Bialgebras in Free Probability

February 1 - April 22, 2011

Workshop on "Combinatorial, Bialgebra, and Analytic Aspects"
February 14 - 25, 2011

organized by M. Aguiar, F. Lehner, R. Speicher, D. Voiculescu

- **Monday, February 21**

  10:00 – 10:30: Coffee

  10:30 – 11:20: M. Aguiar: **Infinitesimal Bialgebras 1**
  Abstract: We will review basic notions pertaining to infinitesimal bialgebras, as well as some new points of view motivated mainly by work of Voiculescu (Free Analysis Questions I and II). These will include:

  - A discussion of the interplay between infinitesimal and Frobenius bialgebras.

  - An algebraic perspective on the algebras of fully matricial functions of Voiculescu.

  11:30 – 12:20: C. Krattenthaler: **Generalized non-crossing partitions for reflection groups and cyclic sieving**

  14:15 – 15:05: T. Banica: **Probabilistic aspects of free quantum groups**

  15:05 – 15:30: Coffee

  15:30 – 16:20: B. Collins: **Free probability, random matrices and quantum information theory**
  Abstract: Operator Algebra - and more recently, probability theory on matrix spaces - have proven to be very useful tools Quantum Information Theory (QIT). We will show how Random Matrix Theory - and in particular, of free probabilistic flavour - can also bring new insight into important questions of QIT. Joint work with Ion Nechita and Serban Belinschi.

  16:30 – 17:00: E. Redelmeier: **Second-order Freeness in the Real Case**
  Abstract: Second-order freeness extends free probabilistic approaches from the moments of large random matrices to their fluctuations. Many important ensembles of complex random matrices asymptotically satisfy a second-order freeness condition. Like freeness, second-order freeness can be used as a way of calculating fluctuations of mixed expressions in independent matrices in terms of moments of individual matrices. In general, however, the real analogues of these complex random matrices do not satisfy this condition. We present a second-order freeness condition satisfied by many important real ensembles. We demonstrate this for several ensembles (real Ginibre, Gaussian orthogonal ensemble, real Wishart, Haar-distributed orthogonal) using genus expansion-type expressions and cartographic techniques for manipulating graphs on nonorientable surfaces.

- **Tuesday, February 22**

  10:00 – 10:30: Coffee

  10:30 – 11:20: M. Aguiar: **Infinitesimal Bialgebras 2**

  11:30 – 12:20: M. Mastnak: **Bialgebras and free multiplicative convolution**
  Abstract: The talk is based on the joint paper with A. Nica entitled "Hopf algebras and the

logarithm of the S-transform in free probability". There we discuss a bialgebra based on non-crossing partitions that encodes some of the combinatorics of the free multiplicative convolution of k-tuples of distributions in a non-commutative probability space. The emphasis of the talk will be on bialgebra aspects of the work in question. Some related work in progress will also be mentioned.

14:15 – 15:05: C. Brouder: **Noncommutative Feynman graphs and Hopf algebra cohomology**
Abstract: Feynman graphs in quantum field theory can be generated as a convolution exponential over a free commutative algebra. We show that, similarly, any linear map f from the tensor algebra $T(V)$ to the scalars (with $f(1) = 1$), can be written as a convolution exponential and can be described with generalized non-commutative Feynman graphs. (joint work with Damien Manuel and Frederic Patras)

15:05 – 15:30: Coffee

15:30 – 16:20: L. Pastur: **Laws of Fluctuations for Spectral Statistics of Random Matrices**
Abstract: We present a review of recent results on the limiting laws for fluctuations of several classes of spectral statistics of random matrices as their size tends to infinity. We pay special attention to random matrices whose randomness is due Haar distributed random matrices of classical groups.

16:30 – 17:20: M. Popa: **Non-commutative functions and some of their applications in free probability**
Abstract: Given two vector spaces, V and W over the complex numbers, a non-commutative function is, briefly, a mapping from a certain class of subsets of the matrix space over V to the matrix space over W satisfying some compatibility conditions: it has to respect direct sums and simultaneous similarities, or equivalently, simultaneous intertwinings. Noncommutative functions have very strong regularity properties and they admit a very nice differential calculus, closely related to some QD-bialgebras arising in free probabilities. Such objects were considered before by J. L. Taylor in his groundbreaking work on the noncommutative spectral theory, and more recently independently by D.-V. Voiculescu in free probability.

- **Wednesday, February 23**

  10:00 – 10:30: Coffee

  10:30 – 11:20: M. Aguiar: **Infinitesimal Bialgebras 3**

  11:30 – 12:20: A. Nica: **On the C*-algebra of the Fock space representation for the q-commutation relations**
  Abstract: For $q \in (-1, 1)$, the q-commutation relations have a natural representation on a deformed Fock space $\mathcal{F}_q$; this was introduced by Bozejko and Speicher in 1991, and was studied by many other authors after that. Let $\mathcal{C}_q \subseteq B(\mathcal{F}_q)$ be the C^*-algebra generated by this representation. For $q = 0$ one has that $\mathcal{C}_0$ is the full Fock space representation of the extended Cuntz algebra. It is widely believed that $\mathcal{C}_q$ is unitarily equivalent to $\mathcal{C}_0$ for all $q \in (-1, 1)$, but at present this is proved only for small values of $|q|$.

