DVR 0065528

Workshop on

"Nonlinear Water Waves - an Interdisciplinary Interface"

November 27 – December 7, 2017

organized by

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Week 1

November 27 – December 1, 2017

- **Monday, November 27, 2017**

  09:00 – 09:30 **Registration**

  09:30 – 10:30 **Jean-Marc Vanden-Broeck**
  Introductory talk:
  Numerical and analytical investigations of nonlinear water waves and related problems

  10:30 – 11:00 *Coffee / Tea Break*

  11:00 – 12:00 **Paul Milewski**
  Understanding Complex Dynamics of Faraday Droplets
  Faraday pilot waves are a newly discovered hydrodynamic structure that consists a bouncing droplet which creates, and is propelled by, a Faraday wave These pilot waves can behave in extremely complex ways and result in dynamics mimicking quantum mechanics. I will show some of this fascinating behaviour and will present a surface wave-droplet fluid model that captures many of the features observed, focussing on the statistical emergence of complex states.

  12:00 – 14:00 *Lunch Break*

  14:00 – 14:45 **Emilian I. Părău**
  Nonlinear hydroelastic waves
  A review of some the results concerning nonlinear hdyroelastic waves will be given. We will present waves forced by moving pressures, periodic and solitary waves on top of a fluid covered by an ice plate, modelled as a thin elastic plate. The fluid is either of constant density of stratified and two-dimensional or three-dimensional problems are considered. Solitary interfacial waves will also be presented and weakly-nonlinear models and fully nonlinear computations will be discussed.

  14:45 – 15:30 **Benjamin Akers**
  Overturned Traveling Interfacial Waves
  Periodic traveling waves are computed on parameterized interfaces, which are not functions of the horizontal coordinate(s). The traveling wave ansatz is discussed in this context.Overturned traveling waves are computed on one and two-dimensional interfaces (corresponding to two and three-dimensional fluids), with surface tension and/or elastic interfacial forces. Numerical continuation procedures are paired with local and global bifurcation theorems. Extreme wave types and bifurcation surfaces are presented. Wilton Ripples are presented in the hydroelastic case. The vortex sheet formulation of the fluid problem is used throughout, which requires numerical evaluation of the Birkhoff-Rott integral.

15:30 – 16:00 *Break*

16:00 – 16:45 **Philippe Guyenne**
Numerical study of solitary wave attenuation in a fragmented ice sheet
A numerical model for phase-resolved simulation of nonlinear ocean waves propagating through fragmented sea ice is proposed. This model solves the full time-dependent equations for nonlinear potential flow coupled with a nonlinear thin-plate formulation for the ice cover. The coefficient of flexural rigidity is allowed to vary spatially so that distributions of ice floes can be directly specified in the physical domain. Two-dimensional simulations are performed to examine the attenuation of solitary waves by scattering through an irregular array of ice floes.

- **Tuesday, November 28, 2017**

09:30 – 10:30 **Andre Nachbin**
Surface waves over highly irregular topographies
Particular interest is given to the three-dimensional (3D) problem of (2D) surface water waves propagating over large-amplitude, non-smooth, bottom topographies. We start by presenting the numerical construction of a 3D Dirichlet-to-Neumann operator for linear water waves in the presence of highly irregular topographies. Next we present the modeling that gives rise to a 2D (weakly nonlinear) Boussinesq-type system in the presence of highly variable ridge-like topographies. Extending the conformal mapping technique to 3D, where the Laplacian is no longer invariant, we generalize the terrain-following Boussinesq system presented in Nachbin (SIAP 2003).

10:30 – 11:00 *Coffee / Tea Break*

11:00 – 12:00 **Bernard Deconinck**
The stability of solutions of integrable equations
Examining the stability of solutions of nonlinear PDEs continues to be an active area of research. Very few instances lend themselves to explicit results for even spectral and linear stability, let alone orbital (nonlinear) stability. Using the Lax pair structure of integrable equations, many of which originate from water wave problems, much progress has been made recently on the stability or instability of solutions of integrable problems.
After introducing the necessary concepts, I will discuss our recent work on the stability of standing wave solutions of the focusing NLS equation. The spectral stability of these solutions was completely characterized recently. The crux of this characterization was the analysis of the non-self adjoint Lax pair for the focusing NLS equation. Although all solutions are unstable in the class of bounded perturbations, different solutions were found to be spectrally stable with respect to certain classes of periodic perturbations, with period an integer multiple of the solution period. We prove that all solutions that are spectrally stable are also (nonlinearly) orbitally stable, using different Krein signature calculations.

