Workshop
Stochastic processes from physics and biology
ESI, November 26-27, 2004

Abstracts

Jiri Cerny

Stock price evolution from microscopic market modelling

We propose a class of Markovian agent based models for the time evolution of a share price in an interactive market. The models rely on a microscopic description of a market of buyers and sellers who change their opinion about the stock value in a stochastic way. The actual price is determined in realistic way by matching (clearing) offers until no further transactions can be performed. Some analytic results for a non-interacting model are presented. We also propose basic interaction mechanisms and show in simulations that these already reproduce certain particular features of prices in real stock markets.

Donald Dawson

Some aspects of Fleming-Viot models with selection, mutation and spatial migration

This will be a review of work in progress with Andreas Greven on some problems in evolutionary theory involving the combination of migration, selection and mutation in a stochastic population model and some related background material.

Michael Eckhoff

Long term behavior and small eigenvalues of metastable, reversible diffusions

For generic $F : \mathbb{R}^d \to \mathbb{R}$ with finitely many local minima the randomly perturbed dynamical system

$$dX_t = -\nabla F(X_t)dt + (2\epsilon)^{1/2}dW_t, \quad \epsilon > 0$$

exhibits several quasi-invariant states corresponding to the local minima on the long term run as ϵ becomes small. They determine the in $1/\epsilon$ exponentially small eigenvalues of the generator $-L_\epsilon \equiv \epsilon\Delta - \nabla F \cdot \nabla$ and are given in a precise way as the inverse of the expected relaxation time of the corresponding quasi-invariant states. Their rescaled distribution function converge uniformly to that of an exponential random variable.

Alison Etheridge

Some spatial population models

Natural populations interact with one another and with their environment in complex ways. No mathematical model can possibly incorporate all such interactions and yet remain analytically tractable. As a result, in order to understand the effects of a feature of a population's dynamics, it is often useful to study 'toy models'. In this talk we describe some results and conjectures for a class of such models which can be viewed as a compromise between fully spatial models which don't appear to be amenable to a rigorous mathematical analysis and interacting particle system models which don't, at present, incorporate all the competitive strategies that a population of, say, plants might adopt. If time permits we will also discuss some of the obstructions that appear in setting up fully spatial models.

Frank den Hollander

Metastable challenges for the lattice gas under Kawasaki dynamics

In this talk we consider the two-dimensional lattice gas subject to Kawasaki dynamics, i.e., particles live on a large box in $\mathbb{Z}^2$ and hop around randomly subject to hard-core repulsion and nearest-neighbor attraction. At the boundary of the box particles are annihilated at rate 1 and are created at rate $\exp[-\Delta/T]$, where T is the temperature and Δ is an activity parameter. Thus, the boundary acts as a reservoir that controls the particle density in the box.

We consider two metastable regimes:

(I) $T \downarrow 0$ and $0 < \Delta < \Delta_c(0)$,

(II) $0 < T < T_c$ and $\Delta \uparrow \Delta_c(T)$,

where T_c and $\Delta_c(T)$ are the critical temperature and critical activity for the gas-liquid phase transition.

For regime (I), we identify the set of critical droplets and obtain sharp estimates for the metastable transition time between the gas phase and the liquid phase [joint work with A. Bovier and F. Nardi].

For regime (II), the objective is to prove that the critical droplet, which grows to a macroscopic size as $\Delta \uparrow \Delta_c(T)$, assumes the equilibrium Wulff shape and that the metastable transition time between the gas phase and the liquid phase can be computed in terms of the free energy of the Wulff droplet [work in progress with A. Bovier and R. Messikh].

Barbara Gentz

Universality of residence-time distributions in non-adiabatic stochastic resonance

Stochastic resonance has been observed in a large variety of biological and physical systems, ranging from the sensory system of crayfish to bistable lasers and climate models. We will consider a Brownian particle in a periodically modulated bistable potential as a simple model exhibiting stochastic resonance. One of the many measures introduced to quantify the phenomenon is the particle's residence-time distribution. Thus we are led to study the noise-induced passage through the unstable periodic orbit separating the domains of attraction.