  In a paper by Dykema and Nica in 1993 it was shown how to construct a unitary $U : \mathcal{F}_q \to \mathcal{F}$ such that $U\mathcal{C}_q U^* \supseteq \mathcal{C}_0$ for all $q \in (-1, 1)$, with equality holding when $|q| < 0.44$.

  In this talk I will present a recent joint work with Matthew Kennedy, where we introduce a unitary $U_{opp} : \mathcal{F}_q \to \mathcal{F}$ (related to the U from the Dykema-Nica paper) which achieves the opposite inclusion:
  $$U_{opp} \, \mathcal{C}_q \, U_{opp}^* \subseteq \mathcal{C}_0, \ \ \forall \, q \in (-1, 1).$$

  As a consequence, it follows that $\mathcal{C}_q$ is an exact C^*-algebra for all q.

  In order to obtain the embedding into $\mathcal{C}_0$ stated above, we prove a "bicommutant type" result (with commutations considered modulo a suitable ideal of compact operators) which gives a

sufficient condition for an operator $T \in B(\mathcal{F})$ to belong to $\mathcal{C}_0$. We then prove this sufficient condition to be satisfied by the generators of $U_{opp} \mathcal{C}_q U_{opp}^*$.

14:15 – 15:05: N. Muraki: **A certain q-interpolation between tensor and free independence**
Abstract: I will construct a certain q-interpolation between tensor and free independence as a universal product for C*-probability spaces. This is a kind of 'q-independence' in the sense that this product produces the Bozejko-Speicher q-Brownian motion in the functional central limit. We remark that this is a kind of approximate notion for true q-independence that cannot exists as a universal product.

15:05 – 15:30: Coffee

15:30 – 16:20: F. Patras: **Noncommutative Spitzer identities**
Abstract: Spitzer identities first appeared in fluctuation theory, allowing to understand algebraically the characteristic functions associated to the extrema of certain sequences of random variables. We derive here functional identities for noncommutative RotaBaxter algebras. Our results generalize the seminal CartierRota theory of classical Spitzer-type identities for commutative RotaBaxter algebras. In the classical, commutative, case, these identities can be understood as deriving from the theory of symmetric functions. Here, we show that an analogous property holds for noncommutative RotaBaxter algebras. That is, we show that functional identities in the noncommutative setting can be derived from the theory of noncommutative symmetric functions. Lie idempotents, and particularly the Dynkin idempotent play a crucial role in the process. Based on joint works with K Ebrahimi-Fard, J Gracia-Bondia and D Manchon.

16:30 – 17:30: D. Evans: **Modular invariants, subfactors and twisted equivariant K-theory**

- **Thursday, February 24**

  10:00 – 10:30: Coffee

  10:30 – 11:20: M. Aguiar: **Infinitesimal Bialgebras 4**

  11:30 – 12:20: G. Chistyakov: **Infinitely divisible approximations of n-fold free convolutions**

  14:15 – 15:05: J-C. Wang: **A new approach to the monotone central limit theorem**
  Abstract: We will discuss the monotone central limit theorem for identical, unbounded summands. We show that the monotone central limit theorem holds under the same conditions of the classical central limit theorem. Our approach is based on the theory of Abel functional equations, rather than the usual combinatorial approach in the literature.

  15:05 – 15:30: Coffee

  15:30 – 16:20: I. Krolak: **General commutation relations - properties of associated algebras and Ornstein-Uhlenbeck semigroup**
  Abstract: We study a certain class of von Neumann algebras generated by selfadjoint elements $\omega_i = a_i + a_i^+$, for a_i, a_i^+ satisfying the general commutation relations:

  $$a_i a_j^+ = \sum_{r,s} t_{js}^{ir} a_r^+ a_s + \delta_{ij} Id.$$

  We assume that operator T for which the constants t_{js}^{ir} are matrix coefficients satisfies the braid relation. The choice of the relations was made since several examples of such structures are investigated in the literature (CAR, q-CCR, twisted commutation relations studied by Pusz and Woronowicz and their modifications which are type III factors). We concentrate on bounds for hypercontractivity of Ornstein-Uhlenbeck semigroup acting on these non-commutative algebras.

  16:30 – 17:30: C. Vargas: **Different sized Haar-unitaries arising from random matrix models**

Abstract: Based on the well known results by Voiculescu on the asymptotic freeness of Haar distributed unitary random matrices, we study the asymptotic joint distribution of these matrices when their sizes grow at different rates. Freeness no longer holds between them, but interesting things can be said when amalgamating with a suitable algebra.

The study of such a setting was inspired by a random matrix model, solved recently by Hoydis, Coulliet and Debbah.

- **Friday, February 25**

  10:00 – 10:30: Coffee

  10:30 – 11:20: M. Gordin: **Formal and Analytic Groups Related to Free Probability**
  Abstract: At the formal level, with every probability distribution on the line a formal group law over reals can be associated. There exists also an anaylitic analogue of this correspondence. The concepts of the monotone and the free convolutions will be considered in the context of such group laws. Relation of the free convolution formalism to Lazard's universal formal group law will be discussed.

  11:30 – 12:20: T. Gannon: **The search for the exotic**

All lectures take place in the ESI Boltzmann Lecture Hall