12:00 – 14:00 *Lunch Break*

14:00 – 14:45 **Karima Khusnutdinova**
On KP and Ostrovsky-type models
In this talk I will overview some results concerning a version of the Kadomtsev-Petviashvili (KP) equation for surface gravity waves related to elliptic-cylindrical geometry, and a system of coupled Ostrovsky equations, derived for strongly interacting internal waves in the presence of background rotation and shear flow. In the first case, we show that there exist non-trivial transformations between different versions of the KP equation associated with the physical problem formulation, and use them to obtain new classes of approximate solutions for the Euler equations for incompressible fluid with free surface and rigid bottom boundary conditions (with and without surface tension). The solutions describe waves with nearly-elliptic front.
In the second case, we study the behaviour of weakly nonlinear oceanic internal waves in the presence of background rotation and shear flow, when two distinct linear long wave modes have nearly coincident phase speeds. The waves are described by a system of coupled Ostrovsky equations, derived from the full set of Euler equations for incompressible density stratified fluid with a free surface and rigid bottom

boundary conditions. We show that in the generic case, in the absence of the shear flow, initial solitary-like waves are destroyed and replaced by two coupled nonlinear wave packets, being the counterpart of the same phenomenon in the single Ostrovsky equation. I will also discuss results obtained when there is a background shear flow, when the dynamics is more diverse, and some other related recent results.
Main references:

1. K.R. Khusnutdinova, C. Klein, V.B. Matveev, A.O. Smirnov, On the integrable elliptic cylindrical Kadomtsev-Petviashvili equation, Chaos 23 (2013) 013126.
2. A. Alias, R.H.J. Grimshaw, K.R. Khusnutdinova, On strongly interacting internal waves in a rotating ocean and coupled Ostrovsky equations, Chaos 23 (2013) 023121.
3. A. Alias, R.H.J. Grimshaw, K.R. Khusnutdinova, Coupled Ostrovsky equations for internal waves in a shear flow, Physics of Fluids 26 (2014) 126603.

14:45 – 15:30 **Olga Trichtchenko**
Stability of periodic travelling wave solutions to Korteweg-de Vries and related equations
In this talk, we explore the simplest equation that exhibits high frequency instabilities, the fifth-order Korteweg-de Vries equation. We show how to derive the necessary condition for an instability of a perturbation of a small amplitude, periodic travelling wave solutions. We proceed by examining how these unstable perturbations change and grow in time as the underlying solution changes. We conclude by commenting on what happens with a different nonlinearity in the underlying equation.

15:30 – 16:00 *Break*

16:00 – 16:45 **Dane Grundy**
Interfacial solitary waves with surface stress
The presence of a low concentration surfactant at the surface of a solitary wave, and also a vertical electric field passing through the fluid body, will lead to a tangential surface stress condition at the fluid interface. Additionally, a no-slip condition at the fluid bottom will cause amplitude decay. The resulting system has a boundary layer at the surface which describes a decay in vorticity from the surface to the main fluid body, and another boundary layer at the fluid bottom describing a decay in tangential velocity. This talk will begin by considering numerical stability criteria for the Korteweg-De Vries equation and move on to discuss recent numerical results for the above system.

- **Wednesday, November 29, 2017**

09:30 – 10:30 **Onno Bokhove**
On variational water wave modelling
I will present the latest progress on variational water wave modelling in domains with water and fixed or flexible structures, such as ships or wind turbine masts. Both the theoretical formulation as well as the consistent numerical discretisation will be discussed.

10:30 – 11:00 *Coffee / Tea Break*

11:00 – 11:45 **Tao Gao**
Numerical investigation on the bifurcation structures of nonlinear water waves
In this talk, the problems of steady water waves propagating on the interface between the fluid and the air are studied numerically. We restrict the main focus in the context of capillary-gravity and flexural-gravity waves. We consider an irrotational flow of a two-dimensional, inviscid and incompressible fluid whose upper free-surface is deformed by a train of waves travelling at a constant speed and needed to be solved as part of the solution. Fully nonlinear (a)symmetric solitary and periodic waves are computed. The detailed bifurcation diagrams are presented.

11:45 – 12:30 **Alex Doak**
Travelling wave solutions on a ferrofluid jet
It has been known since Rayleigh (1879) that a gravity free capillary jet is unstable to long wave perturbations. This instability can be stabilized on jets of ferromagnetic fluid using a magnetic field, generated by a current running through a wire which the ferrofluid coats. Blyth & Părău (2014) computed solitary waves for this problem via a finite difference scheme, assuming the ferrofuid jet is axisymmetric and

surrounded by a vacuum. We extend the computations to find periodic and generalized solitary wave solutions. Furthermore, we shall present a numerical scheme capable of solving for steady solutions when the flow field in the outer fluid is not negligible. This is more akin to the experiments of Bourdin *et al* (2010), who made gravitational forces negligible by surrounding the ferromagnetic jet with a fluid of almost equal density.

