

Programme for the first week of  
**RECENT ADVANCES IN INTEGRABLE SYSTEMS OF HYDRODYNAMIC TYPE**  
organized by A. Constantin and J. Escher (October 12 - 16, 2009)

MONDAY, OCTOBER 12, 2009

- 14h00 — 14h15: Opening
- 14h15 — 15h15: “Nonlinear Resonance Analysis - what is it for?” by E. Kartashova (RISC, Linz)

*Abstract.* Nonlinear resonance analysis (NRA) is presented as a new research field in mathematical physics. The main subject of NRA is nonlinear PDEs possessing resonant solutions. The very special role of resonant solutions has been first demonstrated by Poincaré, for nonlinear ODEs, and generalized to nonlinear PDEs in the frame of Kolmogorov-Arnold-Moser-theory (KAM-theory). The resonances are easy to see in your own kitchen (<http://www.youtube.com/watch?v=fbfjcEzFN2U>), they can be quite disastrous (<http://www.youtube.com/watch?v=3mclp9QmCGs>) and they are quite difficult to describe mathematically, for the problem in its most general formulation is equivalent to the Hilbert’s 10th problem which is proven to be algorithmically unsolvable.

In this lecture we present basic notions of the NRA, fast algorithms for their computing and application of the NRA for describing some real-life phenomena in the Earth atmosphere, oceans and laboratory experiments. At the end of the talk, the most important open questions of the NRA will be formulated, with an appeal to the audience to think about possible answers.

TUESDAY, OCTOBER 13, 2009

- 14h — 15h: “Existence and stability of solitary water waves with weak surface tension” by E. Wahlén (Univ. Lund)

*Abstract.* We prove that two-dimensional solitary water waves with weak surface tension can be constructed by minimising the energy subject to the constraint of fixed momentum. The proof relies on the concentration-compactness method and the main difficulty is to prove that the infimum of the energy is a strictly sub-additive function of the momentum. This is done by a careful analysis of a certain minimizing sequence. The resulting solutions are periodic wave trains modulated by exponentially decaying envelopes and the fact that they are constrained minimisers guarantees some kind of stability. This is joint work with M. Groves.

- 15h — 16h: “On two-dimensional steady edge waves” by M. Ehrnström (Leibniz Univ. Hannover)

*Abstract.* Edge waves are regular, essentially two-dimensional waves propagating along the beach, and vanishing fast in the direction perpendicular to the shoreline. They reside on a semi-infinite domain in the direction away from land. We prove existence and uniqueness of steady two-dimensional edge waves. For small periodic shoreline data, global solutions vanishing in the seaward direction are found. In addition, we prove a priori properties of solutions, well-adapted to the physical background. This is joint work with J. Escher and B. Matioc.

WEDNESDAY, OCTOBER 14, 2009

- 14h — 15h: “Long-time asymptotics for the Camassa-Holm equation” by G. Teschl (Univ. Vienna)

*Abstract.* The Camassa-Holm equation was introduced in 1993 by Camassa and Holm as a model for shallow water waves. As the Korteweg-de Vries equation, it is integrable and can be solved via the inverse scattering method. We will consider decaying solutions with positive momentum, which are known to exist globally, and show how the inverse scattering problem can be reformulated as a Riemann-Hilbert factorization problem. Based on this, we will show how to extract the long-time asymptotics by virtue of the nonlinear steepest descent method.

- 15h — 16h: “Sharp ill-posedness results for the periodic cubic Schrödinger and Benjamin-Ono equations” by L. Molinet (Univ. Tours)

*Abstract.* We establish the ill-posedness of the cubic Schrödinger equation below  $L^2(T)$ . The strategy consists in proving the non continuity for the weak  $L^2$ -topology of the associated flow-map by separating the resonant and non resonant parts of the nonlinear term. Then we will explain how this approach can be applied to get a similar result in the more delicate case of the Benjamin-Ono equation.

THURSDAY, OCTOBER 12, 2009

- 14h — 15h: “Inverse scattering transform for the Degasperis-Procesi equation” by R. I. Ivanov (DIT, Dublin)

*Abstract.* We present the Inverse Scattering Transform method for the Degasperis-Procesi equation. The basic aspects of Inverse Scattering, such as fundamental analytic solutions, the Riemann-Hilbert problem formulation and the Zakharov-Shabat dressing method for the soliton solutions, are outlined.

- 15h — 16h: “Geometric aspects of the KdV and Camassa-Holm equations” by J. Lenells (Univ. Hannover)

*Abstract.* The KdV and Camassa-Holm equations both allow geometric interpretations as Euler equations for geodesic flow on the diffeomorphism group of the circle endowed with a right-invariant metric. A brief introduction to this topic will be given as well as a survey of some known and recent results.

FRIDAY, OCTOBER 16, 2009

- 14h — 15h: “Generalised Fourier Transform and perturbations to soliton equations” by G. Grahovski (Institute for Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences)

*Abstract.* A brief survey of the theory of soliton perturbations is presented. The focus is on the usefulness of the so-called Generalised Fourier Transform (GFT). GFT provides a natural setting for the analysis of small perturbations to an integrable equation: starting from a purely soliton solution one can ‘modify’ the soliton parameters such as to incorporate the changes caused by the perturbation. As illustrative examples the perturbed equations of the KdV hierarchy, in particular the Ostrovsky equation, followed by the perturbation theory for the Camassa-Holm hierarchy will be presented.

- 15h — 16h: “A coordinate-free construction of conservation laws and reciprocal transformations for a class of integrable hydrodynamic-type systems” by A. Sergyeyev (Silesian University)

*Abstract.* Using a (1,1)-tensor  $L$  with zero Nijenhuis torsion and maximal possible number (equal to the number of dependent variables) of distinct, functionally independent eigenvalues we define, in a coordinate-free fashion, the seed systems which are weakly nonlinear semi-Hamiltonian systems of a special form, and an infinite set of conservation laws for the seed systems. The reciprocal transformations constructed from these conservation laws yield a considerably larger class of hydrodynamic-type systems from the seed systems, and we show that these new systems are again defined in a coordinate-free manner, using the tensor  $L$  alone, and, moreover, are weakly nonlinear and semi-Hamiltonian, so their general solution can be obtained by means of the generalized hodograph method of Tsarev. This is joint work with Maciej Błaszak.

All lectures take place in the ESI Boltzmann Lecture Hall.