ESI **The Erwin Schrödinger International Institute for Mathematical Physics**

Boltzmanngasse 9/2
A-1090 Vienna, Austria

ZVR 298034776

Programme for the second week of
RECENT ADVANCES IN INTEGRABLE SYSTEMS OF HYDRODYNAMIC TYPE
organized by A. Constantin and J. Escher (October 19 - 23, 2009)

MONDAY, OCTOBER 19, 2009

- 14h — 15h: "Non-commutative solitons" by O. Lechtenfeld (Univ. Hannover)

  Abstract. Solitonic objects play a central role in gauge and string theory (as monopoles, black holes, D-brains etc.). Certain string backgrounds produce a non-commutative deformation of the low-energy effective field theory, which allows for new types of solitonic solutions. I present the construction, moduli spaces and dynamics of Moyal-deformed solitons, exemplified in the $2+1$-dimensional Yang-Mills-Higgs theory and its Bogomolny system, which is a gauge-fixed to an integrable chiral sigma model (the Ward model). Non-commutative solitons for various $1+1$-dimensional integrable systems (such as sine-Gordon) easily follow by dimensional and algebraic reduction. Supersymmetric extensions exist as well and are related to twistor string theory.

- 15h — 16h: "The Degasperis-Procesi equation as a non-metric Euler equation" by B. Kolev (Univ. Marseille)

  Abstract. In this talk we present a geometric interpretation of the periodic Degasperis-Procesi equation as the geodesic flow of a right invariant symmetric linear connection on the diffeomorphism group of the circle. We also show that for any evolution in the family of b-equations there is neither gain nor loss of the spatial regularity of solutions. This in turn allows us to view the Degasperis-Procesi and the Camassa-Holm equation as an ODE on the Fréchet space of all smooth functions on the circle. This is joint work with J. Escher.

TUESDAY, OCTOBER 20, 2009

14h — 15h: "The Hunter-Saxton system" by M. Wunsch (Kyoto Univ.)

Abstract. The two-component Hunter-Saxton system arises as a generalization of the Hunter-Saxton equation modeling the propagation of orientation waves in nematic liquid crystals. It is moreover the high-frequency limit of the recently derived Camassa-Holm system. In this talk, we address the problem of local existence of periodic solutions, and we demonstrate that there are both solutions losing their initial regularity in finite time, as well as global solutions.

WEDNESDAY, OCTOBER 21, 2009

- 14h — 15h: "The inhomogeneous Camassa-Holm equation and generalized Fourier expansions" by V. Gerdjikov (Institute for Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences)

  Abstract. The inhomogeneous Camassa-Holm equation (CHE) is studied using the expansions over the "squared" solutions of the corresponding Lax operator L. We stress the importance of the expansions over the so-called symplectic basis, which lead to a system of evolution equations for the scattering data, that is easily solved for a generic choice of the inhomogeneity $G(x,t)$. Sufficient (although unexplicit) conditions on the right hand side of CHE are given, which ensure the integrability of the corresponding inhomogeneous CHE. We show that each inhomogeneous CHE allows Lax representation, the Lax operator being provided by L. Unfortunately the corresponding M operator for generic right hand sides possesses pole singularities located on the spectrum of L.

- 15h — 16h: "Inverse spectral/scattering theory associated to the Camassa-Holm equation" by C. Bennewitz (Univ. Lund)

  Abstract. So far the inverse scattering solution method for the Camassa- Holm equation has been implemented by transformation of the associated spectral problem $-u'' + \frac{1}{4}u = \lambda w u$ to the Schrdinger equation. This requires considerable smoothness of the moment w and, more importantly, that $w > 0$. I shall describe a new method to prove uniqueness of the inverse scattering problem without these restrictions.

THURSDAY, OCTOBER 22, 2009

14h — 15h: “Steady periodic water waves with constant vorticity: regularity and local bifurcation” by E. Varvaruca (Imperial College)

Abstract. We study periodic traveling gravity waves at the free surface of water in a flow of constant vorticity over a flat bed. Using conformal mappings the free-boundary problem is transformed into a quasilinear pseudodifferential equation for a periodic function of one variable. The new formulation leads to a regularity result and, by use of bifurcation theory, to the existence of waves of small amplitude even in the presence of stagnation points in the flow. This is joint work with Adrian Constantin.

FRIDAY, OCTOBER 23, 2009

14h — 15h: “A Lipschitz metric for the continuous semigroup of solutions to the Hunter–Saxton equation” by X. Raynaud (Univ. Oslo)

Abstract. The Hunter–Saxton equation is a completely integrable, bi-Hamiltonian equation which arises in the study of liquid crystals. In general, the solutions of the equation break down in finite time. After breakdown, the solutions are no longer unique and they can be prolonged in several consistent ways. By using a change of variable — from Eulerian to Lagrangian coordinates — we show how we can obtain a global semigroup of conservative solutions to the equation. Our main result, which is also going to be the main topic of the talk, is the construction of a metric which makes the semigroup of solutions Lipschitz continuous. This is joint work with Alberto Bressan and Helge Holden.

All lectures take place in the ESI Boltzmann Lecture Hall.