References:
1. Rayleigh , Lord . 1879 *On the capillary phenomena of jets*. Proc. R. Soc. London **29**, 71–97
2. Blyth, M. G. & Pru E. I. 2014 *Solitary waves on a ferrofluid jet*. J. Fluid Mech **750**, 401-420.
3. Bourdin, E. & Bacri, J.-C. & Falcon, E. 2010 *Observation of axisymmetric solitary waves on the surface of a ferrofluid*. Physical review letters **104**, 094502

12:30 – 14:00 *Lunch Break and Free Afternoon*

- **Thursday, November 30, 2017**

  09:30 – 10:30 **Victor Shrira**
  Waves on jet currents: a new paradigm and novel mechanisms of freak wave formation
  The lecture is based on joint work with Alexey Slunyaev (Institute of Applied Physics, Nizhny Novgorod, Russia).
  We develop a new paradigm of how to describe linear and weakly nonlinear dynamics of waves on jet currents with a particular emphasis on new mechanisms of freak wave formation. From numerous seamen accounts and insurer records it has been known for long time that rogue waves events are quite frequent on certain currents, e.g. on notorious in this respect Agulhas current. The theoretical explanation of this fact is still lacking, which is due to our overall poor understanding of wave nonlinear evolution on currents. We address this challenge by developing a new systematic asymptotic theory of waves dynamics on jet currents which does not rely on the WKB approximation. The solutions for jet currents with arbitrary lateral profiles are found by first solving a linear 2D boundary-value problem in terms of an asymptotic series in natural small parameters. In essence, we employ approximate separation of variables, which is justified for the oceanic conditions.
  There are three types of waves on the current: passing through, reflected and trapped. The key role in the new approach is played by the trapped modes. The trapped modes themselves (rather than comprising them harmonic components) participate in the nonlinear interactions. A general weakly nonlinear theory of trapped mode evolution is being developed. In particular, the corresponding interaction coefficients have been derived. The properties of resonant interactions are qualitatively different from those between the waves in the absence of a current. In particular, three-wave interactions are always allowed in deep water and may play an important role in wave field evolution. There are three main advantages of the developed approach: (i) it is systematic and can identify and address the situations where the commonly adopted paradigm is not applicable; (ii) the current could be almost arbitrary, i.e. weak/strong, or smooth/with sharp edges; (iii) the initially 3-D problem is reduced to solving 1D evolution equations with the lateral and vertical dependence being prescribed by the corresponding modal structure. The modes can participate in both three and four- interactions.
  From the perspective of rogue wave occurrence we have identified several new mechanisms which have no analogues in the absence of currents. In particular, in contrast to the case of no currents robust envelope solitons were found both analytically (in the weakly nonlinear setting) and numerically in the full Euler equations. Ongoing laboratory experiments on evolution of trapped modes on jet currents are also discussed.

  10:30 – 11:00 *Coffee / Tea Break*

  11:00 – 12:00 **Wooyoung Choi**
  Spectral formulation for nonlinear water waves and its generalization to two-layer flows
  The spectral formulation of Zakharov (1968) for nonlinear water waves is revisited to explore its relationship with a pseudo-spectral formulation based on unsteady Stokes expansion. Hamiltonian structures under high-order nonlinear approximations are presented and further reductions are made to describe the evolution of uni-directional waves and resonant wave interactions. The spectral formulation is also

extended to a two-layer system with a free surface and some applications are discussed.

12:00 – 14:00 *Lunch Break*

14:00 – 14:45 **Katie Oliveras**
Nonlinear Traveling Internal Waves in Depth-Varying Currents
In this talk, we discuss the nonlinear traveling waves in density stratified fluids with depth varying shear currents. Beginning with the nonlocal formulation of the water-wave problem, we examine the interface between two fluids of differing densities and varying linear shear. We derive as systems of equations depending only on variables at the interface, and numerically solve for periodic traveling wave solutions using numerical continuation considering only branches that bifurcate from solutions where there is no slip in the tangential velocity at the interface for the trivial flow. The spectral stability of these solutions is then determined using a numerical Fourier-Floquet technique. We find that the strength of the linear shear in each fluid impacts the stability of the corresponding traveling wave solutions. Specifically, opposing shears may amplify or suppress instabilities.

