching.

14:30 – 15:15
A level set method for the matching of implicit surfaces
José Iglesias
Universität Wien

A recent trend in shape analysis is to use variational models of nonlinear elasticity, since they manage to balance adequate geometric invariance with being amenable to both analysis and numerical simulation. In this direction, we consider the problem of matching two smooth surfaces embedded in $\mathbb{R}^3$ under large deformations. In particular, we aim to measure the difference between the embeddings, and not only the intrinsic geometry of the surfaces.

A level set formulation for this problem, inspired on variational models for elastic shells, is proposed. Exploiting the information available in a surface matching scenario, we obtain a deformation energy involving only first-order derivatives for which existence of minimizers can be proved, while still reflecting resistance to compression and bending. Injectivity is obtained through volume regularization.

For its numerical solution, we propose a discretization through conforming multilinear finite elements on adaptive octree grids, and a coarse-to-fine multilevel minimization using Sobolev metrics. This approach allows for high-resolution computations on real data.

Research carried out in collaboration with M. Rumpf (Uni. Bonn) and O. Scherzer (Uni. Wien). .

All talks take place at the ESI, Boltzmann Lecture Hall.