While on the exponential scale, accessible to the classical Wentzell–Freidlin theory, all points on a periodic orbit are equally likely to occur as first-exit points, the subexponential asymptotics of the distribution of first-exit points reflects the fact that the unstable orbit is generally not uniformly repelling. As discovered by Day (1992), "cycling" occurs: The distribution of first-exit points rotates around the unstable orbit, periodically in the logarithm of the noise intensity, and thus does not converge in the zero-noise limit.

We show that for a broad range of forcing frequencies and amplitudes, the distribution is close to a periodically modulated exponential one. Remarkably, this modulation is governed by a universal function, depending only on a single parameter related to the forcing period.

(Joint work with Nils Berglund, CPT–CNRS Marseille.)

Grégory Maillard

Chains with complete connections and Gibbs measures

We introduce a statistical mechanical formalism for the study of discrete-time stochastic processes (chains) with which we give: General properties of extremal chains, conditions for the uniqueness of the consistent chain and results on loss of memory and mixing properties for chains in the Dobrushin regime. We discuss the relationship between chains and one-dimensional Gibbs measures: We establish conditions for a chain to define a Gibbs measure and vice versa and we discuss the equivalence of uniqueness criteria for chains and fields.

Heinrich Matzinger

Optimal sequence alignment and the Longest Common Subsequence problem

Optimal sequence alignment is one of the main tools in computational molecular biology as well as in computational linguistics. Optimal alignment of random sequences can be formally described as an (oriented) first passage percolation problem. Unlike the classical first passage percolation model, however, the weights of the edges are correlated in optimal alignment. Because of complicated dependencies, a large part of the theory developed for first passage percolation is not immediately useful for the alignment problem. We investigated the fluctuation of the optimal alignment-score and several other related statistics. The examined variables exhibit a very interesting behavior: a small change in the parameters of the model gives rise to highly different properties. There are, however, many other interesting issues. For example, to name a few:
-Typically, is the optimal alignment unique in most places of the text or not?
-Are there only local variants of the same optimal alignment or do macroscopically strongly different optimal alignment coexist?
-For a fixed point in one text, what is the range on which it gets mapped by different optimal alignments (keeping the texts identical)?
-How much does this possible mapping-region fluctuate when the texts are redrawn?

Rongfeng Sun

Convergence of Coalescing Nonsimple Random Walks to the Brownian Web

(Joint work with C. M. Newman and K. Ravishankar)

The Brownian Web (BW) is a family of coalescing Brownian motions starting from every point in space and time $\mathbb{R} \times \mathbb{R}$. It was first introduced by Arratia, and later analyzed in detail by Tóth and Werner. More recently, Fontes, Isopi, Newman and Ravishankar gave a characterization of the BW, and general convergence criteria allowing either crossing or noncrossing paths, which they verified for coalescing simple random walks. Later Ferrari, Fontes, and Wu verified these criteria for a two dimensional Poisson Tree. In both cases, the paths are noncrossing. To date, the general convergence criteria of FINR have not been verified for any case with crossing paths, which appears to be significantly more difficult than the noncrossing paths case. Accordingly,in this paper, we formulate new convergence criteria for the crossing paths case, and verify them for non-simple coalescing random walks satisfying a finite fifth moment condition. Several corollaries are presented, including an analysis of the scaling limit of voter model interfaces that extends a result of Cox and Durrett.

Alexander Tikhomirov

Limit theorems for spectra of random matrices with martingale structure

We study two classical ensembles of random matrices introduced by Wigner and Wishart. We discuss Stein's method for the asymptotic approximation of expectations of functions of the normalized counting measure of high dimensional matrices. The method is based on differential equation for densities of the limit laws.

Anita Winter

Subtree prune and regraft: a reversible real tree valued Markov process

The real trees form a class of metric spaces that extends the class of trees with edge lengths by allowing behavior such as infinite total edge length and

vertices with infinite branching degree. A well-known example for an R-tree is David Aldous's Brownian continuum random tree (CRT), i.e. the tree inside a standard Brownian excursion.

The Brownian CRT arises as the scaling limit as $N \to \infty$ of a critical finite variance Galton-Watson tree conditioned to have total population size N.

We use Dirichlet form methods to construct and analyze a reversible Markov process whose stationary distribution is the Brownian CRT. This process is inspired by the subtree pruning and regrafting Markov chain that appears in phylogenetic analysis.

A key technical ingredient in this work is the use of a Hausdorff–Gromov type distance to metrize the space whose elements are compact real trees equipped with a probability measure.