14:45 – 15:30 **Magda Carr**
Investigation of Mode-2 Internal-Solitary-Like Waves
(joint work with Marek Stastna, David Deepwell & Peter. A. Davies)
Internal solitary-like waves (ISWs) are a common feature in stratified flows. In the ocean, the majority of ISWs observed are of the first baroclinic mode (mode-1). ISWs of mode-1 displace isopycnals in one direction only. Mode-2 ISWs, on the other hand, displace isopycnals in opposite directions and most typically are convex (upper isopcynals are displaced upward and lower isopycnals are displaced downward). Recent observations of convex mode-2 ISWs suggest that they may be more prevalent than previously thought (Shroyer *et al.*, 2010, Yang *et al.*, 2009) a nd that even though mode-2 waves have less energy than mode-1 waves, the magnitude of the wave-localized turbulent dissipation can be similar in both cases. Mode-2 ISWs may, therefore, have a significant effect on eroding the pycnocline and hence on the vertical fluxes of heat and nutrients in the water column.
In this talk some recent studies (numerical and experimental) on mode-2 ISW dynamics will be presented. Topics will include (i) shoaling mode-2 waves, (ii) mode-2 waves interacting with topographic features, (iii) mode-2 waves travelling on pycnoclines not located at mid-depth and (iv) mode-2 waves colliding head-on with mode-1 ISWs.
References:
Shroyer, E. L., Moum, J. N., & Nash, J. D. 2010 Mode 2 waves on the continental shelf: Ephemeral components of the nonlinear internal wavefield. *J. Geophys. Res.* 115, C07001.
Yang, Y. J., Fang, Y. C., Chang, M-H., Ramp, S. R., Kao, C-C., & Tang, T. Y. 2009 Observations of second baroclinic mode internal solitary waves on the continental slope of the northern South China Sea. *J. Geophys. Res.* 114, C10, 2156-2202.

15:30 – 16:00 *Break*

16:00 – 16:45 **Ricardo Barros**
Large amplitude internal waves in three-layer flows
Large amplitude internal waves in a three-layer flow confined between two rigid walls will be examined in this talk. The mathematical model under consideration arises as a particular case of the multi-layer model proposed by Choi (2000) and is an extension of the two-layer MCC (Miyata-Choi-Camassa) model. The model can be derived without imposing any smallness assumption on the wave amplitudes and is well-suited to describe internal waves within a strongly nonlinear regime. We will investigate its solitary-wave solutions and unveil some of their properties by carrying out a critical point analysis of the underlying dynamical system.

- **Friday, December 1, 2017**

09:30 – 10:30 **Richard Kollár**
How Extensions of Eigenvalue Problems Can Help To Study Stability Problems and Decay Rates of Internal Waves in Stratified Fluids

We study decay rates of internal waves in stratified critical fluids without surface tension. The resulting Rayleigh-Taylor problem for the decay rates reduces to a nonlinear eigenvalue problem. Although it is quite technical to analyze the eigenvalue problem directly, an extension of the problem to multiparameter setting significantly simplifies the analysis and shows that there are two types of internal modes - the fast decaying viscous modes and the slowly decaying creeping modes caused by the stratification.
An analogous higher dimensional extension can be applied to study spectral stability of nonlinear waves in Hamiltonian systems, where it helps to identify the Krein signature directly from the Evans function without evaluation of the eigenfunctions using the concept of geometric Krein signature. We propose a simple explanation for the magic of the extension and suggest a mathematical framework for general stability studies. The motivation comes from the reduction method quasi-steady state approximation traditionally used in mathematical chemistry.

10:30 – 11:00 *Coffee / Tea Break*

11:00 – 11:45 **Dominic Amann**
Numerical approximation of an asymptotic expansion for steady periodic water waves
Recently, a constructive algorithm based on asymptotic expansions was proposed for computing water waves of large amplitude, in the absence of stagnation points. Here, we perform and analyze a numerical implementation of this algorithm. We ensuring efficiency by reducing the computational domain to one dimension and show unique solvability as well as convergence rates. Numerical examples verify the analytical results and display some characteristics of large amplitude waves.

11:45 – 12:30 **Berry Bakker**
Spatial Dynamics in Nonlocal Equations
The spatial dynamics approach is typically applied to study specific solutions of a PDE by posing it as an (infinite dimensional) dynamical system in a spatial variable. Coherent states can then be found by locating bounded solutions, such as periodic, homoclinic, and heteroclinic orbits. However, typical nonlocal equations do not allow for an obvious reformulation in terms of spatial dynamics. This is due to the nonlocal interaction entering in the time-like variable, resulting in a forward-backward delay equation. In this talk we discuss a novel approach to study nonlinear waves in nonlocal equations, using functional analytic and topological techniques whilst drawing inspiration from the spatial dynamics approach. As an application to nonlinear water waves we consider the Whitham equations. We show that the spatial evolution of coherent states in Whitham equations is governed by Hamiltonian identities, which implies the existence of solitons and periodic waves. This is joint work with Arnd Scheel.

All talks take place at ESI, Boltzmann Lecture Hall!